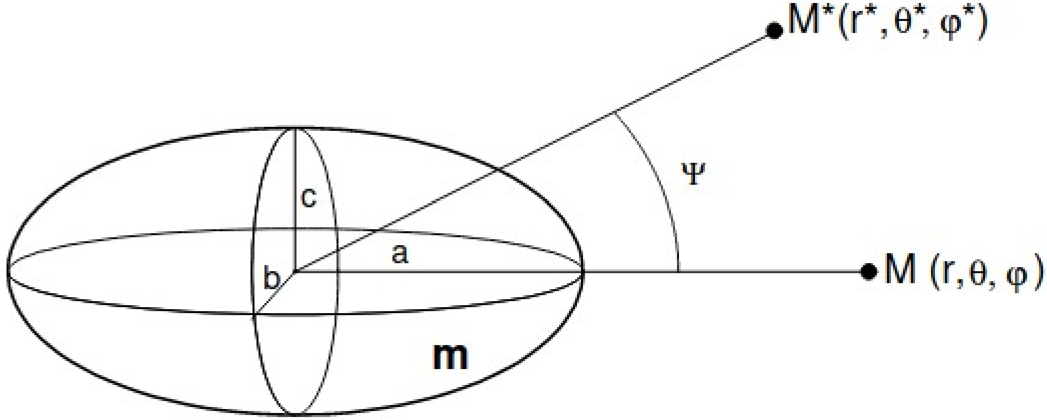
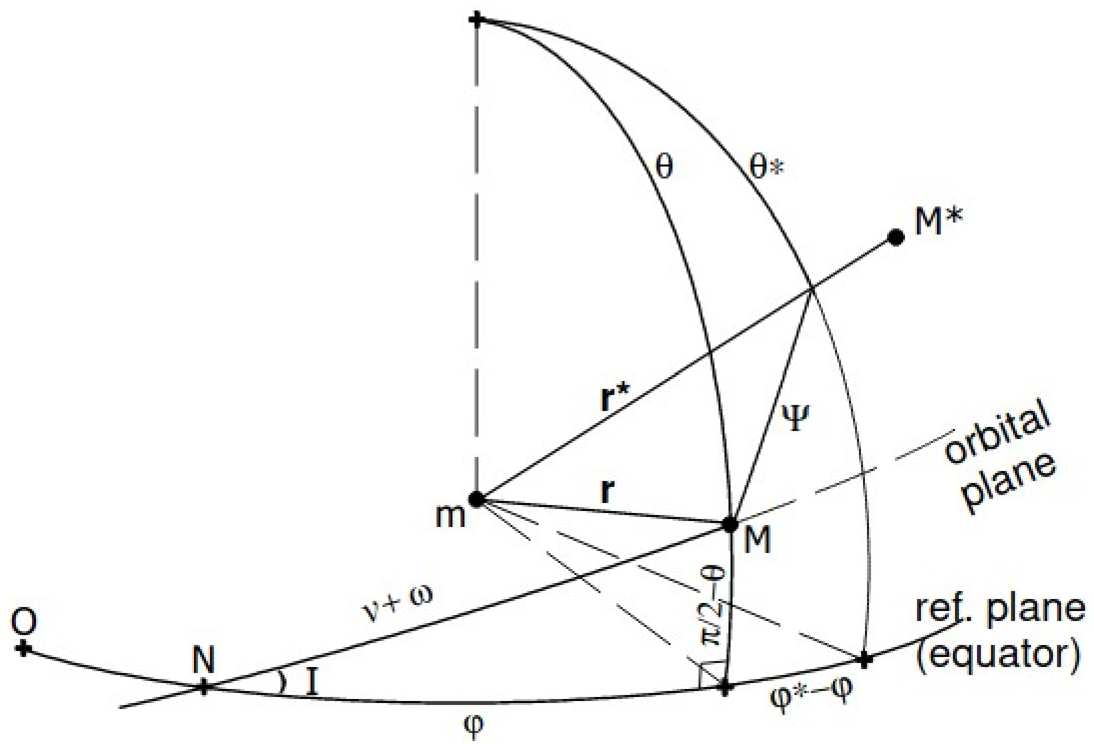


Potencial de maré

$$U = -\frac{Gm}{r^*} - \frac{k_f G M R^5}{2r^3 r^{*3}}(3\cos^2\Psi - 1);$$



Desenvolvendo até segunda ordem em excentricidade e inclinação:

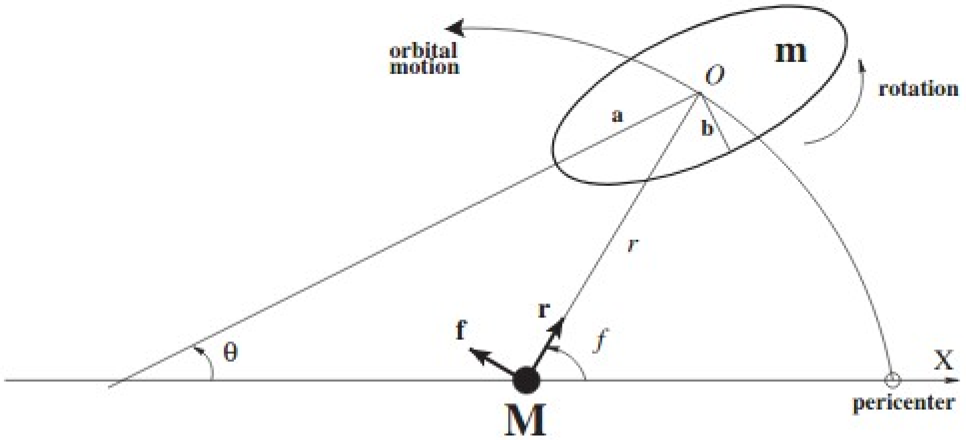
$$U_2 = -\frac{3k_f GMR^5}{4a^3 r^{*3}} \Bigg[-\frac{2}{3} - e^2 + \left(1 + \frac{3}{2}e^2 - \frac{1}{2}S^2\right)P^2$$

$$+\left(1 - \frac{5}{2}e^2 - \frac{1}{2}S^2\right)P^2 \cos(2\varphi^* - 2\ell - 2\omega) + \frac{7}{2}eP^2 \cos(2\varphi^* - 3\ell - 2\omega)$$

$$-\frac{1}{2}eP^2 \cos(2\varphi^* - \ell - 2\omega) + \frac{17}{2}e^2P^2 \cos(2\varphi^* - 4\ell - 2\omega)$$

$$-\left(2 - 3P^2\right)e \cos \ell - \left(3 - \frac{9}{2}P^2\right)e^2 \cos 2\ell$$

$$+QS\Big(\sin \varphi^* - \sin(\varphi^* - 2\ell - 2\omega)\Big) + \frac{1}{2}P^2S^2\Big(\cos 2\varphi^* + \cos(2\ell + 2\omega)\Big)\Bigg]$$



6 Tidal forces acting on the tide generating body

The perturbing force acting on a point of mass M^* placed in $\mathbf{r}^* \equiv (r^*, \theta^*, \varphi^*)$, due to the disturbing potential U_2 is given by

$$\mathbf{F} = -M^* \mathrm{grad}_{\mathbf{r}^*} U_2 = \underbrace{-M^* \frac{\partial U_2}{\partial r^*}}_{F_1} \hat{\mathbf{r}}^* \underbrace{- \frac{M^*}{r^*} \frac{\partial U_2}{\partial \theta^*}}_{F_2} \hat{\theta}^* \underbrace{- \frac{M^*}{r^* \sin\theta^*} \frac{\partial U_2}{\partial \varphi^*}}_{F_3} \hat{\varphi}^* \qquad (20) \quad (21)$$

Usando as equações de Gauss é possível calcular as variações médias dos elementos orbitais, conhecendo as componentes das forças de maré (*F1, F2, F3*)

$$< \dot{a} > \;=\; \frac{1}{\sqrt{\mu a}(1-e^2)\pi} \int_0^{2\pi} \Big[eR\sin f + T(1 + e\cos f) \Big] r^2 df$$

$$< \dot{e} > \;=\; \frac{1}{2\pi\sqrt{\mu a^3}} \int_0^{2\pi} \left[R\sin f + T\left(\cos f + \frac{1}{e}\left(1 - \frac{r}{a}\right)\right)\right] r^2 df$$

SISTEMAS DE 1 PLANETA

Evolução orbital média

As variações médias de semi-eixo maior e excentricidade são:

$$\langle \dot{a} \rangle = -\frac{2}{3} n a^{-4} [(2 + 46e^2)\hat{s} + 7e^2 \hat{p}]$$

$$\langle \dot{e} \rangle = -\frac{1}{3} n e a^{-5} (18\hat{s} + 7\hat{p}).$$

$$\hat{s} = \frac{9}{4} \frac{k_{d*}}{Q_*} \frac{m_p}{m_*} R_*^5 \quad ; \qquad \hat{p} = \frac{9}{2} \frac{k_{dp}}{Q_p} \frac{m_*}{m_p} R_p^5$$

1/Q → **dissipação**

Ferraz-Mello + (2008)
Rodríguez & Ferraz-Mello (2010)

3.1 Semi-major axis

The decreasing timescale of the semi-major axis can be defined as $\tau_a \equiv a/|\dot{a}|$. Using equation (2.6),

$$\tau_a = \frac{3}{2} n^{-1} a^5 [(2 + 46e^2)\hat{s} + 7e^2\hat{p}]^{-1}. \tag{3.1}$$

If we need to know the timescale only due to planetary tides, it is enough to put $\hat{s} = 0$ (or $Q_*^{-1} = 0$) in the above equation to obtain

$$\tau_a^p = \frac{3n^{-1}a^5}{14e^2\hat{p}}. \tag{3.2}$$

Note that $\lim \tau_a^p = \infty$ as $e \to 0$, indicating that, when the only tide is the planetary, the semi-major axis stops decreasing after circularization. The contribution of the stellar tide follows a similar analysis putting $\hat{p} = 0$ (or $Q_p^{-1} = 0$). The result is

$$\tau_a^* = \frac{3n^{-1}a^5}{2(2 + 46e^2)\hat{s}}. \tag{3.3}$$

Note that $\lim \tau_a^* = \frac{3n^{-1}a^5}{4\hat{s}} < \infty$ as $e \to 0$. This shows that after circularization the semi-major axis continues to decrease due to the stellar tide.

Rodríguez & Ferraz-Mello (2010)

3.2 Eccentricity

The timescale of orbital circularization can be defined as $\tau_e \equiv e/|\dot{e}|$.

$$\tau_e = \frac{3n^{-1}a^5}{18\hat{s} + 7\hat{p}}.$$

$$\tau_e^p = \frac{3n^{-1}a^5}{7\hat{p}}; \qquad \tau_e^* = \frac{3n^{-1}a^5}{18\hat{s}}.$$

Note that τ_e^p and τ_e^* are independent of e. Moreover, we see that

$$\frac{\tau_e^p}{\tau_e^*} = \frac{18}{7}\frac{\hat{s}}{\hat{p}}.$$

$$\frac{\hat{p}}{\hat{s}} \approx 50\frac{k_{dp}}{k_{d*}}\frac{Q_*}{Q_p}.$$ → (p = Júpiter)
(* = Sol)

$$2 \times 10^7 < Q_*/k_{d*} < 1.5 \times 10^9 \qquad 2.5 \times 10^4 < Q_p/k_{dp} < 2.5 \times 10^5$$ (Valores típicos)

$$\frac{k_{dp}}{k_{d*}}\frac{Q_*}{Q_p} \geq 1 \text{ and } \frac{\hat{p}}{\hat{s}} \gg 1. \qquad \tau_e^p \ll \tau_e^*,$$ **Evolução governada pela maré no planeta**

4 Orbital decay for different type of planets

4.1 Critical eccentricity for orbital decay

As we have done in the case of timescales for orbital circularization, we may compare the timescales for orbital decay due to planetary and stellar tides. For that sake, using equations (3.2) and (3.3) we obtain

$$\frac{\tau_a^p}{\tau_a^*} = \frac{(2 + 46e^2)}{7e^2} \frac{\hat{s}}{\hat{p}}. \tag{4.1}$$

Note that the above timescales ratio depends on the eccentricity, which is a time-dependent quantity. We can compute a critical value of e for which equation (4.1) is equal to unity. It is

$$e_c = \sqrt{\frac{2}{7\hat{p}/\hat{s} - 46}} \quad . \tag{4.2}$$

If $e > e_c$ then $\tau_a^p < \tau_a^*$ and the planetary tide dominates. On the contray, if $e < e_c$ then $\tau_a^p > \tau_a^*$ and the stellar tide becomes more effective to produce orbital decay.

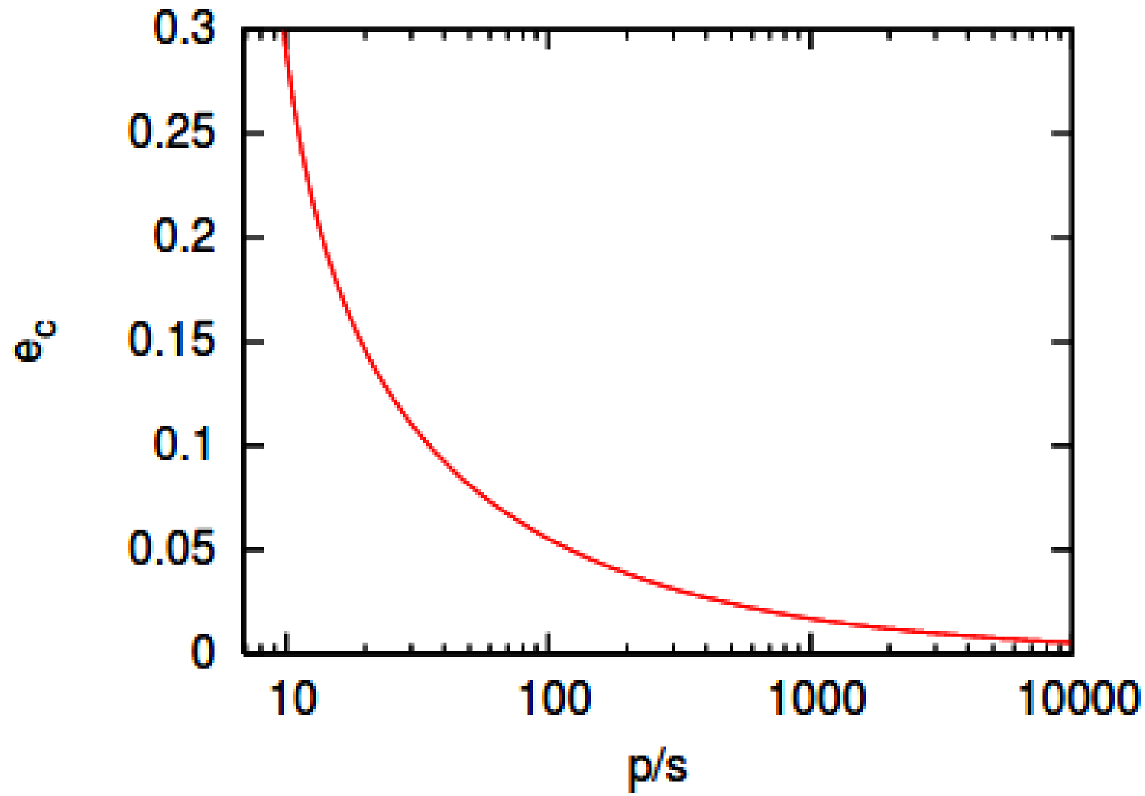


Figure 1 is a plot of e_c as a function of $\hat{p}/\hat{s}$. The minimum value of $\hat{p}/\hat{s}$ for which equation (4.2) has a possible solution ($e_c < 1$) is 48/7. Note that, for high values of $\hat{p}/\hat{s}$, it is sufficient a small value of eccentricity for that the orbital decay is dominated by the planetary tide ($\tau_a^p < \tau_a^*$). On the other hand, for very small values of $\hat{p}/\hat{s}$, an eccentricity close to unity is necessary to have comparable evolutions due to planetary and stellar tides (remember that the model is valid up to second order in e).

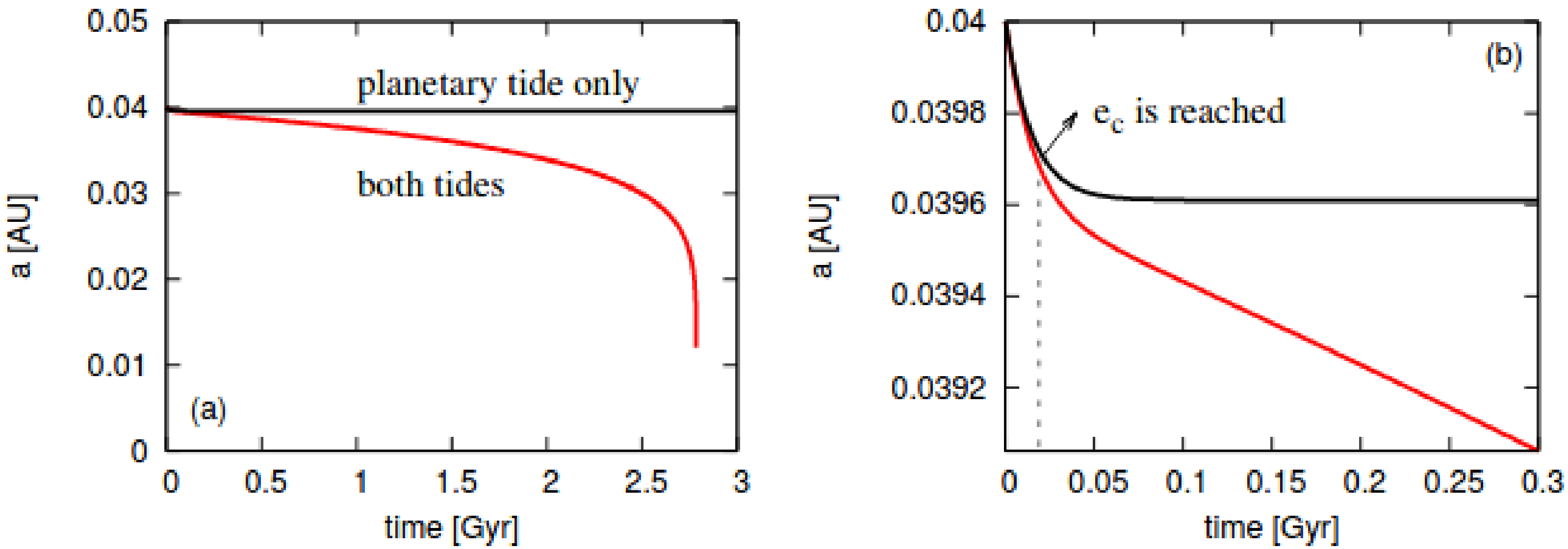
4.2 Hot Jupiters and hot Neptunes

In this section we analyze the tidal evolution considering two different types of close-in planets, depending on the planetary mass. In a general way we call *hot Jupiter* any close-in planet for which $m_p \sim m_J$, and *hot Neptune* planets for which $m_p \sim m_N$, being m_J and m_N the mass of Jupiter and Neptune, respectively. From equations (3.1) and (3.4) we know that

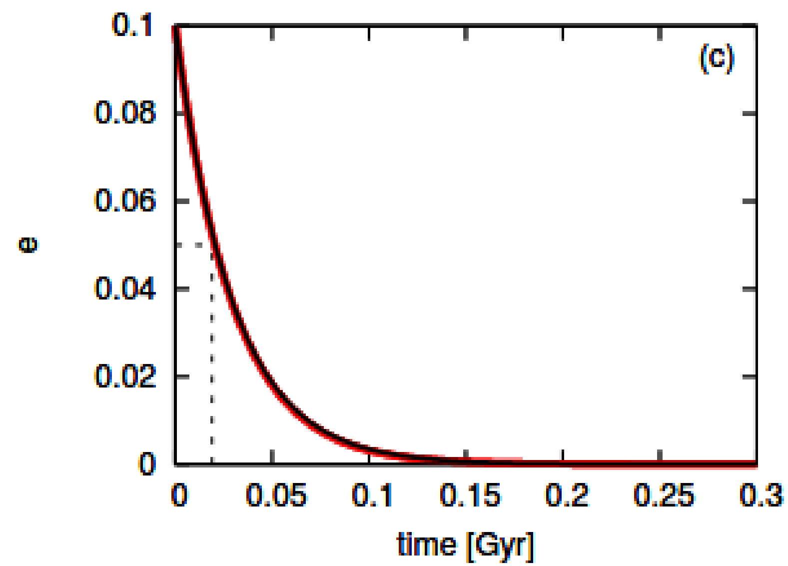
$$\frac{\tau_a}{\tau_e} = \frac{1}{2} \frac{18\hat{s} + 7\hat{p}}{[(2 + 46e^2)\hat{s} + 7e^2\hat{p}]}. \tag{4.3}$$

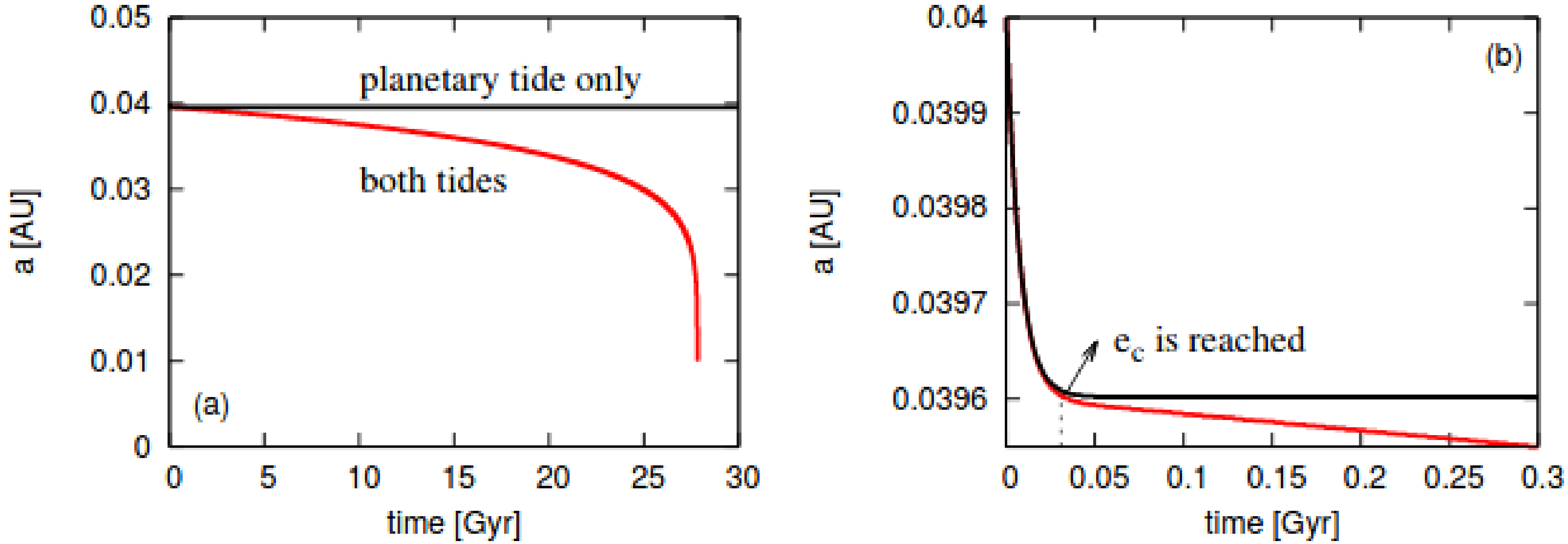
It is clear from the above equation that, for small values of e, $\tau_a \gg \tau_e$. This indicates that the effect of tides circularize the orbit before producing any appreciable decrease in semi-major axis. Note that this fact minimizes the contribution due to the planetary tide because after circularization, only the stellar tide contributes to produce the orbital decay (see equation (2.6)). Equation (2.4) shows that the stellar tide contribution is proportional to the planet mass m_p, where the coefficient of proportionality depends on stellar paramaters only. Hence, massive planets are more effective to have orbital decay due to the stellar tide, indicating that hot Jupiters must decay faster than hot Neptunes.

Planet	$m_p(m_J)$	Q_p	a_0 (AU)	e_0	$\hat{p}/\hat{s}$	e_c
Jupiter-like	1.0	1×10^5	0.04	0.1	125	0.05
Neptune-like	0.1	1×10^4	0.04	0.1	2.40×10^3	0.01



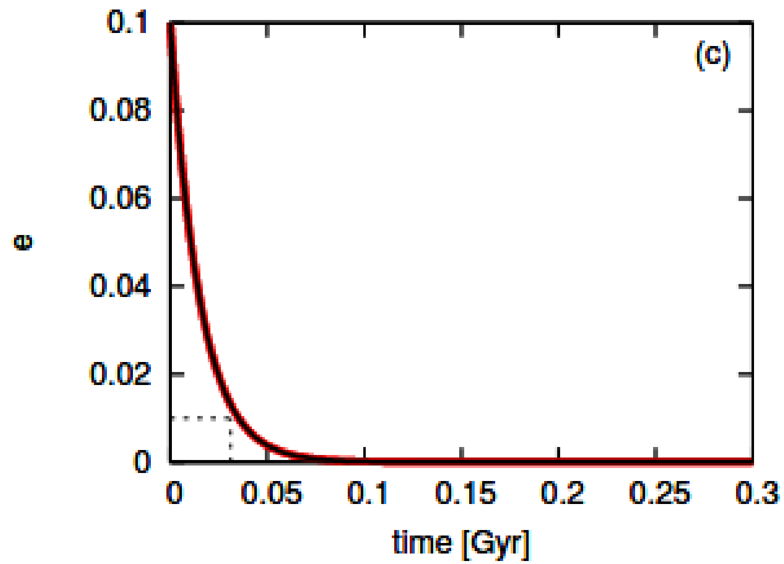
Sol - Júpiter





0.05
0.04
0.03
0.02
0.01
0
a [AU]
planetary tide only
both tides
(a)
0
5
10
15
20
25
30
time [Gyr]
0.04
0.0399
0.0398
0.0397
0.0396
a [AU]
(b)
e_c is reached
0
0.05
0.1
0.15
0.2
0.25
0.3
time [Gyr]

Sol - Netuno



0.1
0.08
0.06
0.04
0.02
0
e
(c)
0
0.05
0.1
0.15
0.2
0.25
0.3
time [Gyr]

Evolução orbital média

As variações médias (conservativas!) de excentricidades e pericentros são:

$$\frac{de_p}{dt} = -\frac{15}{16} n_p e_c \left(\frac{m_c}{m_*}\right) \left(\frac{a_p}{a_c}\right)^4 \frac{\sin(\varpi_p - \varpi_c)}{(1 - e_c^2)^{5/2}},$$

Legendre expansion in the ratio a_p/a_c

$$\frac{de_c}{dt} = \frac{15}{16} n_c e_p \left(\frac{m_p}{m_*}\right) \left(\frac{a_p}{a_c}\right)^3 \frac{\sin(\varpi_p - \varpi_c)}{(1 - e_c^2)^2},$$

$$\frac{d\varpi_p}{dt} = \frac{3}{4} n_p \left(\frac{m_c}{m_*}\right) \left(\frac{a_p}{a_c}\right)^3 (1 - e_c^2)^{3/2} \left[1 - \frac{5}{4}\left(\frac{a_p}{a_c}\right)\left(\frac{e_c}{e_p}\right) \frac{\cos(\varpi_p - \varpi_c)}{1 - e_c^2}\right]$$

and

$$\frac{d\varpi_c}{dt} = \frac{3}{4} n_c \left(\frac{m_p}{m_*}\right) \left(\frac{a_p}{a_c}\right)^2 \left(1 - e_c^2\right)^{-2} \left[1 - \frac{5}{4}\left(\frac{a_p}{a_c}\right)\left(\frac{e_p}{e_c}\right) \frac{\left(1 + 4e_c^2\right)}{\left(1 - e_c^2\right)} \cos(\varpi_p - \varpi_c)\right],$$

Mardling (2007)

A solução secular do sistema é bem conhecida:

- oscilações das excentricidades ao redor de soluções de equilíbrio (pontos fixos)

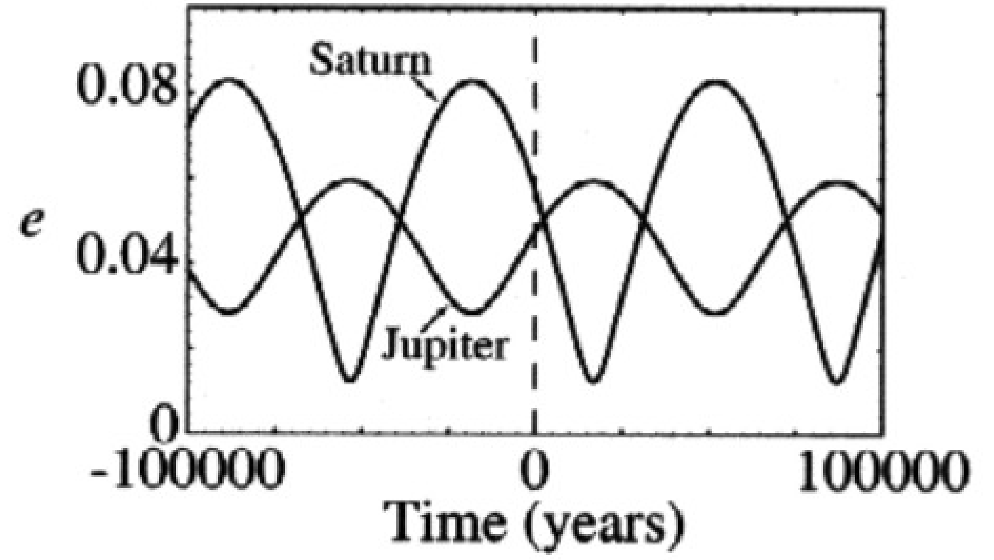
- circulação ou oscilação de w2 – w1 ao redor de 0^0 ou 180^0.

$$\langle \mathcal{R}_D \rangle = C_0 + C_1(e^2 + e'^2) + C_2 s^2 + C_3 e e' \cos(\varpi' - \varpi),$$

$$\left(\frac{da}{dt}\right)_{sec} = 0, \qquad \left(\frac{de}{dt}\right)_{sec} = n\alpha(m'/m_c) C_3 e' \sin(\varpi - \varpi'),$$

$$\left(\frac{d\varpi}{dt}\right)_{sec} = n\alpha(m'/m_c)\left[2C_1 + C_3(e'/e)\cos(\varpi - \varpi')\right],$$

$$(\Delta e)_{sec} = \left|(n\alpha/\dot{\varpi})(m'/m_c) C_3 e'\right|,$$



Murray & Dermott (1999)

Incluindo as variações devidas à maré:

$$\dot{e}_{\rm p} = -W_{\rm o}\, e_{\rm c}\, \sin\eta - W_{\rm T}\, e_{\rm p},$$

$$\dot{e}_{\rm c} = W_{\rm c}\, e_{\rm p}\, \sin\eta,$$

$$\dot{\eta} = \overline{W}_{\rm q} - W_{\rm o}\left(\frac{e_{\rm c}}{e_{\rm p}}\right)\cos\eta,$$

→

$$W_{\rm o} = \frac{15}{16} n_{\rm p} \left(\frac{a_{\rm p}}{a_{\rm c}}\right)^4 \left(\frac{m_{\rm c}}{m_*}\right) \varepsilon_{\rm c}^{-5},$$

$$W_{\rm c} = \frac{15}{16} n_{\rm c} \left(\frac{m_{\rm p}}{m_*}\right) \left(\frac{a_{\rm p}}{a_{\rm c}}\right)^3 \varepsilon_{\rm c}^{-4},$$

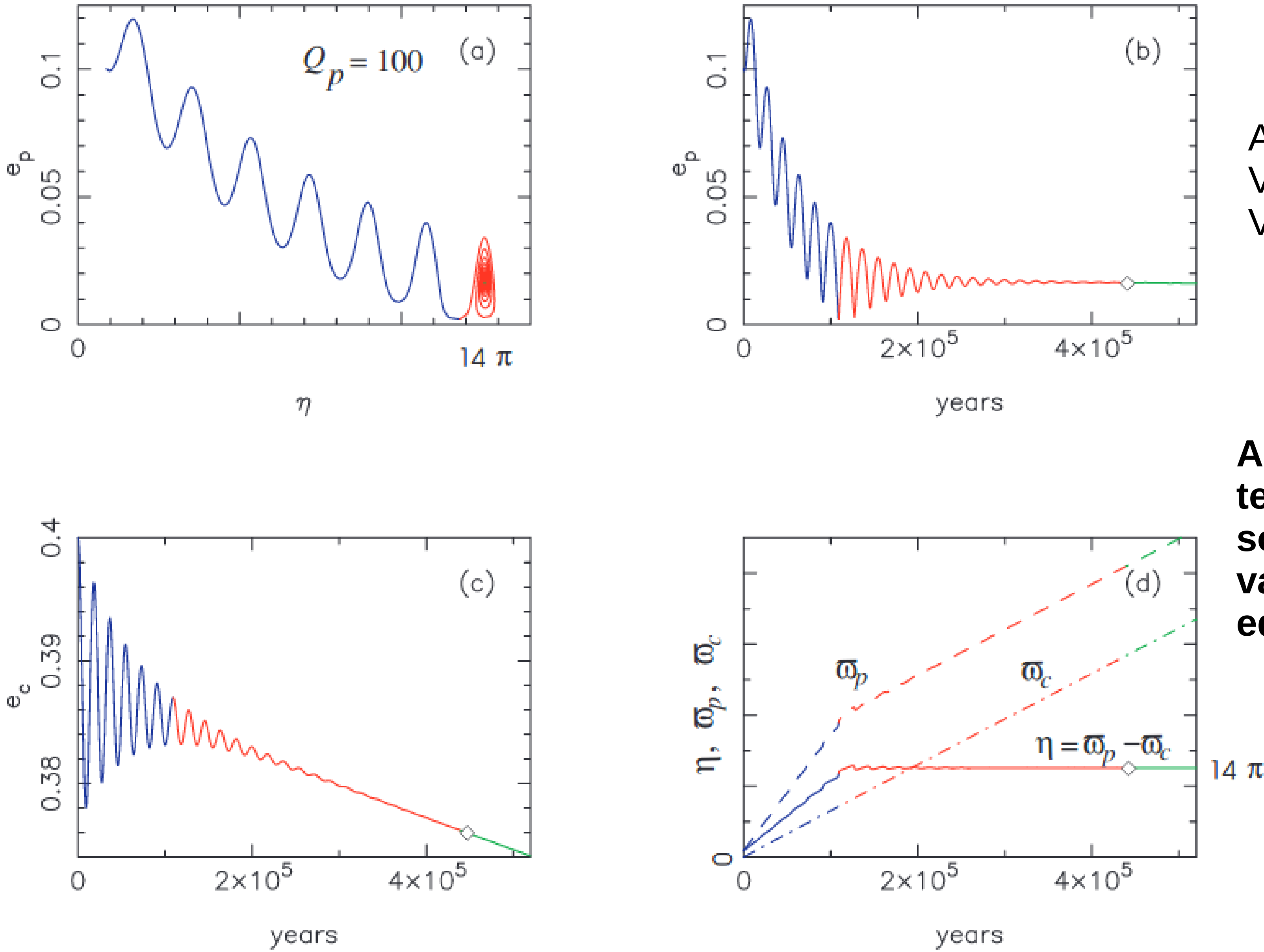
$$\eta \equiv \varpi_{\rm p} - \varpi_{\rm c} \qquad \varepsilon_{\rm c} = \sqrt{1 - e_{\rm c}^2}.$$

$$\overline{W}_{\rm q} = \frac{3}{4} n_{\rm p} \left(\frac{a_{\rm p}}{a_{\rm c}}\right)^3 \left(\frac{m_{\rm c}}{m_*}\right) \varepsilon_{\rm c}^{-3} \left[1 - \sqrt{\frac{a_{\rm p}}{a_{\rm c}}} \left(\frac{m_{\rm p}}{m_{\rm c}}\right) \varepsilon_{\rm c}^{-1} + \gamma \varepsilon_{\rm c}^3\right],$$

$$W_{\rm T} = \frac{21}{2} n_{\rm p} \left(\frac{k_{\rm p}}{Q_{\rm p}}\right) \left(\frac{m_*}{m_{\rm p}}\right) \left(\frac{R_{\rm p}}{a_{\rm p}}\right)^5 = \tau_{\rm circ}^{-1}$$

$$e_{\rm p}^{\rm (eq)} = e_{\rm c} W_{\rm o}/|\overline{W}_{\rm q}| = \frac{(5/4)(a_{\rm p}/a_{\rm c})\, e_{\rm c}\, \varepsilon_{\rm c}^{-2}}{\left|1 - \sqrt{a_{\rm p}/a_{\rm c}}(m_{\rm p}/m_{\rm c})\varepsilon_{\rm c}^{-1} + \gamma \varepsilon_{\rm c}^3\right|}.$$

→ **Excentricidade de equilíbrio**



Azul = Criculação
Vermelho = Libração
Verde = *Longterm*

A dissipação (maré) tende a levar as soluções para seus valores de equilíbrio.

Longterm evolution

$$\dot{e}_c = -\frac{W_c W_o W_T}{\overline{W}_q^2} e_c$$
$$= -\left(\frac{25}{16}\right)\left(\frac{m_p}{m_c}\right)\left(\frac{a_p}{a_c}\right)^{5/2} W_T \frac{e_c}{F(e_c)}$$
$$= -\left(\frac{\lambda}{\tau_{circ}}\right)\frac{e_c}{F(e_c)}$$

where

$$\lambda = \left(\frac{25}{16}\right)\left(\frac{m_p}{m_c}\right)\left(\frac{a_p}{a_c}\right)^{5/2} \quad \text{and} \quad F(e_c) = \varepsilon_c^3\left(1 - \alpha\varepsilon_c^{-1} + \gamma\,\varepsilon_c^3\right)^2$$

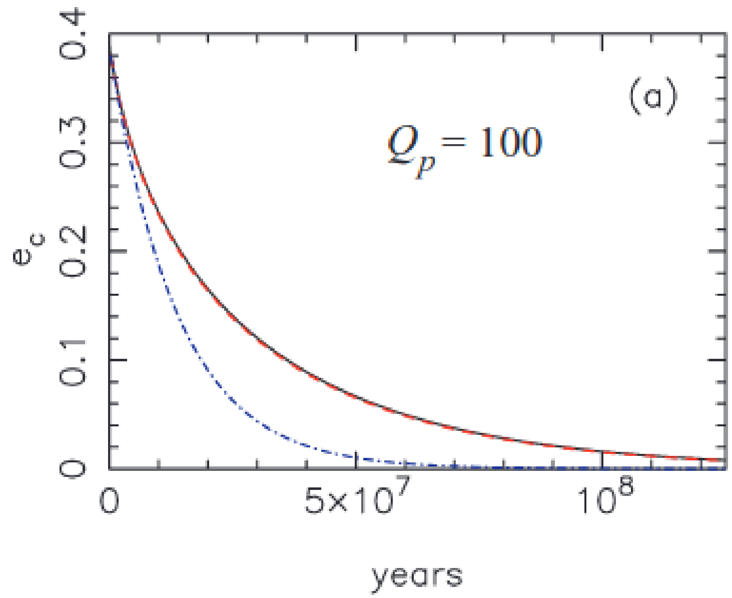
$$\alpha = \sqrt{a_p/a_c}(m_p/m_c)$$

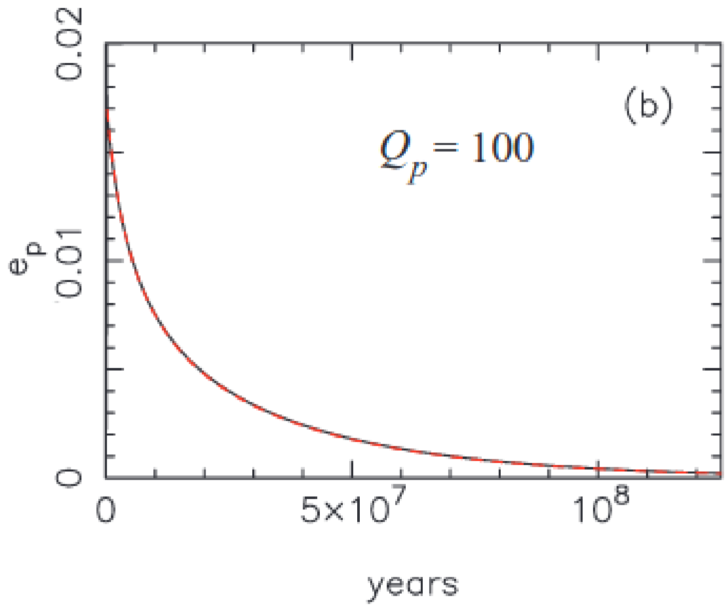
→

$$e_c(t) = e_c(t_c)\,e^{-(t-t_c)/\tau_c},$$

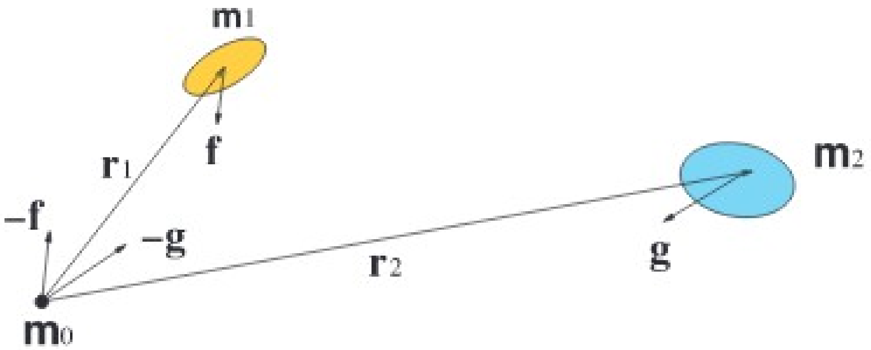
where

$$\tau_c = \left(F\left(e_c^*\right)/\lambda\right)\tau_{circ}$$





EQUAÇÕES EXATAS



$$\ddot{\boldsymbol{r}}_1 = -\frac{G(m_0+m_1)}{r_1^3}\boldsymbol{r}_1 + Gm_2\left(\frac{\boldsymbol{r}_2-\boldsymbol{r}_1}{|\boldsymbol{r}_2-\boldsymbol{r}_1|^3} - \frac{\boldsymbol{r}_2}{r_2^3}\right) + \frac{(m_0+m_1)}{m_0 m_1}(\boldsymbol{f}+\boldsymbol{f}_{\rm rel}) + \frac{\boldsymbol{g}}{m_0},$$

$$\ddot{\boldsymbol{r}}_2 = -\frac{G(m_0+m_2)}{r_2^3}\boldsymbol{r}_2 + Gm_1\left(\frac{\boldsymbol{r}_1-\boldsymbol{r}_2}{|\boldsymbol{r}_1-\boldsymbol{r}_2|^3} - \frac{\boldsymbol{r}_1}{r_1^3}\right) + \frac{(m_0+m_2)}{m_0 m_2}\boldsymbol{g} + \frac{(\boldsymbol{f}+\boldsymbol{f}_{\rm rel})}{m_0},$$

$$\boldsymbol{f}_{\rm rel} = \frac{Gm_1m_0}{c^2r_1^3}\left[\left(4\frac{Gm_0}{r_1} - v_1^2\right)\boldsymbol{r}_1 + 4(\boldsymbol{r}_1\cdot\boldsymbol{v}_1)\boldsymbol{v}_1\right]$$ → Relatividade

$$\boldsymbol{f} = -3k_1\Delta t_1\frac{Gm_0^2R_1^5}{r_1^{10}}\left[2\boldsymbol{r}_1(\boldsymbol{r}_1\cdot\boldsymbol{v}_1) + r_1^2(\boldsymbol{r}_1\times\boldsymbol{\Omega}_1+\boldsymbol{v}_1)\right],$$

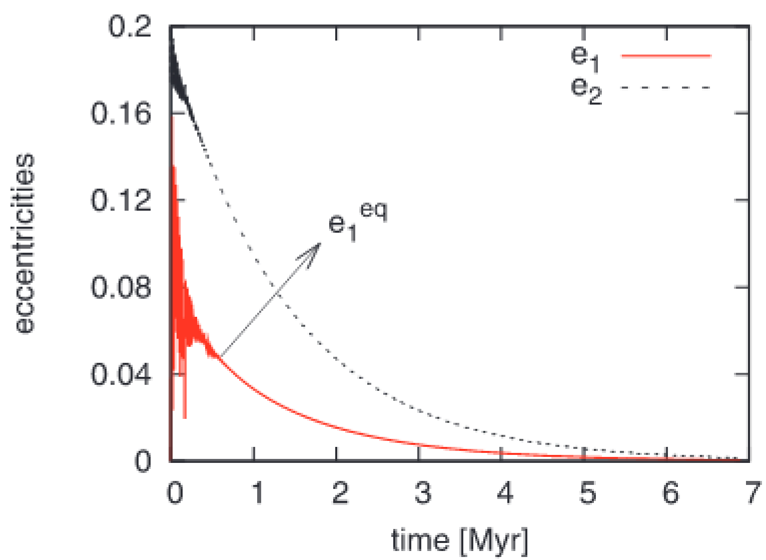
$$\boldsymbol{g} = -3k_2\Delta t_2\frac{Gm_0^2R_2^5}{r_2^{10}}\left[2\boldsymbol{r}_2(\boldsymbol{r}_2\cdot\boldsymbol{v}_2) + r_2^2(\boldsymbol{r}_2\times\boldsymbol{\Omega}_2+\boldsymbol{v}_2)\right],$$

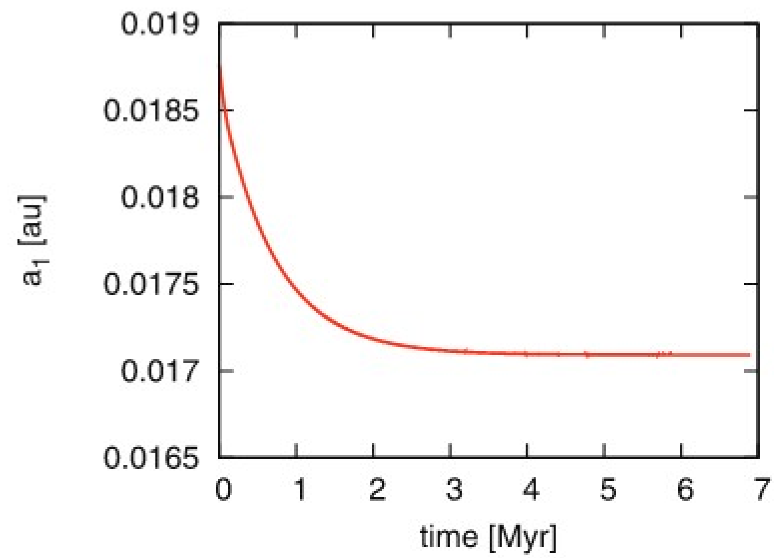
→ Maré

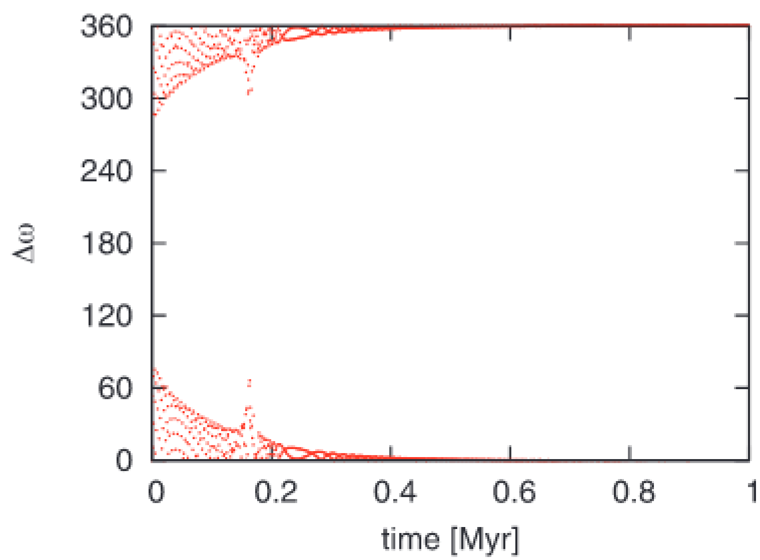
Rodríguez + (2011)

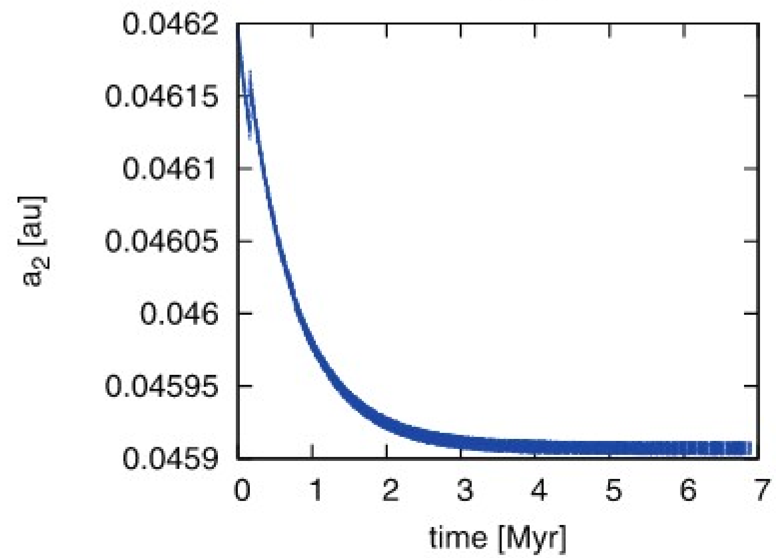
Body	m_i	R_i	$a_{i\,\mathrm{current}}$ (au)	$e_{i\,\mathrm{current}}$	Q'_i
0	$0.93\,\mathrm{m}_\odot$	$0.87\,\mathrm{R}_\odot$	–	–	–
1	$8.0\,m_\oplus$	$1.58\,R_\oplus$	0.017	0	100
2	$13.6\,m_\oplus$	$2.39\,R_\oplus$	0.046	0	100

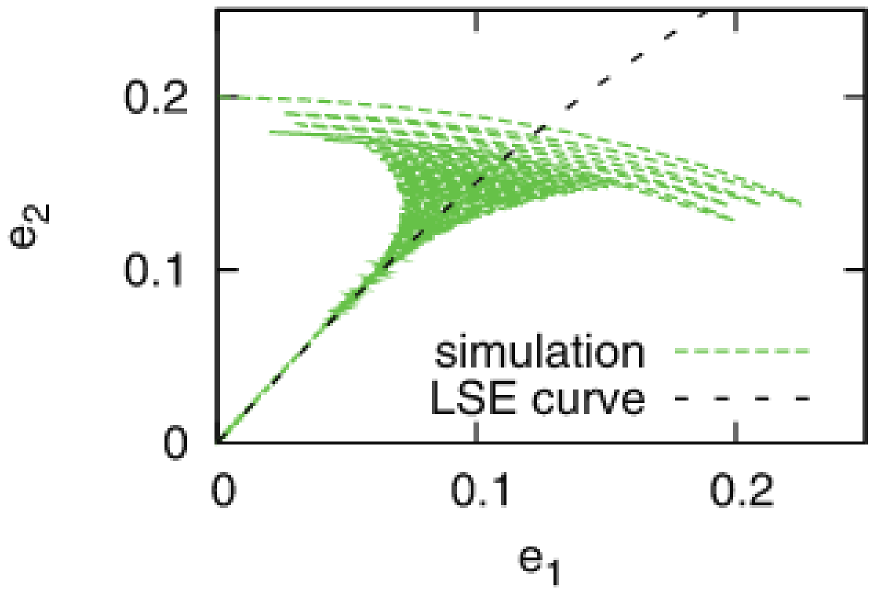
Sol
super Terra
Super Terra







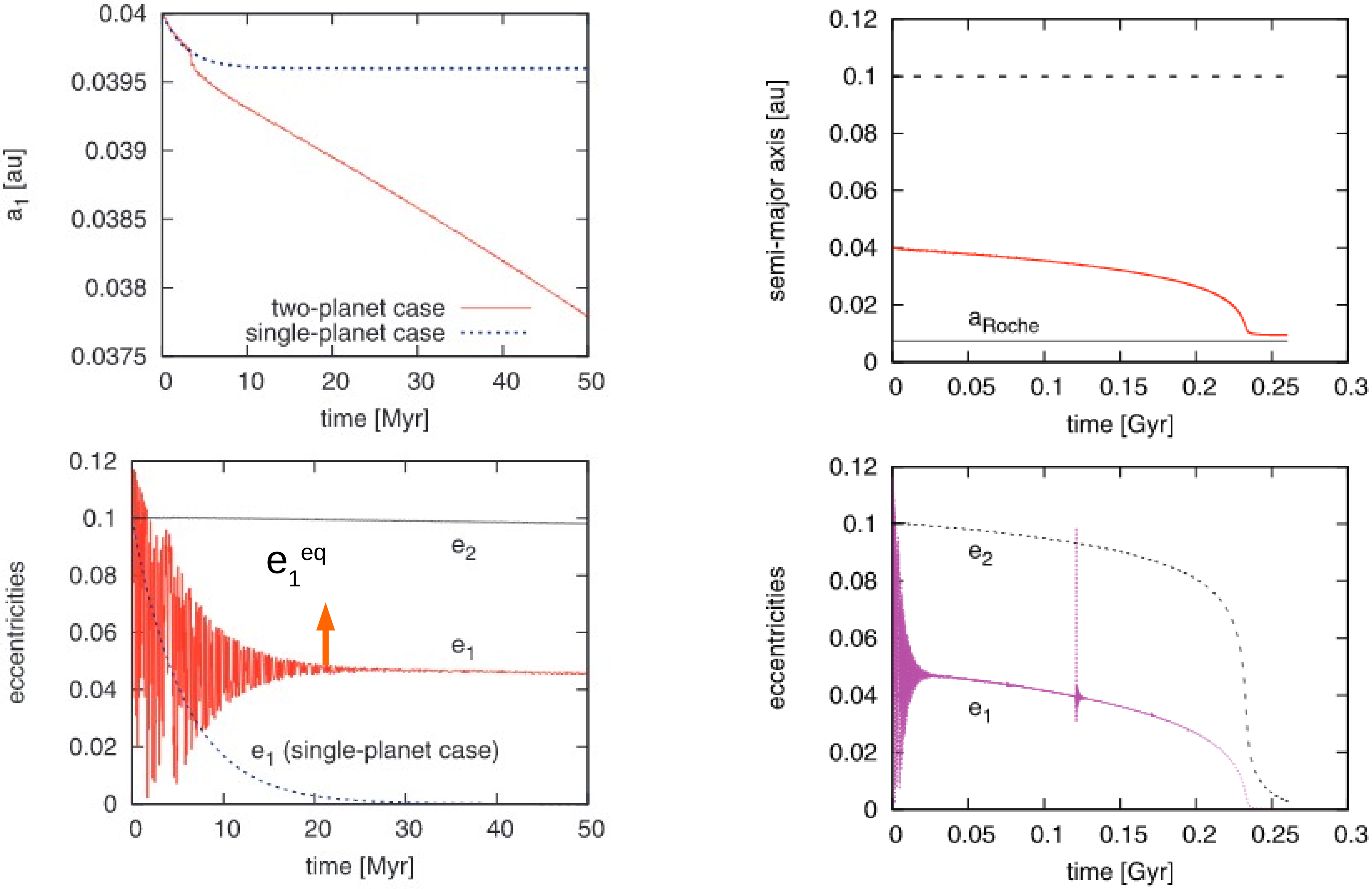




LSE: local das soluções estacionárias
(calculadas de forma semi-analítica)

Body	m_i	R_i	a_i (au)	e_i	Q'_i
0	$1\,M_\odot$	$1\,R_\odot$	–	–	–
1	$5\,m_\oplus$	$5^{1/3}R_\oplus$	0.04	0.1	100
2	$1\,m_J$	–	0.1	0.1	–

Sol
super Terra
Júpiter



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K. Goździewski, A. Niedzielski and J. Schneider (eds)
EAS Publications Series, **42** *(2010) 411-418*
www.eas.org

TIDAL DECAY AND CIRCULARIZATION OF THE ORBITS OF SHORT-PERIOD PLANETS

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Celest Mech Dyn Astr (2008) 101:171–201
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ORIGINAL ARTICLE

Tidal friction in close-in satellites and exoplanets: The Darwin theory re-visited

Sylvio Ferraz-Mello · Adrián Rodríguez · Hauke Hussmann

Mon. Not. R. Astron. Soc. **382,** 1768–1790 (2007) doi:10.1111/j.1365-2966.2007.12500.x

Long-term tidal evolution of short-period planets with companions

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Mon. Not. R. Astron. Soc. **415,** 2349–2358 (2011) doi:10.1111/j.1365-2966.2011.18861.x

Tidal decay and orbital circularization in close-in two-planet systems

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