

# Resonancias orbitales

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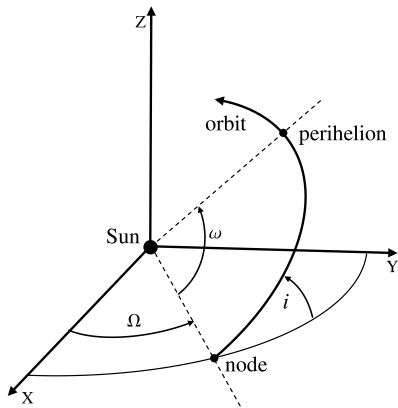
Curso Dinámica Orbital 2018

# Orbital motion

# Oscillating planetary orbits

# Orbital Resonances

Commensurability between **frequencies** associated with orbital motion: mean motion, nodes and pericenters

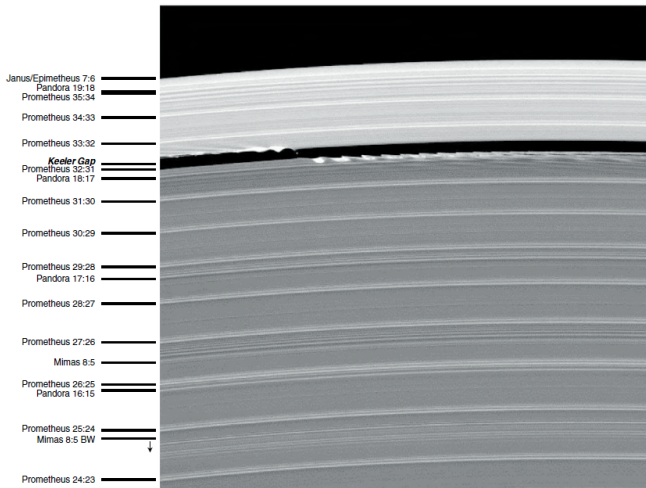


- two-body resonances ( $\lambda, \lambda_p$ )
- three-body resonances ( $\lambda, \lambda_{p1}, \lambda_{p2}$ )
- secular resonances ( $\Omega, \varpi$ )
- Kozai-Lidov mechanism ( $\omega$ )

# Some examples

- Io-Europa-Ganymede
- Saturn satellites
- Saturn rings
- Uranus satellites
- asteroids with Jupiter, Mars, Earth, Venus...
- Trans Neptunian Objects with Neptune
- Pluto - Neptune
- comets - Jupiter
- Pluto satellites: Styx, Nix, and Hydra

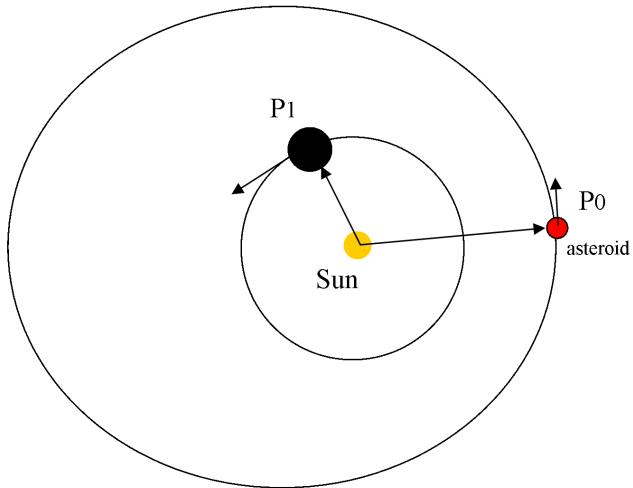
# Saturn rings



Lissauer and de Pater, Fundamental Planetary Science

# Two-body resonance

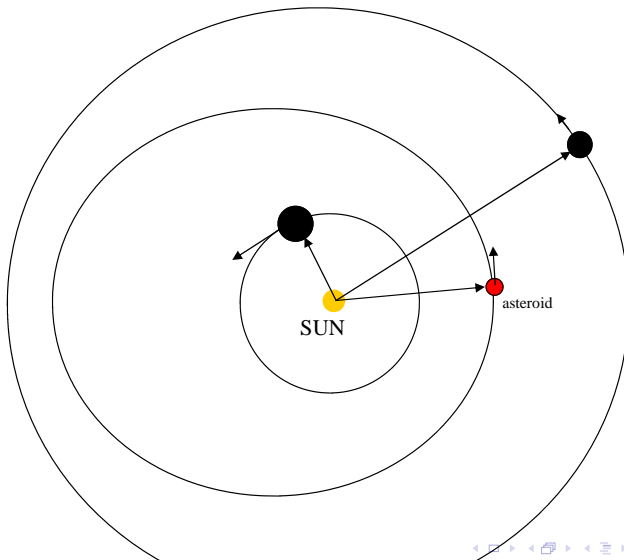
$$k_0 n_0 + k_1 n_1 \simeq 0$$



# Three-body resonance

$$k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0$$

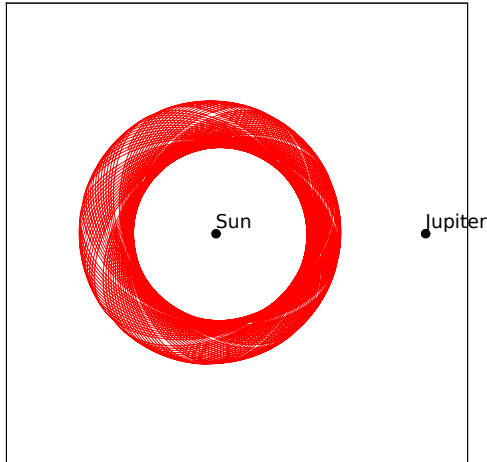
only the asteroid feels the resonance



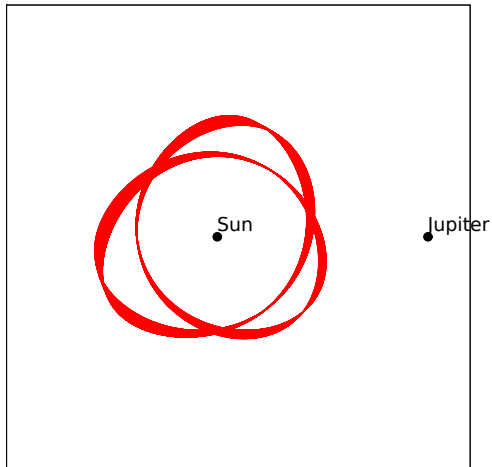


# Non resonant asteroid: relative positions

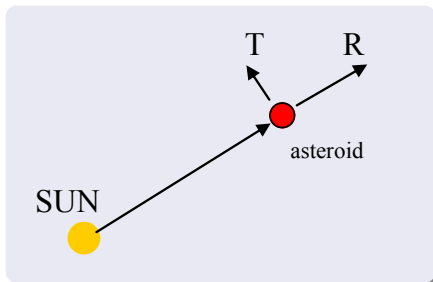
Mean perturbation is radial: Sun-Jupiter



Mean perturbation has a **transverse** component.



# from Gauss equations



$$F_{\text{perturb}} = (R, T, N)$$

$$\frac{da}{dt} \propto (R, T)$$

$$\left\langle \frac{da}{dt} \right\rangle \propto T$$

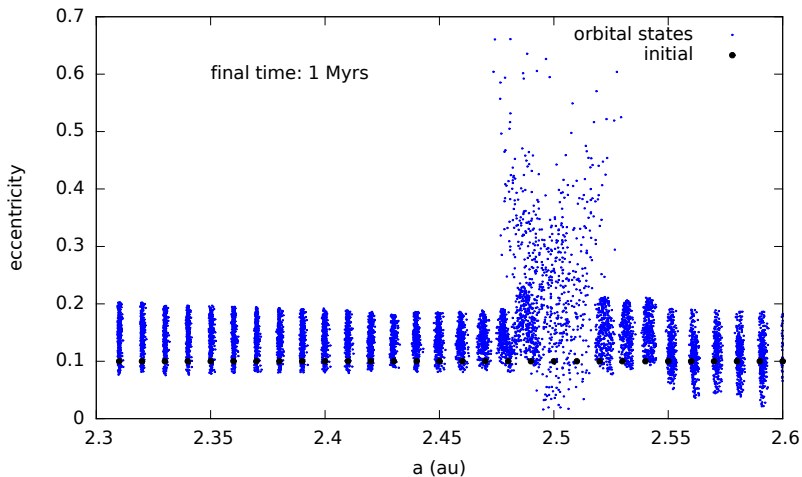
**Non resonant**

$$T = 0 \Rightarrow a = \text{constant}$$

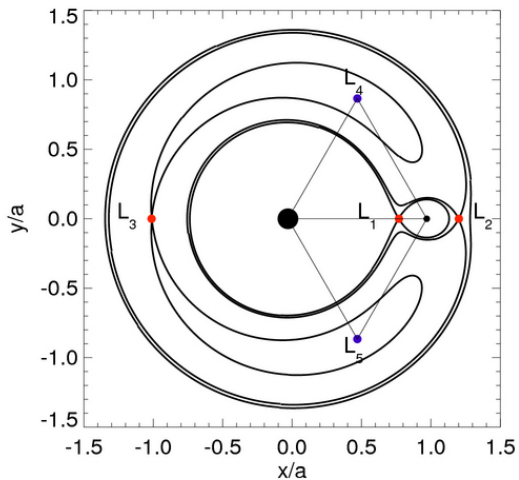
**Resonant**

$$T \neq 0 \Rightarrow a = \text{oscillating}$$

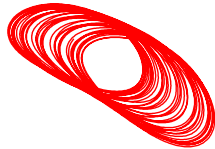
# Dynamical effects: a numerical exercise



# 1772: Lagrange equilibrium points



# 1906: (588) Achilles by 500 yrs



Sun  
●

Jupiter  
●

# 1784: Laplacian resonance



$$3\lambda_{Europa} - \lambda_{Io} - 2\lambda_{Ganymede} \simeq 180^\circ$$

$$3n_{Europa} - n_{Io} - 2n_{Ganymede} \simeq 0$$

They are also in commensurability by pairs:

$$2n_{Europa} - n_{Io} \simeq 0$$

$$2n_{Ganymede} - n_{Europa} \simeq 0$$



It must be the consequence of some physical mechanism.

# 1846: discovery of Neptune

**quasi resonance** Uranus - Neptune:

$$n_{Uranus} \sim 2n_{Neptune}$$

**quasi resonance** Saturn - Uranus:

$$n_{Saturn} \sim 3n_{Uranus}$$

**quasi resonance:** Jupiter - Saturn

$$2n_{Jupiter} \sim 5n_{Saturn}$$

Why the planets are close to resonance?

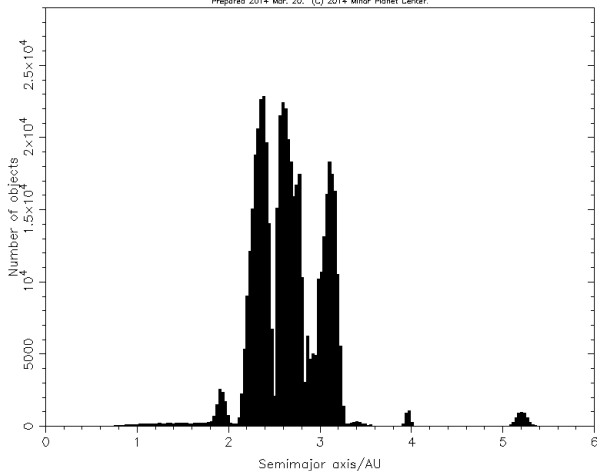
Hint: **planetary migration**



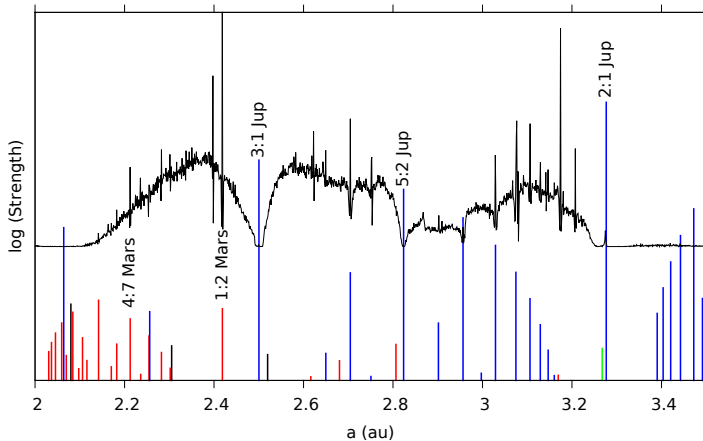
# 1866: Kirkwood gaps

Distribution of the Minor Planets: Semimajor axis

Prepared 2014 Mar. 20. (C) 2014 Minor Planet Center.



# Kirkwood gaps at present



Main belt of asteroids is *sculpted* by resonances.

$$k_0 n_{ast} = k_1 n_{Jup}$$
$$a_{ast} \simeq \left(\frac{k_0}{k_1}\right)^{2/3} a_{Jup}$$

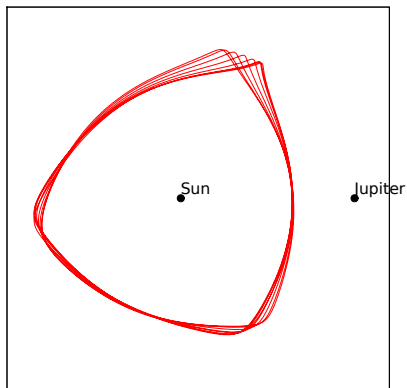
There is an infinite number of resonances...

**which are the relevant ones?**

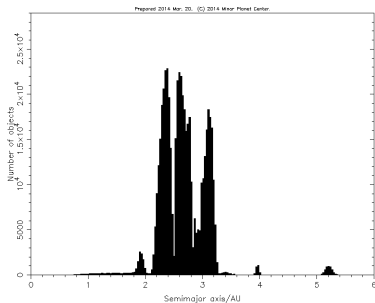
# 1875: resonant asteroids (153) Hilda 3:2

$$2n_{Hilda} = 3n_{Jup}$$

$$a_{Hilda} = \left(\frac{2}{3}\right)^{2/3} a_{Jup} = 3.97 \text{ua}$$

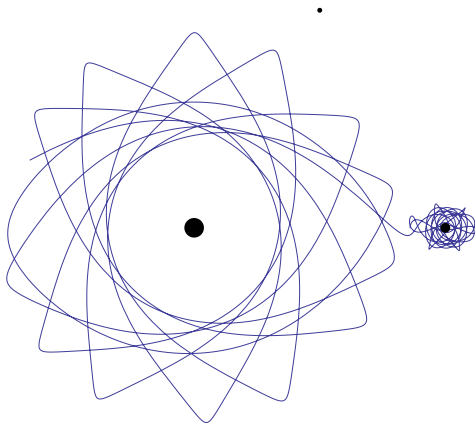


Distribution of the Minor Planets: Semimajor axis



# Hildas and Trojans

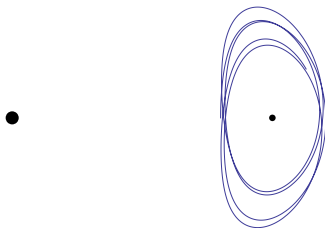
# Temporary satellite capture



the most probable origin of the irregular satellites

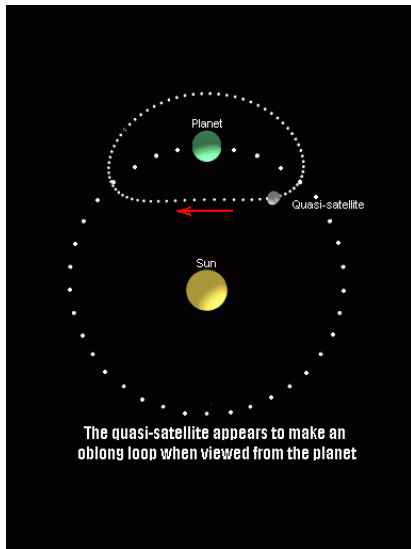
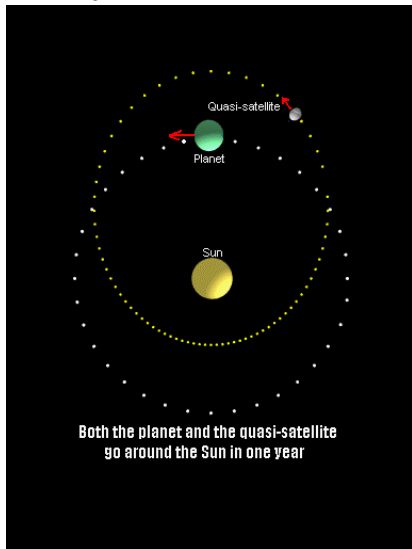
# Quasi satellite, resonance 1:1

it is not orbiting around the planet, it is synchronized 1:1 with the planet



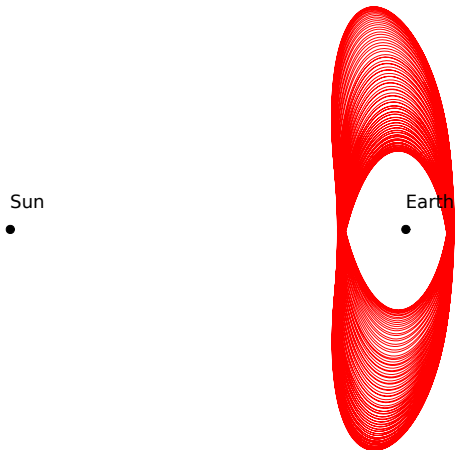
# Quasi satellite

from Wiegert website:

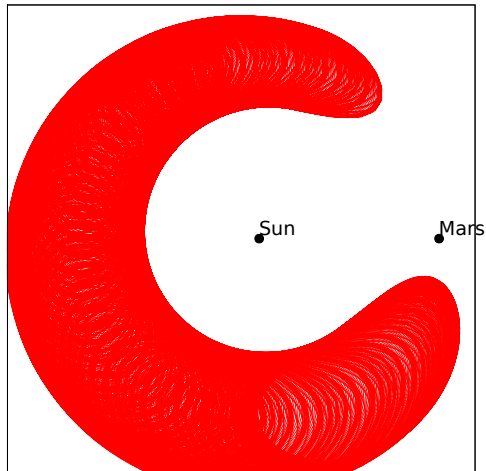




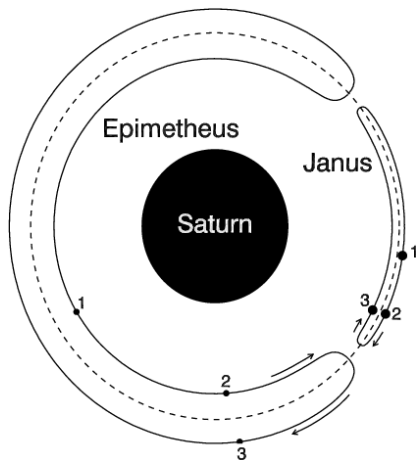
# 2004 GU9: Earth quasi satellite, resonance 1:1



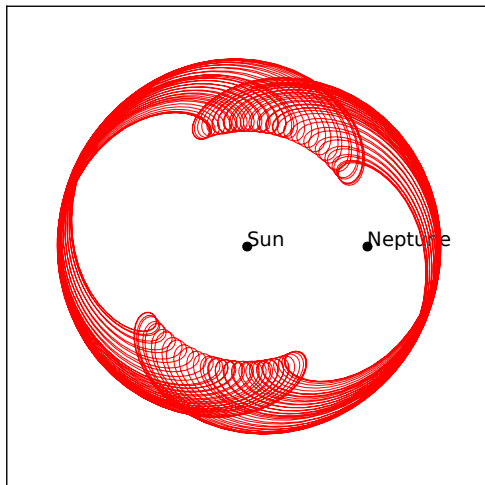
# 1999 ND43: Mars horseshoe, resonance 1:1

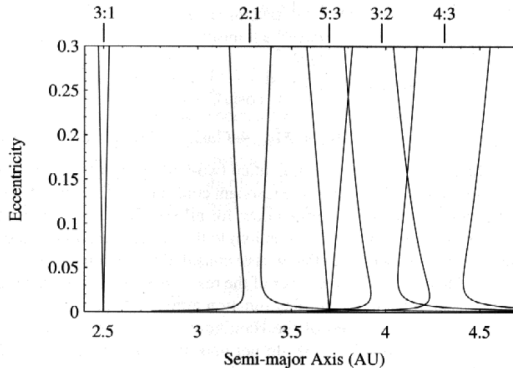


# Janus - Epimetheus 1:1



# (134340) Pluto in exterior resonance 2:3



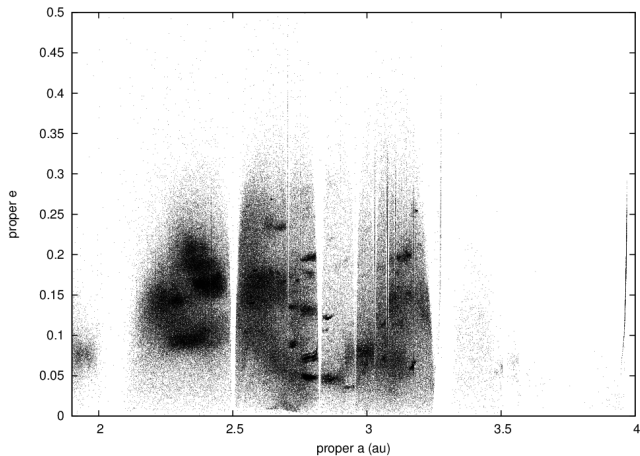


Murray and Dermott in Solar System Dynamics

## Chaos:

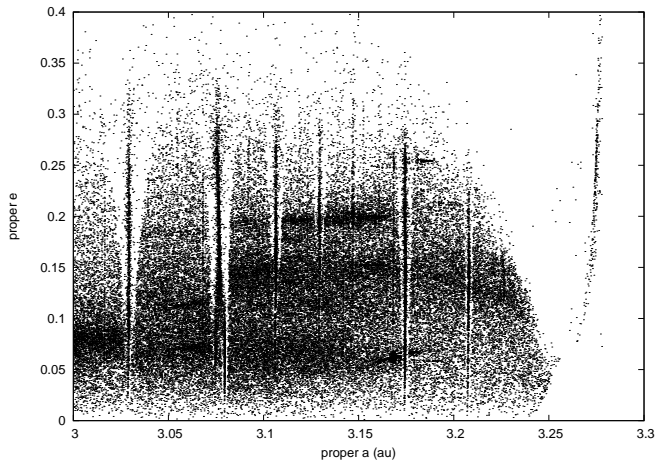
- at resonance borders
- superposition of resonances

# 350.000 asteroids (proper elements)



AstDyS database

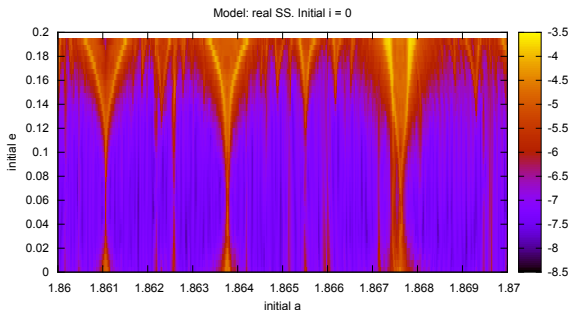
# Resonant structure (zoom)



AstDyS database

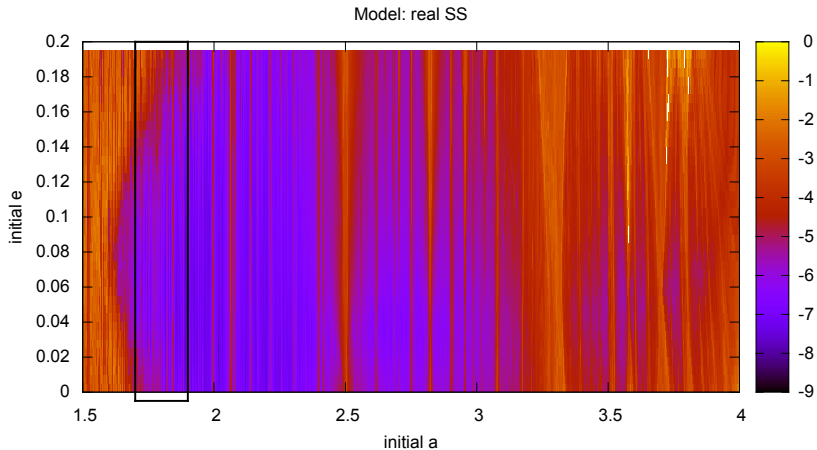
# Dynamical Maps

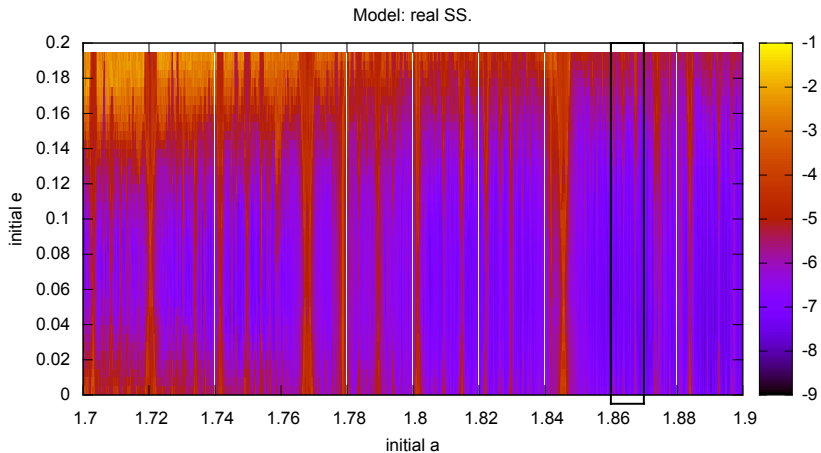
- take set of initial values  $(a, e)$
- integrate for some 10.000 yrs
- surface (color) plot of  $\Delta a(a, e)$

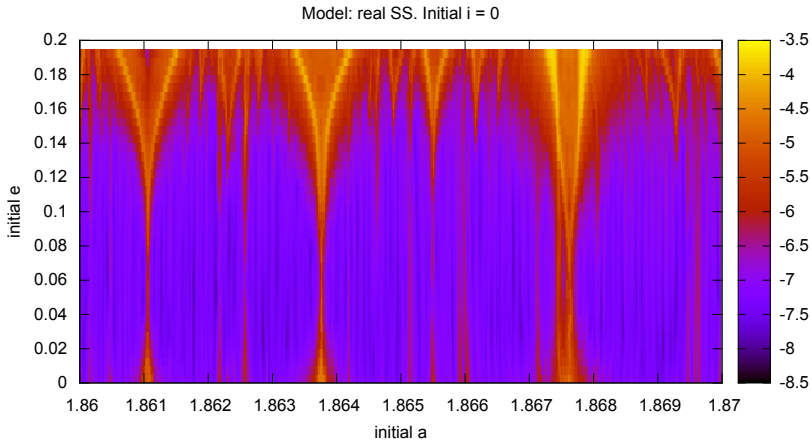




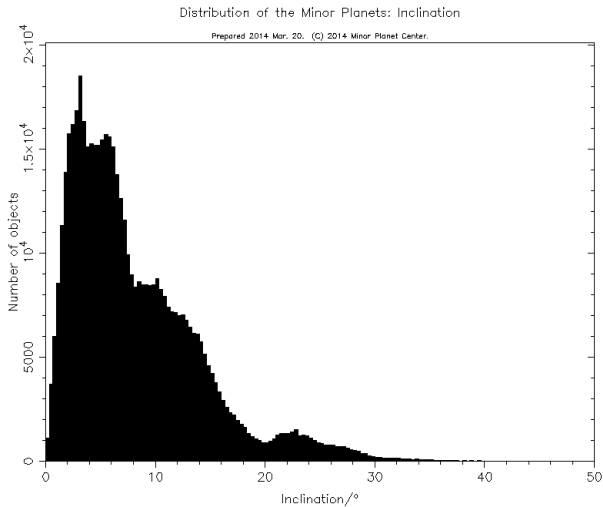
# Asteroid region



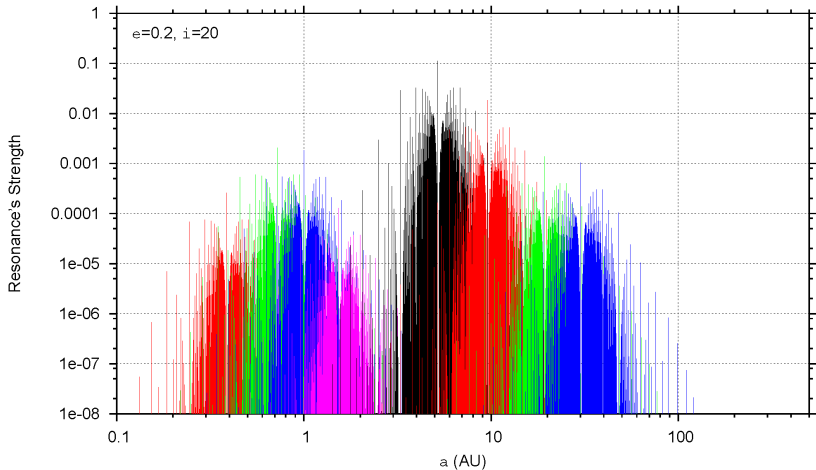




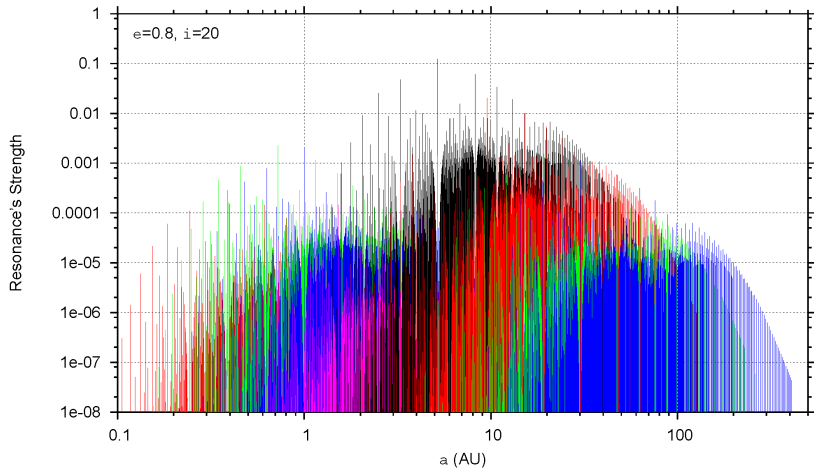
# Orbital inclinations of asteroids



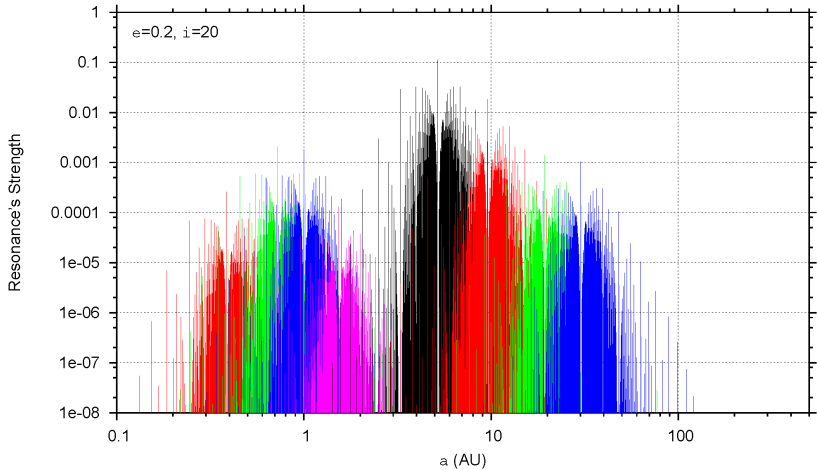
# Atlas of resonances in the Solar System, low $e$



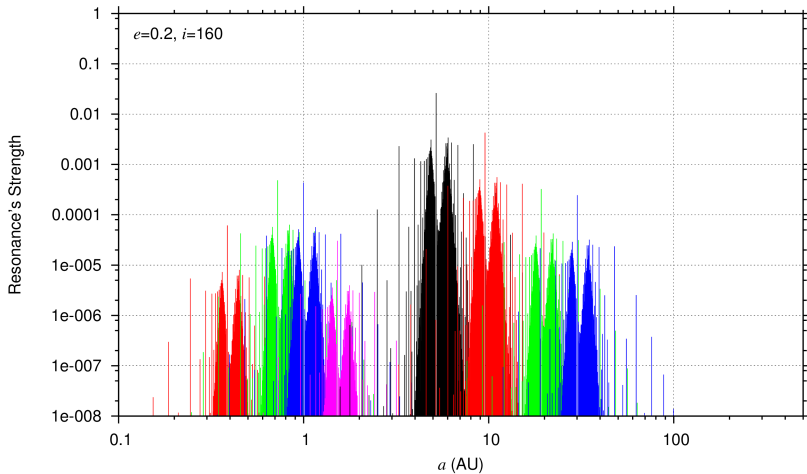
# Atlas of resonances in the Solar System, high $e$



# Atlas for DIRECT orbits

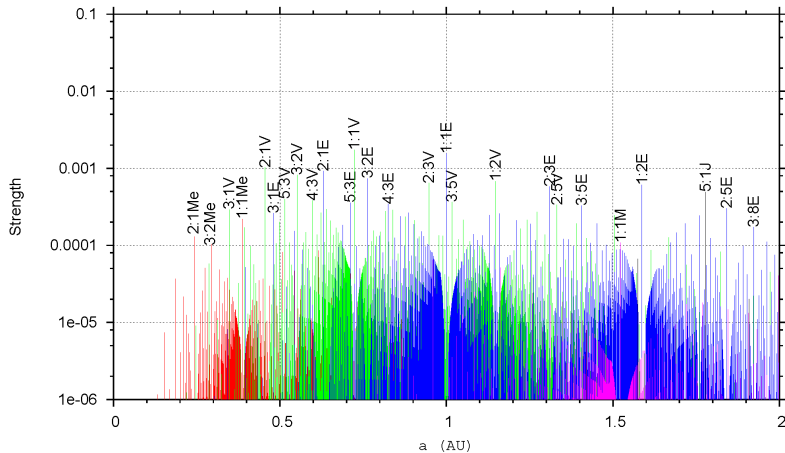


# Atlas for RETROGRADE orbits



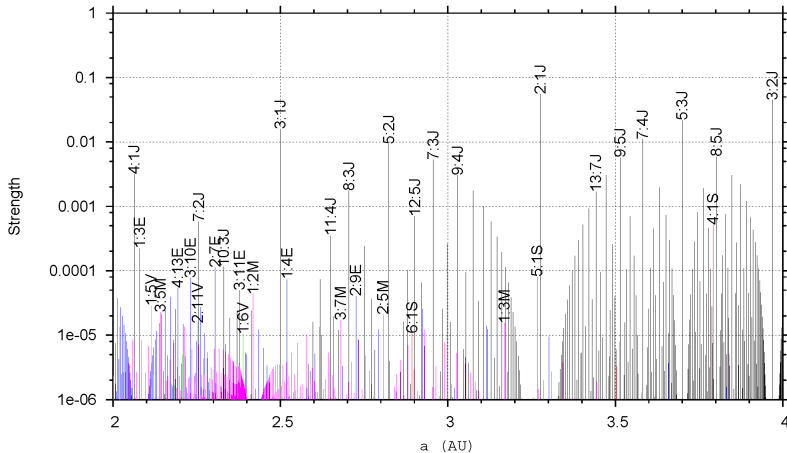


# Atlas from 0 to 2 au



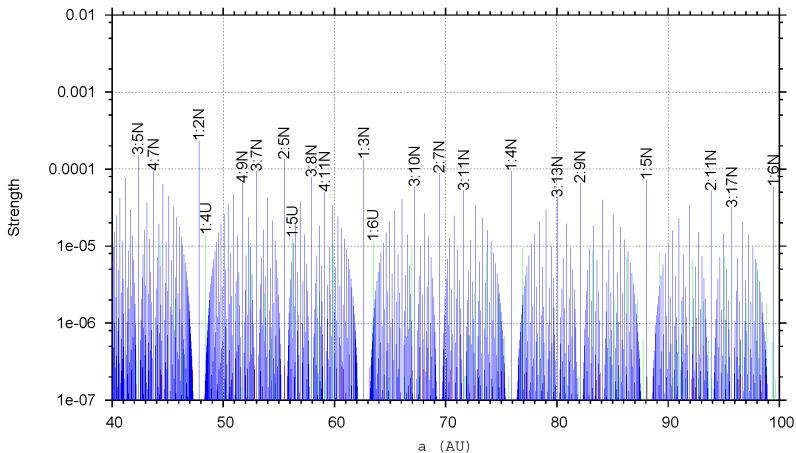
Gallardo 2006

# Atlas in the asteroids region



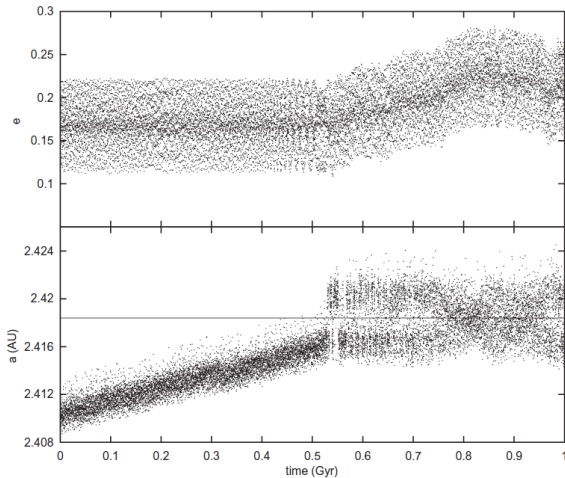
Gallardo 2006

# Atlas in the trans-Neptunian Region



Gallardo 2006

# Stickiness: ability to capture particles

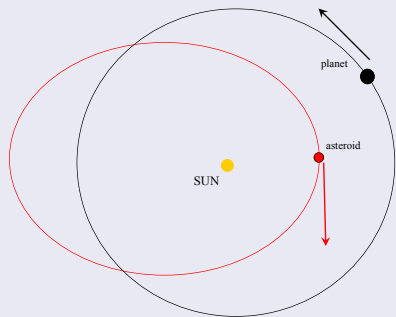


Gallardo et al. 2011

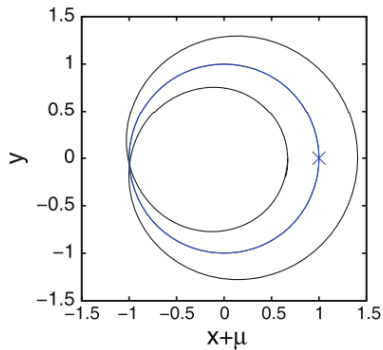
# Bizarre worlds...

# Coorbital retrograde

## heliocentric motion



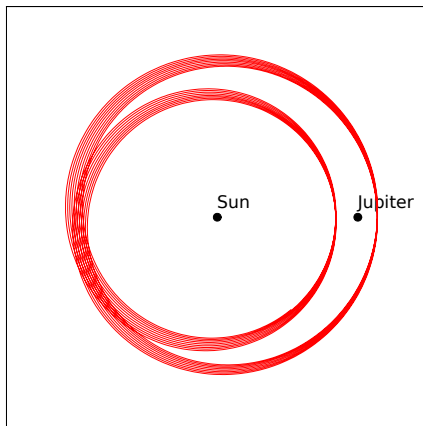
## relative motion



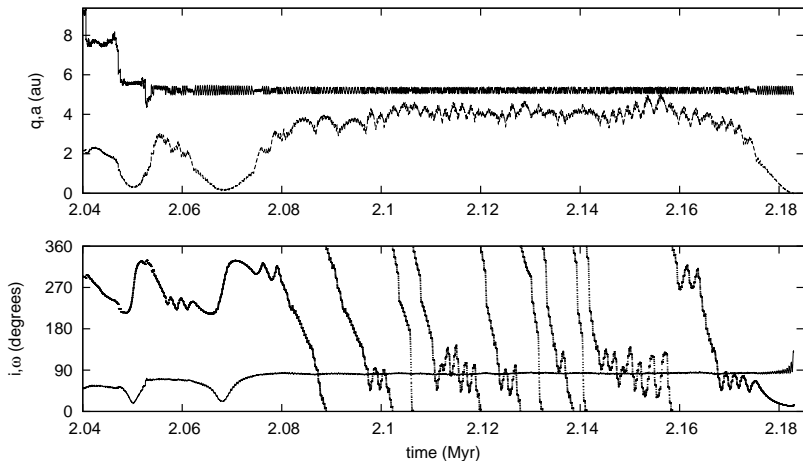
Morais and Namouni 2013

# 2015 BZ509: discovered in January 2015

$$a = 5.12 \text{ au}, e = 0.38, i = 163^\circ$$



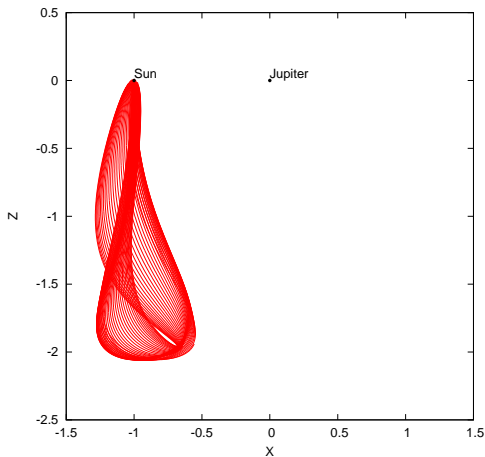
# Bizarre extreme: fictitious particle in resonant polar orbit



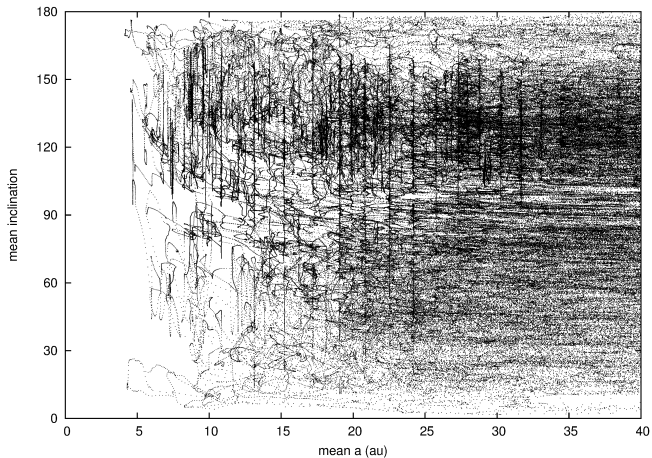


# Bizarre extreme: fictitious particle in resonant polar orbit

collision with the Sun in the rotating frame

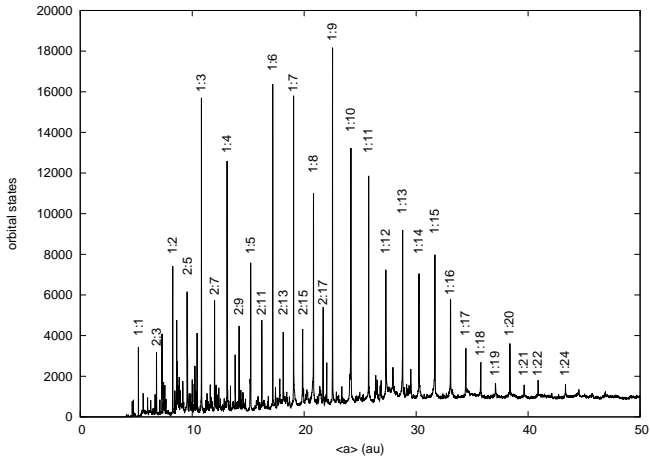


# Resonances of Long Period Comets



Fernandez et al. 2016

# Resonances of Long Period Comets

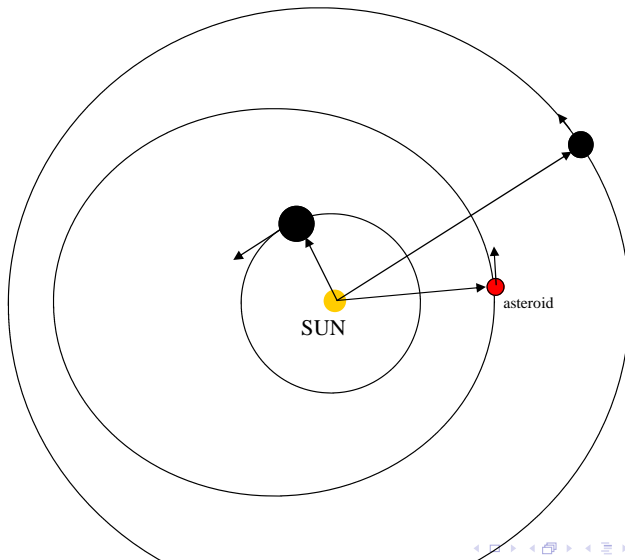


Fernandez et al. 2016

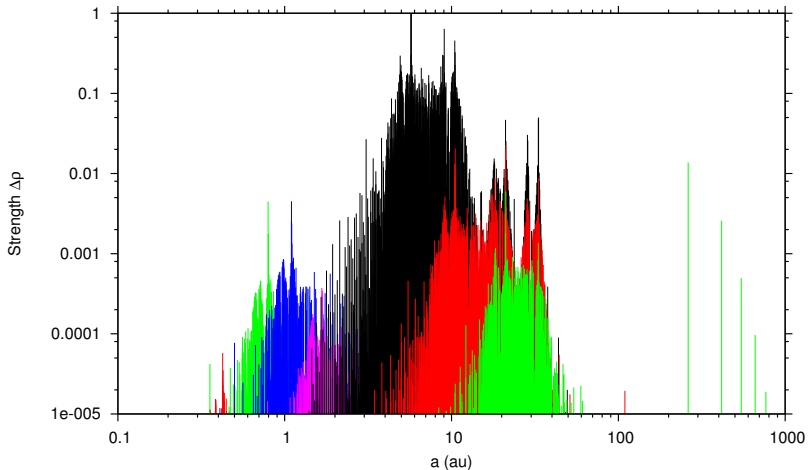
# Three-body resonance

$$k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0$$

only the asteroid feels the resonance

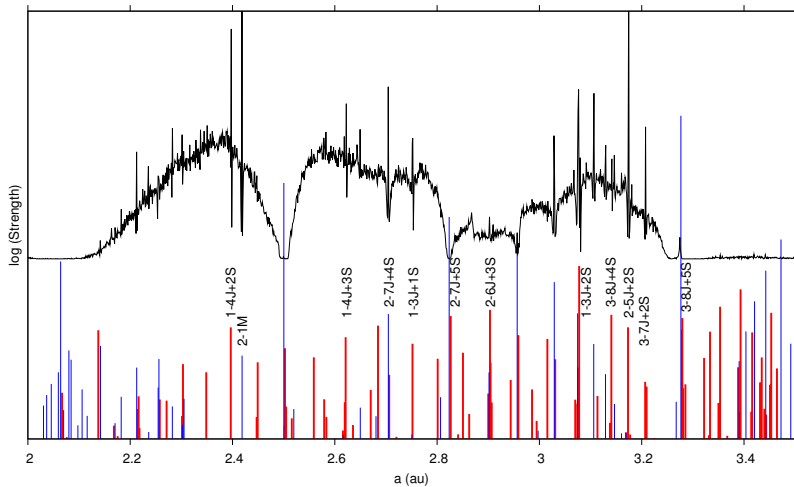


# Atlas of TBRs: global view (for $e = 0.15$ )



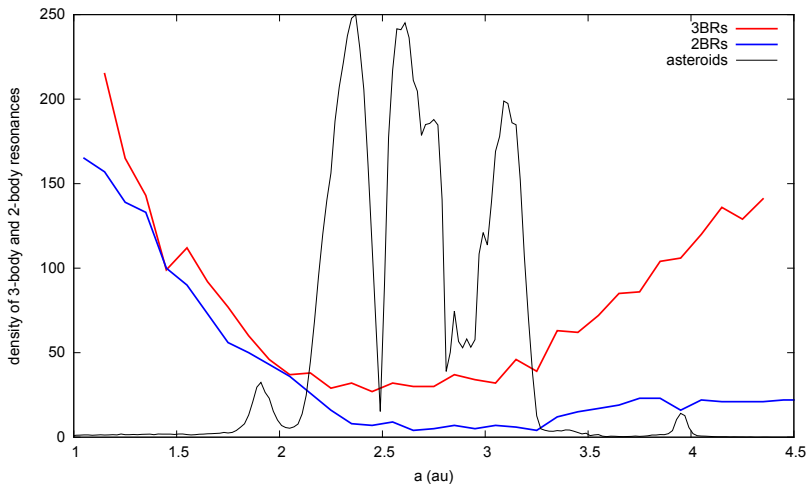
Gallardo 2014

# Effects on the distribution of asteroids



Gallardo 2014

# Density of resonances versus density of asteroids

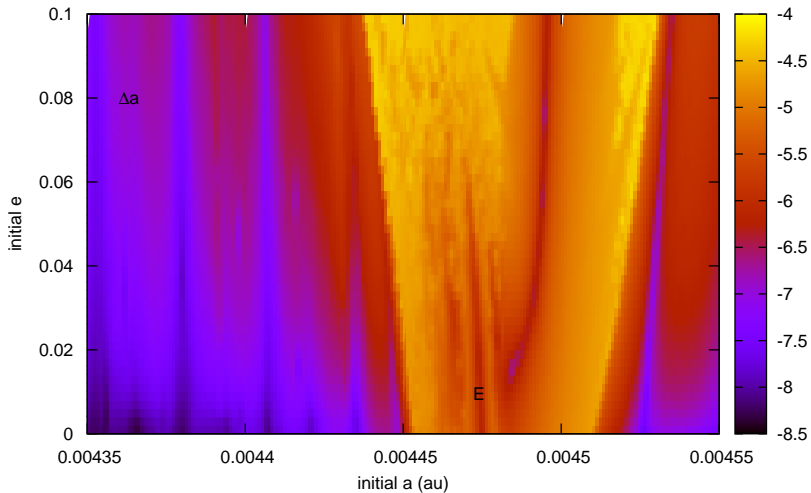


# Galilean satellites





# Europa: dynamical map



Gallardo 2016

- 1772: Lagrange equilibrium points
- 1799: Lagrange planetary equations, stability of the SS
- 1784: Laplacian resonance  $3\lambda_E - \lambda_I - 2\lambda_G \simeq 180^\circ$
- Laplace quasi resonances: great inequality  $2\lambda_{Jup} - 5\lambda_{Sat}$
- 1846: Neptune discovered
- 1866 Kirkwood gaps
- 1875: first resonant asteroid (153) Hilda 3:2
- 1882: secular resonances (Tisserand,  $\nu_6$ )
- 1906: first Trojan asteroid (588) Achilles
- 1930: Pluto and exterior resonance 2:3 Neptune-Pluto
- 1962: Lidov-Kozai mechanism
- 1993: resonant TNOs (2:3 plutino)
- planetary systems in 2BRs and 3BRs
- minor bodies in 3BRs
- high inclination resonant orbits
- retrograde resonances

- *Efectos dinámicos de las resonancias orbitales en el Sistema Solar*. Gallardo 2016, BAAA 58, 291.
- *Resonances in the asteroid and TNO belts: a brief review*. Gallardo 2018, Planetary and Space Science 157, 96.
- <http://www.fisica.edu.uy/~gallardo/atlas/>