The new Hamiltonian has now been found (to the order ε) and so has the determining function, where

$$S_1 = \int F_{1p} dl = (v - l), \tag{J.7.15}$$

the equation of the center. With

$$F^*(L^*, G^*) = \frac{\mu^2}{2L^{*2}} + \varepsilon \frac{\mu^2}{L^{*3}G^*}, \tag{J.7.16}$$

Hamilton's equations have the solution $L^* = \text{const.}$, $G^* = \text{const.}$,

$$l^* = l_0 + \left(\frac{\mu^2}{L^{*3}} + \varepsilon \frac{3\mu^2}{L^{*4}G^*} \right) t, \quad g^* = g_0 + \varepsilon \frac{\mu^2}{L^{*3}G^{*2}} t, \tag{J.7.17}$$

giving the secular perturbations. Then, from

$$S = L^* l + G^* g + \varepsilon (v - l) \tag{J.7.18}$$

we have

$$L = L^* + \varepsilon \left(\frac{\partial v}{\partial l} - 1 \right), \ G = G^*, \ l^* = l + \varepsilon \frac{\partial v}{\partial L}, \ g^* = g + \frac{\partial v}{\partial G}, \tag{J.7.19}$$

to the first order in ε. $\frac{\partial v}{\partial L}$ can be found at once from the integral expression, (J.7.16), for S_1. The remaining derivatives can be found by applying the chain rule using formulas for elliptic motion. This is done in Ref. 19, where the approach just used, is applied to the problem of the motion of an artificial satellite. This method was proposed by Poincaré, and applied to orbits of minor planets by von Zeipel; it is often referred to as the "von Zeipel method."

Bibliography and References

Chapter 1

There are many first-rate introductory texts in astronomy available, and a perusal of some of these will provide pleasure as well as information. For more detailed information on the astronomical background pertinent to the dynamics of the solar system and space flight, see:

1. Baker, R.M.L., and Makemson, M.W. *An Introduction to Astrodynamics.* New York: Academic Press, 1967.

For information on spherical astronomy, see:

2. Smart, W.M. *Textbook on Spherical Astronomy.* Cambridge, England: Cambridge University Press, 1956.
3. Woolard, E.W. and Clemence, G.M. *Spherical Astronomy.* New York: Academic Press, 1966.

For a gold mine of practical information, see:

4. *Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac.* London: Her Majesty's Stationery Office. First published in 1960, but updated in more recent printings.

Also refer to:

5. *The Astronomical Almanac* for the current year, issued by the Nautical Almanac Offices in the U.S. and Great Britain, and available through Washington: U.S. Government Printing Office, or London: Her Majesty's Stationery Office.

Chapter 2

Many texts include introductions to vectors. But beware of those that define vectorial properties in terms of their components. The references quoted here are:

6. Pars, L.A. *Introduction to Dynamics.* Cambridge, England: Cambridge University Press, 1953.

7. Rutherford, D.E. *Vector Methods.* New York: Interscience Publishers, 1954.
8. Milne, E.A. *Vectorial Mechanics.* New York: Interscience Publishers, 1948.

Chapter 3

See references such as 6, 7, and 8.

Chapter 4

Accounts of central orbit theory will be found in most texts on mechanics, such as 6, or the following:

9. Ramsey, A.S. *Dynamics.* (Parts one and two.) Cambridge, England: Cambridge University Press, 1946.
10. Whittaker, E.T. *Analytical Dynamics.* (4th edition.) Cambridge, England: Cambridge University Press, 1959.
11. Moulton, F.R. *An Introduction to Celestial Mechanics.* New York: Macmillan, 1914. Now available through Dover Publications, New York.

Chapter 5

Much of the material in this chapter is based on:

12. Ramsey, A.S. *Newtonian Attraction.* Cambridge, England: Cambridge University Press, 1949.

See also 8. Discussions of the figures rotating and tidally distorted masses, with application to astronomy, are given in:

13. Jeans, J.H. *Problems of Cosmogony and Stellar Dynamics.* Cambridge, England: Cambridge University Press, 1919.
14. Jeans, J.H. *Astronomy and Cosmogony.* Cambridge, England: Cambridge University Press, 1928.
15. Chandrasekhar, S. *Ellipsoidal Figures of Equilibrium.* New Haven: Yale University Press, 1969.

Chapter 6

Discussions of the problem of two bodies are found in 11, and many other texts including:

16. Baker, R.M.L. *Astrodynamics: Applications and Advanced Topics.* New York: Academic Press, 1967.
17. Bate, R.B., Mueller, D.D. and White, J.E. *Fundamentals of Astrodynamics.* New York: Dover Publications, 1971.
18. Battin, R.H. *Astronautical Guidance.* New York: McGraw Hill, 1964.

19. Brouwer, D. and Clemence, G.M. *Methods of Celestial Mechanics.* New York: Academic Press, 1961. This is the authoritative text on classical celestial mechanics.
20. Escobal, P.R. *Methods of Orbit Determination.* New York: Wiley, 1965.
21. Escobal, P.R. *Methods of Astrodynamics.* New York: Wiley, 1968.
22. Fitzpatrick, P.M. *Principles of Celestial Mechanics.* New York: Academic Press, 1970.
23. Gauss, C.F. *Theoria Motus.* New York: Dover Publications. Originally published in 1809, this makes vivid reading even today, and communicates some of the flavor of Gauss' genius.
24. Herget, P. *The Computation of Orbits.* Ann Arbor, MI: Edwards Bros., Inc., 1948. The outstanding work of its time in this area by the outstanding practitioner in this field. It is by no means outdated.
25. Herrick, S. *Astrodynamics, I* and *II.* New York: Van Nostrand, 1971, 1972.
26. Plummer, H.C. *An Introductory Treatise on Dynamical Astronomy.* New York: Dover Publications, 1960. First published in 1918, this is a classic, and still, for me, one of the best references available.
27. Pollard, H. *Mathematical Introduction to Celestial Mechanics.* Englewood Cliffs, N.J.: Prentice Hall, 1966. Chapters I to III appear in *Celestial Mechanics*, Mathematical Association of America, 1976. This text is carefully presented to be mathematically rigorous; it is beautifully written, and very clear. In particular, it is accessible to those with no initial background in mechanics.
28. Roy, A.E. *Orbital Motion.* Bristol, England: Adam Hilger, Ltd., 1982.
29. Smart, W.M. *Celestial Mechanics.* Cambridge, England: Cambridge University Press, 1953.
30. Taff, L.G. *Celestial Mechanics.* New York: Wiley, 1985.
31. Watson, J.C. *Theoretical Astronomy.* Philadelphia: Lippincott, 1986.

For a discussion of the solution of Kepler's equation, with different initial guesses, see:

32. Danby, J.M.A. and Burkardt, T.M. The Solution of Kepler's Equation, I and II. *Celestial Mechanics*, **31**, 95–107, 317–328, 1983; **40**, 303–312, 1987.

K. Stumpff published his paper on universal variables in:

33. Stumpff, K. Neue Formeln and Hilfstafeln zur Ephemeridenrechung, *Astr. Nach*, **275**, 108–128, 1947.

S. Herrick worked independently, and published a paper that will be more accessible to most readers:

34. Herrick, S. Universal Variables. *Astron. J.* **70**, 309–315, 1965.

A fundamental paper is

35. Goodyear, W.H. Completely General Closed Form Solution for Coordinates and Partial Derivatives of the Two-Body Problem. *Astron. J.*, **70**, 189–192, 1965.

Invention in the problem of two bodies is far from dead. For a recent original method see:

36. Shepperd, S.W. Universal Keplerian State Transition Matrix. *Celestial Mechanics*, **35**, 129–144, 1985.

Formulas for the Julian Date, and the solar coordinates are taken from:

37. *Almanac for Computers, 1986.* Nautical Almanac Office, U.S. Naval Observatory. Washington: U.S. Government Printing Office.

More extensive formulas for the Sun, Moon and Planets can be found in:

38. Van Flandern, T.C. and Pulkkinen, K.F. Low-Precision Formulas for Planetary Positions. *Ap. J. Supp.*, 391–411, Nov., 1979.

Chapter 7

Many of the texts mentioned for Chapter 6 contain material on this subject. In particular, see 24. Also:

39. Williams, K.P. *The Calculations of Orbits of Asteroids and Comets.* Bloomington, IN: The Principia Press, 1934.
40. Tattersfield, D. *Orbits for Amateurs with a Microcomputer.* New York: Wiley, 1984.

The section on Gauss' method owes much to B.G. Marsden. His paper, which contains many additional references, and historical notes, is:

41. Marsden, B.G. Initial Orbit Determination: the Pragmatist's Point of View. *Astron., J.*, **90**, 1541–1547, 1985.

Herget's method for finding a preliminary orbit with the use of more than three observations is described in:

42. Herget, P. Computation of Preliminary Orbits. *Astron. J.*, **70**, 1–3, 1965.

Calculation of partial derivatives specifically for Keplerian motion is described in 19 and 24, with a very elegant set of formulas in 35.

Some discussion on the method of least squares will be found in 19 and 24.

Chapter 8

See 11, 19, 26 and 27. Also:

43. Szebehely, V. *Theory of Orbits.* New York: Academic Press, 1967.
44. Wintner, A. *The Analytical Foundations of Celestial Mechanics.* Princeton: Princeton University Press, 1947.
45. Lundberg, J., Szebehely, V., Nerem, R.S. and Beal, B. Surfaces of Zero Velocity in the Restricted Problem of Three Bodies. *Celestial Mechanics*, **36**, 191–205, 1985.

Chapter 9

See 27 for an outstanding introduction to qualitative properties of motion in the problem of n bodies.

A discussion of the practical applications of the equations of motion to the solar system is given by:

46. Porter, J.G. A Comparative Study of Perturbation Methods. *Astron. J.*, **63**, 405–406, 1958.

Chapter 10

A great deal of basic information can be found in:

47. *Interpolation and Allied Tables.* Great Britain: Her Majesty's Nautical Almanac Office, 1956.

For further information, and worked examples, see:

48. *Planetary Coordinates for the Years 1960–1980.* Great Britain: Her Majesty's Nautical Almanac Office, 1958.

See also 19 and 24.

Texts on numerical analysis include:

49. Conte, S.D. and De Boor, C. *Elementary Numerical Analysis.* New York: McGraw Hill, 1972.
50. Henrici, P. *Discrete Variable Methods in Ordinary Differential Equations.* New York: Wiley, 1963.

Fehlberg's RK 4(5) method was published in:

51. Fehlberg, E. *Classical Fifth-, Sixth-, Seventh-, and Eighth-order Runge-Kutta formulas with stepsize control.* NASA Technical Report 278, 1968.

This is described in detail, with program listings in FORTRAN, PASCAL and BASIC in:

52. Danby, J.M.A. *Computing Applications to Differential Equations.* Reston, VA: Reston Publishing Company, 1985.

Chapter 11

See texts such as 16, 17, 26 and 28, but especially, 19 and 24. Many worked examples can be found in 48. For excellent sources of derivations, formulas and algorithms, see:

53. Moyer, T.D. *Mathematical Formulation of the Double-Precision Orbit Determination Program (DPODP).* NASA Technical Report 32–1527. Pasadena, CA: Jet Propulsion Laboratory, 1971.
54. Spier, G.W. *Design and Implementation of Models for the Double Precision Trajectory Program (DPTRAJ).* NASA Technical Memorandum 33–451. Pasadena, CA: Jet Propulsion Laboratory, 1971.

See also:

55. Brown, E.W. *An Introductory Treatise on the Lunar Theory.* New York: Dover Publications, 1960. (First published in 1896).
56. Brown, E.W. and Shook, C.A. *Planetary Theory.* Cambridge, England: Cambridge University Press, 1933.

Hansen's method for perturbations is described in 19. See also 48 and:

57. Musen, P. Modified Formulae for Hansen's Special Perturbations. *Astron. J.*, **63**, 426–429, 1958.

The planetary equations are used to solve, to the first order, the problem of the motion of an artificial satellite in the field of an oblate planet in:

58. Kozai, Y. The Motion of a Close Earth Satellite. *Astron. J.*, **64**, 367–377, 1959.

Herrick's method for the variation of parameters is given in:

59. Herrick, S. A Modification of the "Variation of Constants" Method for Special Perturbations. *P.A.S.P.* **60**, 321–323, 1948.

He applied his method, and gives a general discussion of other methods in:

60. Herrick, S. Icarus and the Variation of Parameters. *Astron. J.*, **58**, 156–164, 1953.

Herrick gives a discussion of the integral for M in:

61. Herrick, S. The Mean Longitude or Mean Anomaly in Perturbations by Variation of Constants. *Astron. J.*, **56**, 186–188, 1951.

Merton's method is given in:

62. Merton, G. A Modification of the Perturbation-of-Elements Method. *M.N.R.A.S.*, **109**, 421–435, 1949.

Poincaré's method was first applied in:

63. Henon, M. and Heiles, C. The Application of the Third Integral: Some Numerical Experiments. *Astron. J.*, **60**, 73–79, 1964.

Some applications to astronomical topics can be found in:

64. Danby, J.M.A. High-Energy Orbits in Barred Spirals. *Astron. J.*, **72**, 919–925, 1967.
65. Danby, J.M.A. Motion of a Satellite of a Very Oblate Planet. *Astron. J.*, **73**, 1031–1038, 1968.

Chapter 12

See 55; also 19, 26 and 29.

Chapters 13 and 14

See 26, 29, and many texts on mechanics, including:

66. Routh, E.J. *Advanced Dynamics of a System of Rigid Bodies.* New York: Dover Publications, 1955.

Also see:

67. Jeffreys, H. *The Earth.* Cambridge, England: Cambridge University Press, 1952.
68. Munk, W.H. and MacDonald, J.F. *The Rotation of the Earth.* Cambridge, England: Cambridge University Press, 1960.
69. Lambeck, K. *The Earth's Variable Rotation.* Cambridge, England: Cambridge University Press, 1980.
70. Kopal, Z., Ed. *Physics and Astronomy of the Moon.* New York: Academic Press, 1971.

Appendix

71. Marsden, B.G. *Catalogue of Cometary Orbits.* Minor Planet Center, Smithsonian Astrophysical Observatory, Cambridge, MA, 1982.

72. Bretagnon, P. Théorie du mouvement de l'ensemble des planètes. Solution VSOP82. *Astronomy and Astrophysics*, **114**, 278–288, 1982.
73. Meeus, J. *Astronomical Formulae for Calculators.* Richmond, VA: Willmann-Bell, 1979.
74. Bretagnon, P. and Simon, J. *Planetary Programs and Tables from -4000 to +2800.* Richmond, VA: Willmann-Bell, 1986.
75. Carusi, A., Kresak, L., Perozzi, E. and Valsecchi, G.B., *Long-term Evolution of Short-period Comets.* Bristol, England: Adam Hilger, 1986.

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