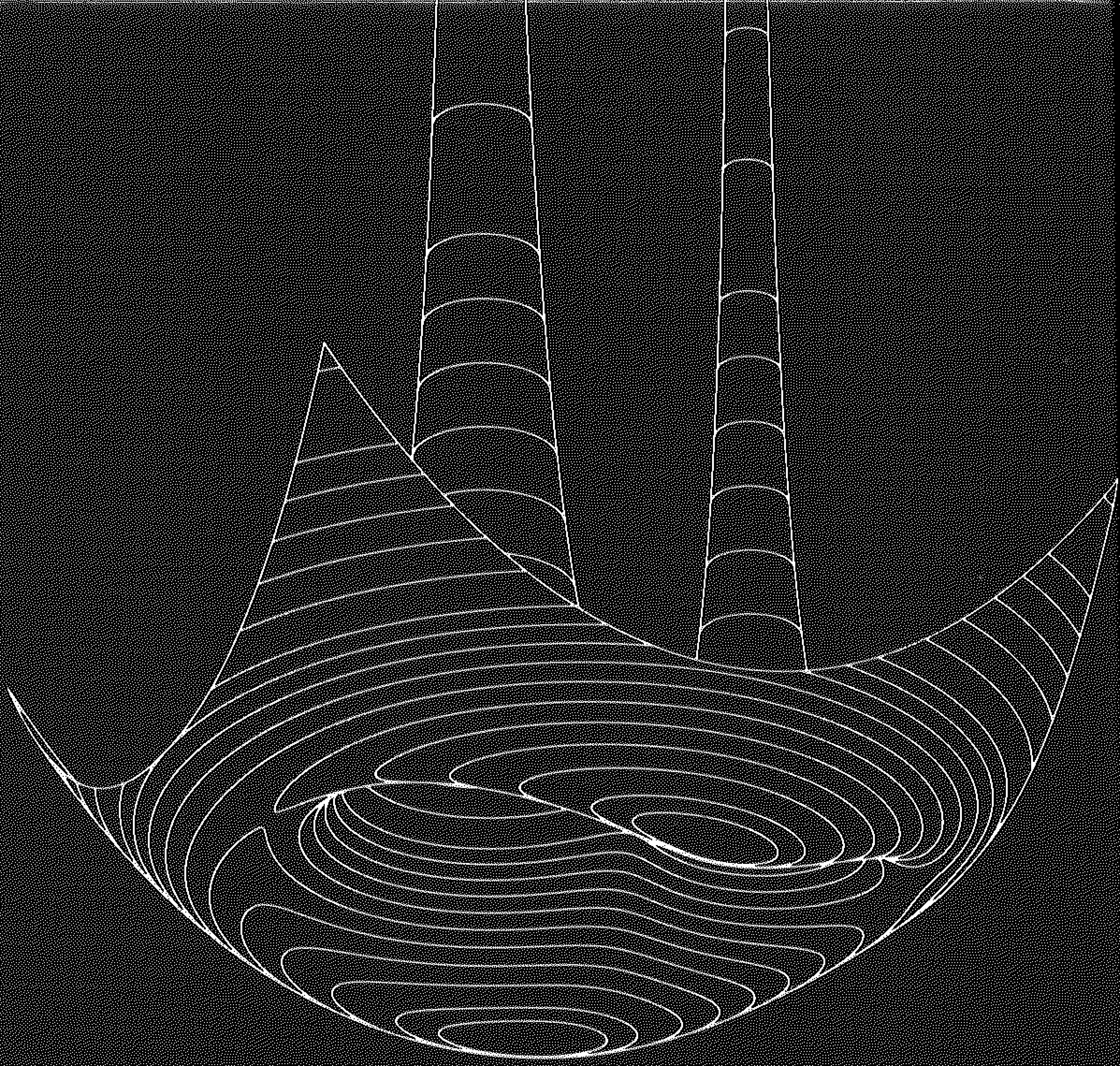

Fundamentals of

Celestial Mechanics

J.M.A. Danby

Second Edition
Revised & Enlarged

About The Author

Tony Danby was born in 1929, and studied mathematics at Oxford and Manchester University. He has taught in the U.S. since 1957, before which he was for a time, a professional musician, as first chair oboist in the London Philharmonic Orchestra. His specialty is celestial mechanics, and he is concerned with innovative ways of teaching in the age of the computer. He is the author of *Computer Applications to Differential Equations* which includes 80 projects for the computer, many of them involving orbital motion.

About This Book

This is an introductory text that should be accessible to reader having a background in calculus and elementary differential equations. The original edition (1962) has been radically revised, and emphasis is placed on computation. The numerical analysis needed for the computations is derived, and sample programs (run on a PC) are included. There are introductory chapters on the astronomical background and on vectorial mechanics. Sections dealing with the problem of two bodies include the use of universal variables, several methods (including that of Laguerre) for solving Kepler's equation, and problems. The chapter on the determination of orbits includes two versions of Gauss' method, the application of least squares and an introduction to recursive methods. The chapter on numerical methods includes three methods for the numerical integration of differential equations, one of which has full stepsize control. There are also chapters on perturbations, the three- and *n*-body problems, the motion of the Moon and the rotations of the Earth and Moon. The appendix includes numerical tables and derivations of properties of conic sections that are used in the text. The text includes several hundred problems, and suggested computer projects–a diskette that can be used in conjunction with this text is separately available as detailed on the last page of this book.

From the Reviews

The select group of scientists and engineers who dedicate their career to celestial mechanics consider the author's book of fundamental importance. The original edition of 1962 appeared when the author worked at Yale University with several of the giants in our field (Brouwer, Clemence, Eckert, Hagihara, Herget, etc.). The considerable influence of the original edition on our field is surpassed by the present second, revised and significantly enlarged edition ... This excellent introductory text book is strongly recommended for undergraduate courses in celestial mechanics, orbit dynamics and astrodynamics.

Mathematical Reviews – American Mathematical Society

A new edition of *Fundamentals of Celestial Mechanics* is a welcome sight to those of us who teach this subject to advanced undergraduates or beginning graduate students ... One of the best features of the original was its large variety of problems and exercises, and the new edition has even more ... The expanded sections and new material have broadened the text's applications ... this second edition of Danby's *Fundamentals of Celestial Mechanics* will also become a classic.

Sky & Telescope magazine

This second edition has been substantially revised and enlarged and demonstrates in a dramatic manner how the use of computers of all sizes has allowed teachers and students alike to gain a much deeper understanding of a subject requiring substantial numerical computation ... (it) is most certainly a valuable addition to any teacher's personal library. It is also cheap enough for students, who are interested in the finer details of numerical computation of orbits, to purchase their own copy.

The Observatory

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PREFACE

The first edition of this text was written over twenty-five years ago. At that time, "computing" could be time-consuming and harrowing. (The actual computation might be over in a second, but the "turn-around time" could be measured in days.) I well remember as eminent an authority as Dirk Brouwer advising me against giving computing assignments in my courses. So I wrote the text to provide background and learning over a wide area, but I made no attempt to give details of practical calculation.

That time now seems to be as far away as the logarithm table! *Then* one might just talk about a calculation; *now* it takes hardly any longer to sit down and perform it. From my own experience I am convinced that when calculation is routine in and out of class, the learning process becomes much more efficient than it used to be. More can be covered and learned, with greater purpose, increasing satisfaction and enjoyment. In making the changes for this edition I have assumed that a reader will have ready access to a computer terminal allowing rapid debugging, and running, and immediate viewing of output.

Because of the expense of adding new material, changes in the old text have had to be kept to a minimum. Were I able to have started again from scratch, alterations would have been much more radical. The big changes are in the chapters on the two-body problem, the determination of orbits, and numerical procedures. I have tried to make these chapters practical and up-to-date. The resulting text can be used by beginning students who are concerned with general applications of dynamics to astronomy (with or without computation), or by those who want to learn the fundamental practical details of orbital work. I hope also that the selection of material may make the text useful as a reference.

I will certainly be reproached for actually listing some programs. I make no claim for any personal merit as a programmer. An experienced programmer is entitled to view my programs with ridicule — but he is also entitled to ignore them. My first obligation is to students, some of whom will only be starting to write programs. I have found from experience that students are greatly helped if they can see sample programs, and that *a lot of time is saved if such programs are available.* The programs are, of course, for adapting and improving, not for copying. My choice of BASIC as the language for program listing will also be controversial. I am looking for maximum understanding: any programmer can follow BASIC, even if he or she will not admit to using it. But BASIC, especially in some modern adaptations, is an excellent language for scientific computation. (I also considered using FORTRAN or PASCAL. But FORTRAN is regarded with furious scorn by some partisans of PASCAL, and the feeling is returned with interest by many users of FORTRAN. I wish to remain neutral.) The programs listed here have all been run on an IBM PC. Emphasis is placed on the use of double precision.

The text is accessible to undergraduates or graduate students in mathematics, physics, engineering or astronomy. As minimum requirements, a user is assumed to have taken introductory courses in calculus, differential equations and mechanics. Other topics, such as numerical analysis, are developed in the text as needed. There is plenty of material for two semesters; so a one semester course can be designed in many ways (even with the exclusion of all computing).

The first three chapters are introductory, and can be omitted provided that the material is known. I try to stress the geometrical interpretations of vector operations (especially the cross-product) and the fact that these are independent of any coordinate system. I also stress the application of vectors to rotating systems.

Chapter 4 deals with fundamental mechanics. Some of the material may already be familiar to a reader, and some (such as sections 4.6–4.9) can be omitted without detriment to the understanding of later material.

In a course concerned with the fundamentals of orbital calculation, much of the contents of Chapter 5 could be omitted. This chapter can always be visited later, especially if the final three chapters are to be considered.

Chapter 6 is concerned with the core problem of celestial mechanics. It has been expanded to include computational solutions of the initial value problem and the two-point boundary value problem; the use of universal variables is emphasized. Four methods are introduced in section 6.6 for the solution of a nonlinear equation; all are used later. I cannot recommend any omissions from this chapter!

Much of Chapter 7 is occupied with the classical problem of determining an orbit from three observations. This is often dismissed as irrelevant, and it is certainly less important than the problem of improving an orbit using many observations (the problem addressed in the final two sections of this chapter). But I have found that many students are fascinated by the problem, so I like to discuss it, particularly since, in my opinion, modern refinements of Gauss's method provide some of the most elegant mathematics in celestial mechanics. If time is short, I recommend that Laplace's method just be described in general terms, and the main emphasis placed, deservedly, on Gauss.

Anyone claiming to know anything about orbital mechanics must know something about the problem of three bodies: an introduction is offered in Chapter 8. This material merits at least a survey, and the same can be said about Chapter 9 (the final three sections of which should on no account be omitted).

The numerical solution of differential equations is essential for most work on orbits. Three methods are described in Chapter 10, and programs are listed. These are: Fehlberg's Runge-Kutta method of order 5, with stepsize control; a predictor-corrector method of arbitrary order for a system of first-order equations; the classical method, also of arbitrary order, for a system of second-order equations. The section on interpolation has also been expanded.

Together with Chapter 6, Chapter 11 contains the most important material of the text. Since the subject of perturbations could itself occupy several texts, some sections in Chapter 11 are bound to be somewhat thin, but I would be reluctant to dispense with any of the material. Some derivations, as in 11.9, might be omitted, and Encke's method and the numerical solution of the equations for the variation of parameters may be obsolete (although they are still often mentioned).

The final three chapters have not been changed in this edition. Chapter 12 contains the minimum of information about the orbit of the Moon that an educated astronomer or celestial mechanician should know. Chapters 13 and 14 contain important information on the rotations of the Earth and Moon that is not easily available elsewhere.

The section in the Appendix on conics has been expanded; this may be helpful since the emphasis placed on conics in courses containing analytical geometry is decreasing. Also in the Appendix are listings of programs for the solution of a linear system and for matrix inversion. Listings of orbital elements of comets and asteroids may help in some of the projects. Personally, I am in favor, especially in the classroom, of making up my own orbits, or getting students to do so. For instance, students can make up their own orbits, and generate "observations"; these are exchanged, with the recipients responsible for recovering as well as possible the original elements. For verisimilitude, errors should be added to the calculated observations; these require the generation of sample values of a normal distribution. The generation of these "Gaussian deviates" is described also in the Appendix.

Some of the problems have been taken from the following texts:

L. A. Pars, *An Introduction to Dynamics* (Cambridge)

A. R. Ramsey, *Dynamics*, Parts 1 and 2 (Cambridge)

A. R. Ramsey, *Newtonian Attraction* (Cambridge)

W. M. Smart, *Spherical Astronomy* (Cambridge)

E. T. Whittaker, *Analytical Dynamics* (Cambridge)

F. R. Moulton, *An Introduction to Celestial Mechanics* (Dover)

K. P. Williams, *The Calculation of the Orbits of Asteroids and Comets* (Principia Press)

R. Kurth, *Introduction to the Mechanics of the Solar System* (Pergamon Press)

D. E. Rutherford, *Vector Methods* (Oliver and Boyd)

I am grateful to the publishers and authors for their kind permission to reproduce the problems. I would also like to thank Dr. P. K. Seidelmann, Director of the Nautical Almanac Office, and Dr. B. G. Marsden, of the Minor Planet Center, for permission to reproduce numerical data. Dr. Marsden has also been generous in providing information on orbit determination procedures. I would like to thank Dr. L. E. Doggett who has not only been helpful in providing data, but also deserves the credit for persuading the publisher to produce this new edition. I would like to thank my wife, Phyllis, who has labored beyond the call of marriage vows in proof-reading not only the original edition, but also this version. Finally, I would like to thank Mr. Perry Remaklus, Jr. of Willmann-Bell, Inc., for all of his work and help.

I recall with great affection the students that I have known, and look forward to meeting many more. To all of these, and to those I can only meet through my writing, this text is dedicated.

This new printing contains two additions to the Appendix. The section on random variables is intended to fill the gap created by my assumptions, made in Chapter 7, of prior knowledge in this area: up to now I have had to hand this out to my students as additional material. The section on Hamiltonian mechanics appears in response to a critic's complaint. It is a highly condensed introduction; but, I hope, better than nothing.

J.M.A. Danby

Raleigh, North Carolina
May 1991

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Chapter 1

The Astronomical Background

1.1 Introduction

Much of the theory in this text is that of conventional mechanics, but since its applications will concern celestial objects, we shall be using terms that may not be familiar to readers who lack a background in astronomy. This chapter is intended for such readers. It is confined to a bare introduction to the astronomical terms used in the text. Readers are strongly advised to refer, in addition, to some general text on descriptive astronomy.

The fundamental law of celestial mechanics is Newton's law of gravitation. Most applications concern the solar system, but the theory can be applied anywhere in the universe. For very small distances, comparable with the size of the atom, and for very large distances, of the order of a billion light years, this law may, perhaps, not be useful, but we shall not be concerned with such extremes here. It is important to realize that, apart from some small modifications (as in the case of Mercury's orbit), Newton's law holds good in the sense that it gives the right answers; furthermore it is the only reasonable law in elementary mechanics that gives the right answers. In practice, the law is used not only in work on the solar system but also in the mechanics of multiple star systems, of the galaxy, and even of clusters of galaxies.

In celestial mechanics we are primarily concerned with things as we find them. We need to explain observed motion and to provide accurate predictions for the future. But when a rocket is launched, some control is exercised over its orbit, even if only by a judicious choice of the initial conditions of launching. An orbit is chosen in advance; in addition to the forces due to gravity, thrusts can be imposed; if the rocket does not follow its chosen orbit precisely, corrections must be applied. These circumstances fall into the province of *astronautics* (which can be said to include "experimental celestial mechanics"). Actually, the methods of celestial mechanics can be applied to the problems of astronautics, and the basic theory of the two subjects is the same.