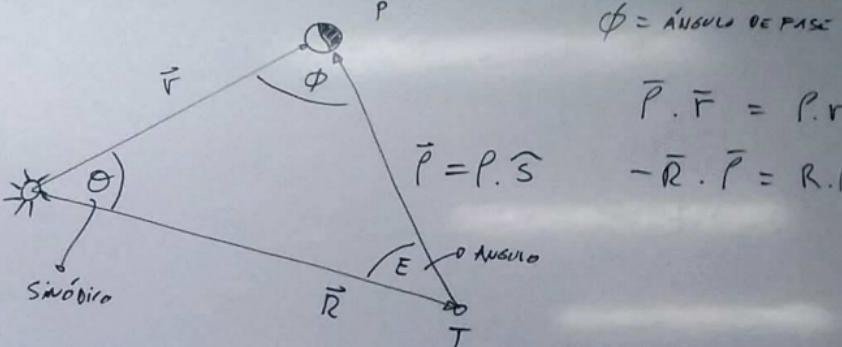


Movimiento Aparente Planetario (^{órbitas circulares:} $\vec{R} = \text{un. } \hat{R}$, $F = a \cdot F$)

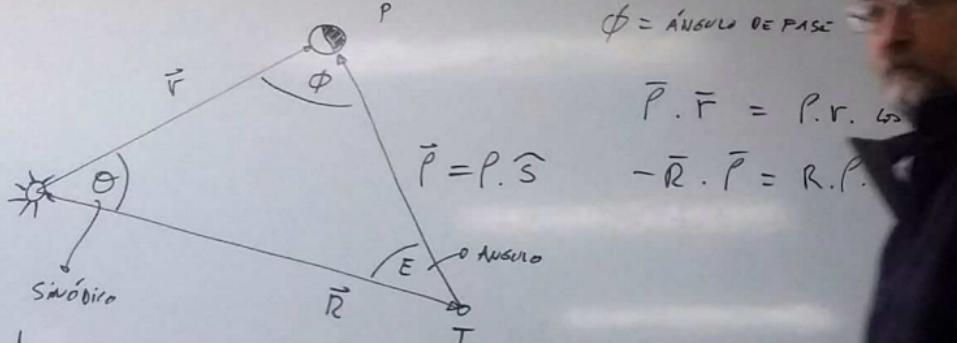


ϕ = ÁNGULO DE FASE

$$\vec{P} \cdot \vec{F} = P \cdot r \cdot \omega \phi$$

$$-\vec{R} \cdot \vec{P} = R \cdot P \cdot \omega E$$

Movimiento Aparente Planetario (Órbitas circulares:
 $\vec{R} = r_m \hat{r}$ ó $\vec{r} = R_m \hat{r}$)



$$\theta(t) \propto t$$

ϕ = ÁNGULO DE FASE

$$\bar{P} \cdot \bar{F} = P_r r \omega$$

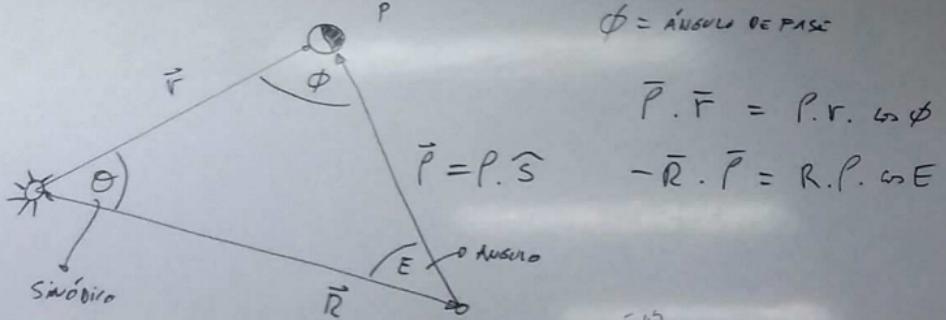
$$-\bar{R} \cdot \bar{P} = R_p P$$

$$M = \sqrt{\mu/a^3} = h/a\omega$$

$$\mu = h^2(M_p + m_p) \approx h^2$$

↑ VEL. ANGULAR
RAD/S/DÍA

Movimiento Aparente Planetario (órbitas circulares:
 $\vec{R} = l m \cdot \hat{\vec{R}}$, $F = a \cdot \vec{F}$)



ϕ = ÁNGULO DE FASE

$$\bar{P} \cdot \vec{F} = P_r r \cos \phi$$

$$-\bar{R} \cdot \bar{P} = R_p P \cos E$$

$$\theta(t) \propto t$$

$$\dot{\theta} = \omega_p - \omega_r = \frac{2\pi}{T_p} - \frac{2\pi}{l m} \Rightarrow \frac{1}{S} = \frac{1}{T_p} - 1$$

$$M = \sqrt{l^3/a^3} = k/a^{3/2}$$

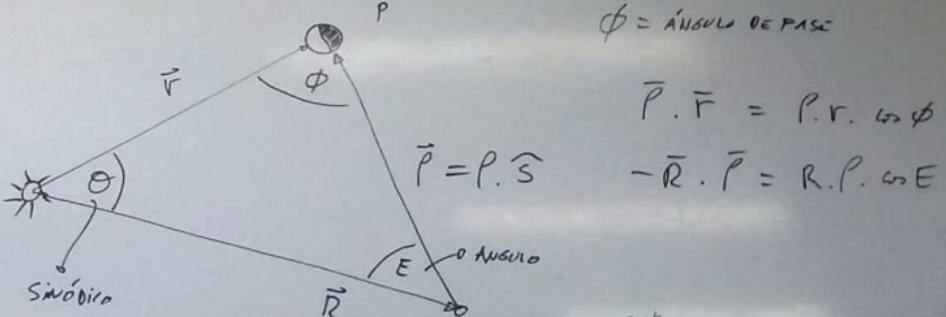
$$k = h^2(m_p + m_r) \approx h^2$$

↑ VEL. ANGULAR
RAD/S/DÍA

$S = \text{Período siderico}$

EN $\frac{1}{T_p}$

MOVIMIENTO APARENTE PLANETARIO (Órbitas circulares: $\vec{R} = t \omega \cdot \hat{R}$, $\vec{F} = a \cdot \hat{F}$)



$$\dot{\theta} = \omega_p - \omega_e = \frac{2\pi}{T_p} - \frac{2\pi}{T_{sin}} \Rightarrow \frac{1}{S} = \frac{1}{T_p} - 1$$

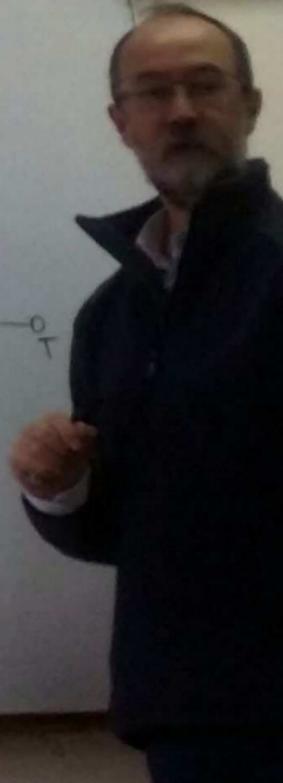
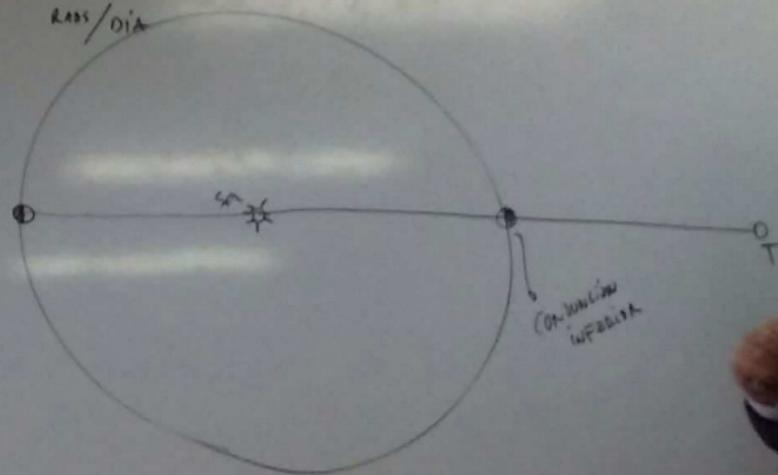
$$M = \sqrt{r/a^3} = \frac{h}{\sqrt{GM}}$$

↑ Vel. angular

Rads/Día

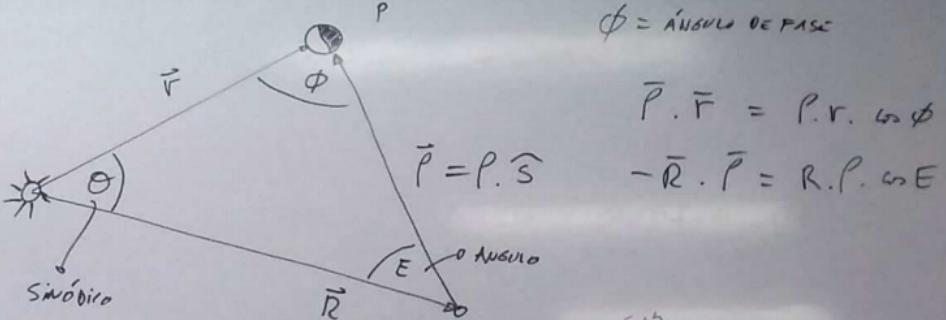
S = Período sideral

Cón. Sup.



MOVIMIENTO APARENTE PLANETARIO (ÓRBITAS CIRCULARES: $\vec{r} = r \omega \hat{R}$, $\vec{F} = a \cdot \vec{F}$)

ϕ = ÁNGULO DE FASE



$$\vec{P} \cdot \vec{F} = P \cdot r \cos \phi$$

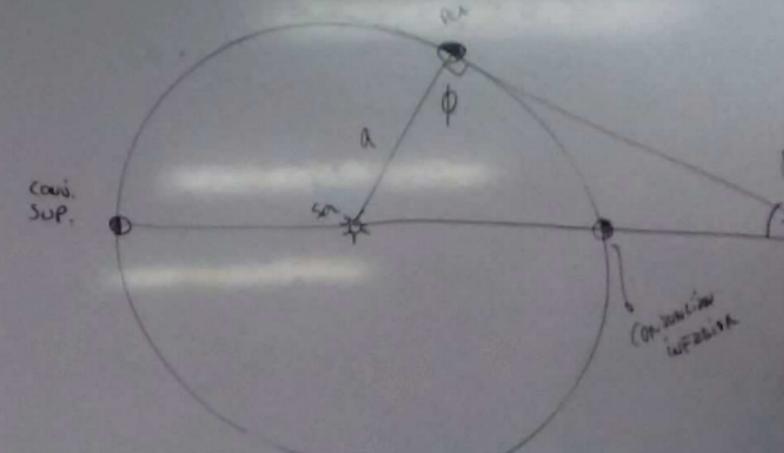
$$-\vec{R} \cdot \vec{P} = R \cdot P \cos E$$

$$\theta(t) \propto t$$

$$\dot{\theta} = M_p - M_T = \frac{2\pi}{T_p} - \frac{2\pi}{T_{anio}} \Rightarrow \left[\frac{1}{S} \right] = \frac{1}{T_p} - 1$$

PLAETA INFERIOR

$$M_p E_{max} = \frac{a}{1}$$

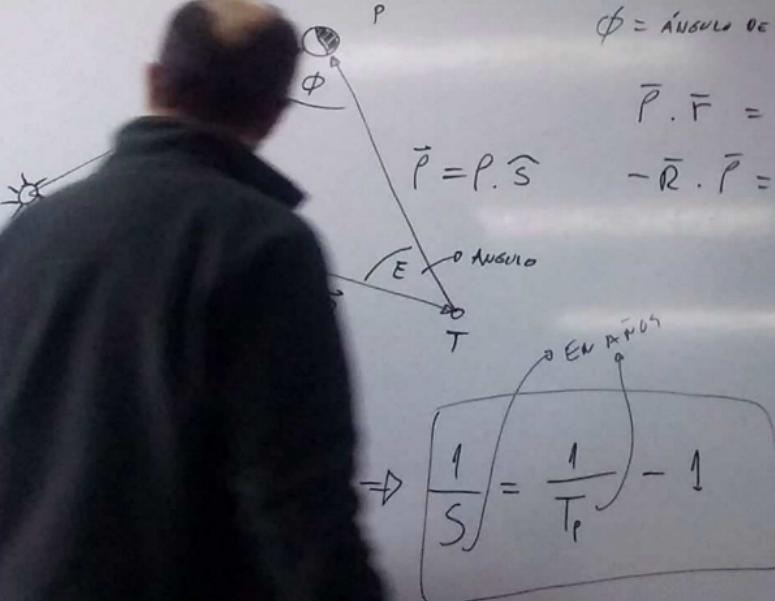


MOVIMIENTO APARENTE PLANETARIO (Órbitas circulares: $\vec{R} = \text{un. } \hat{R}$, $\vec{F} = a \cdot \hat{F}$)

$\phi = \text{ÁNGULO DE FASE}$

$$\bar{P} \cdot \bar{F} = P \cdot r \cdot \omega \phi$$

$$-\bar{R} \cdot \bar{P} = R \cdot P \cdot \omega E$$



PLAETA INFERIOR

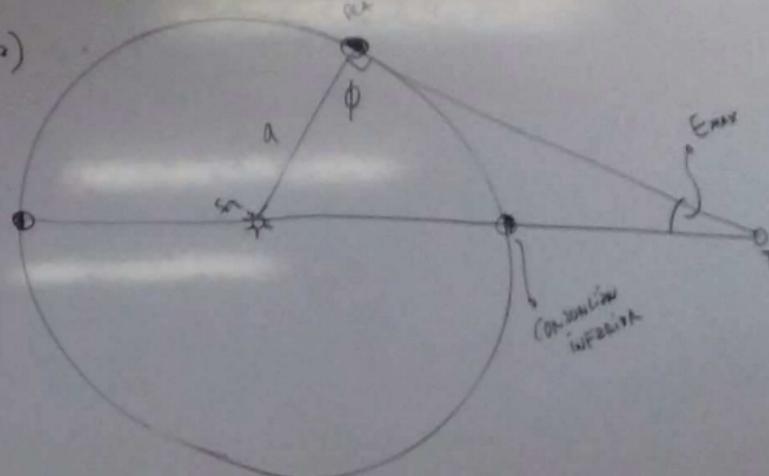
VENUS: $a = 0.723 \text{ UA}$

$\Rightarrow E_{\text{MAX}} = \text{ARCSIN}(0.723)$

46°

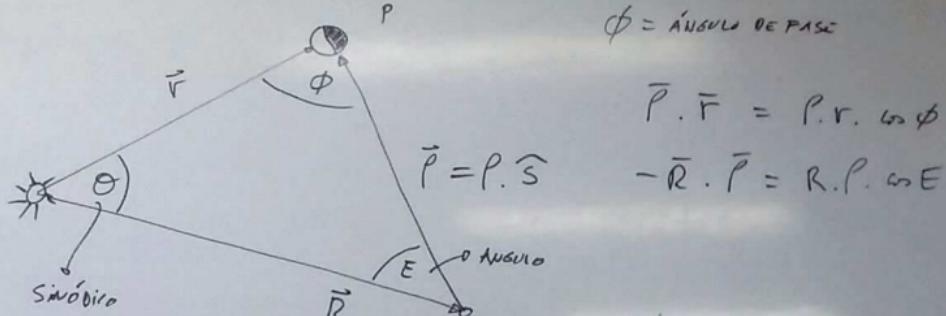
Cou. SUP.

$$\frac{2\pi}{S} =$$



$$M_E E_{\text{MAX}} = \frac{a^2}{1}$$

MOVIMIENTO APARENTE PLANETARIO (ÓRBITAS CIRCULARES: $\vec{r} = r_m \hat{R}$, $\vec{F} = a \cdot \vec{F}$)



$$\theta(t) \propto t$$

$$\Theta = M_p - M_s = \frac{2\pi}{T_p} - \frac{2\pi}{T_{solo}} \Rightarrow \frac{2\pi}{S} = \frac{1}{T_p} - \frac{1}{T_{solo}}$$

$$\left(\frac{1}{S} \right) = \left(\frac{1}{T_p} \right) - 1$$

PLAETA INFERIOR

VENUS: $a = 0.723 \text{ UA}$

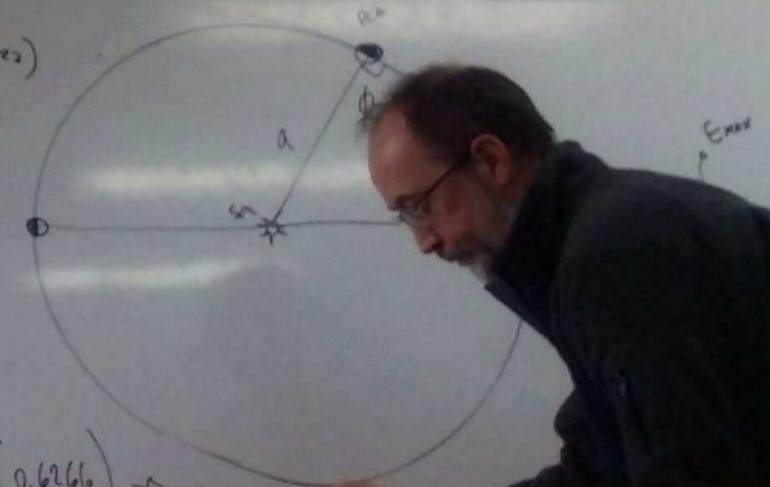
$\Rightarrow E_{max} = \arcsen(0.723)$

46°
Caud. SUP.

$$\frac{2\pi}{S} = \frac{k}{a^m} - \frac{k}{1}$$

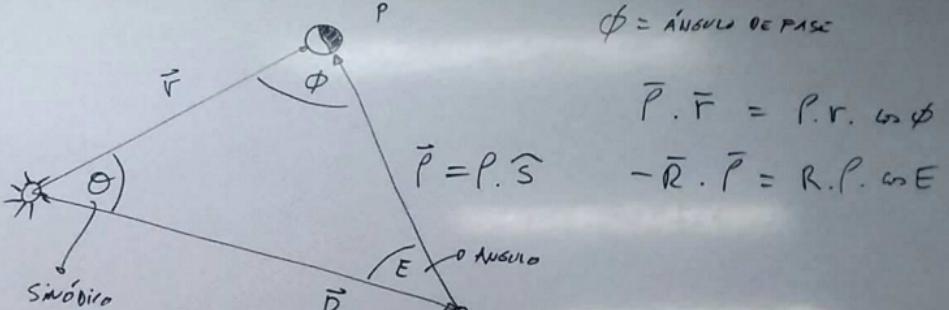
$$\frac{2\pi}{S} = \frac{k}{(0.723)^m} - k = k(0.6266) \Rightarrow \frac{1}{S} =$$

$$M_p E_{max} = \frac{a^m}{1}$$



MOVIMIENTO APARENTE PLANETARIO (Órbitas circulares: $\vec{r} = r \omega \cdot \hat{R}$, $\vec{F} = a \cdot \vec{F}$)

ϕ = ÁNGULO DE FASE



$$\theta(t) \propto t$$

$$\dot{\theta} = M_p - M_T = \frac{2\pi}{T_p} - \frac{2\pi}{T_{\text{ano}}} \Rightarrow \frac{2\pi}{S} = \frac{1}{T_p} - \frac{1}{T_{\text{ano}}}$$

$$\frac{1}{S} = \frac{1}{T_p} - 1$$

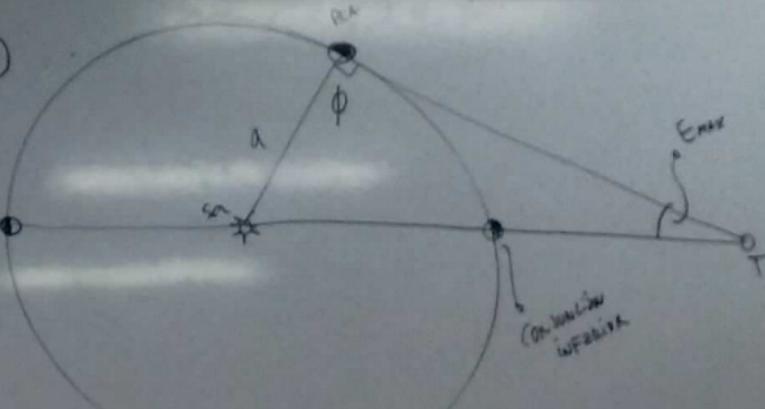
PLAETA INFERIOR

VENUS: $a = 0.723 \text{ UA}$

$\Rightarrow E_{\text{max}} = \arccos(0.723)$

46°

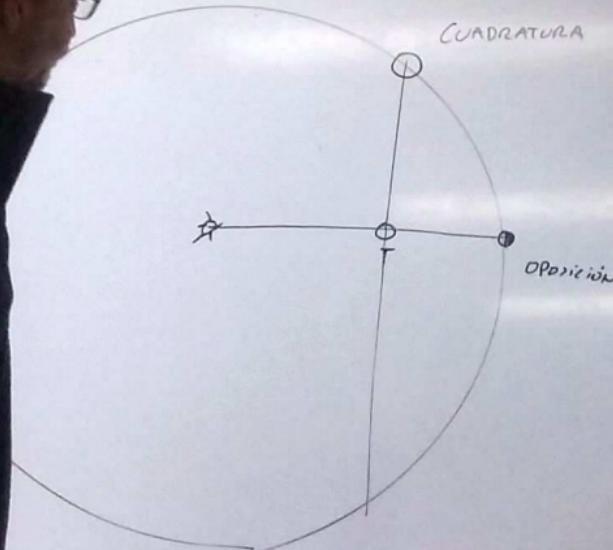
Caud. SUP.



$$\frac{2\pi}{S} = \frac{k}{a^{2n}} - \frac{k}{1}$$

$$\frac{2\pi}{S} = \frac{k}{0.723^{2n}} - k = k(0.6266) \Rightarrow \frac{1}{S} = 1.7 \times 10^{-3} \text{ días}^{-1} \Rightarrow S = 585 \text{ días}$$

MENSAJE APARENTE PLANETARIO (órbitas circulares:
 $\pi = 1m \cdot R$, $F = a \cdot F$)



PLAETA INFERIOR

VENUS: $a = 0.723 \text{ UA}$

$E_{\max} = \arcsen(0.723)$

46°

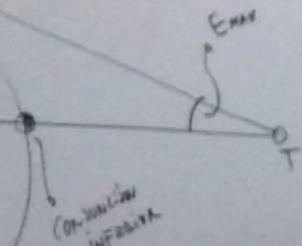
Cou.
SUP.

$$\frac{2\pi}{S} = \frac{k}{a^{3/2}} - \frac{k}{1}$$

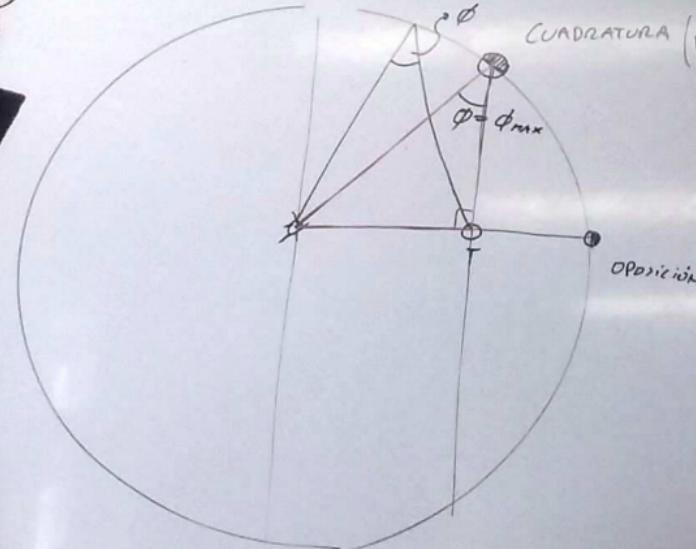
v_{Venus}

$$\frac{2\pi}{S} = \frac{k}{0.723^{3/2}} - k = k(0.6266) \Rightarrow \frac{1}{S} = 1.7 \times 10^{-3} \text{ días}^{-1} \Rightarrow S = 585 \text{ días}$$

$$E_{\max} = \frac{a}{1}$$



MOVIMIENTO APARENTE PLANETARIO (órbitas circulares: $\vec{r} = r \omega \cdot \hat{R}$, $F = a \cdot F$)



PLAETA INFERIOR

VENUS: $a = 0.723$ UA

$\Rightarrow E_{\max} = \arccos(0.723)$

46°

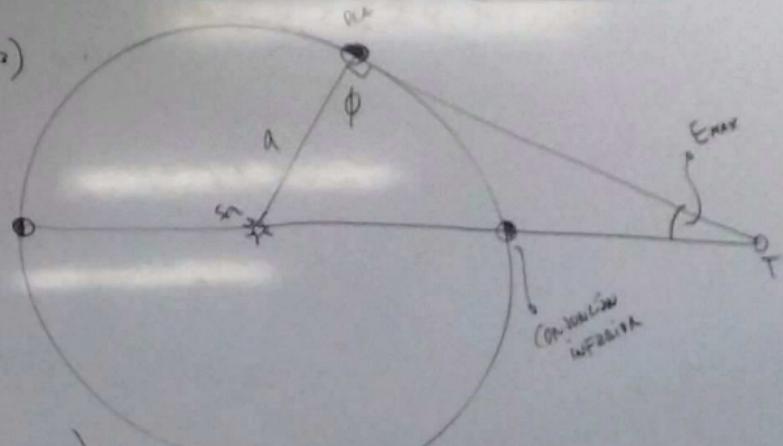
Casi. SUP.

$$\frac{2\pi}{S} = \frac{k}{a^{3/2}} - \frac{k}{1}$$

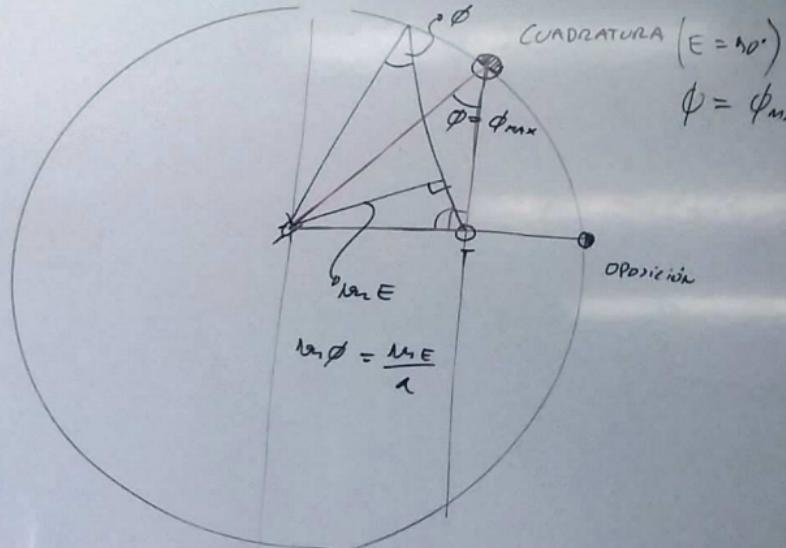
v_{Venus}

$$\frac{2\pi}{S} = \frac{k}{0.723^{3/2}} - k = k(0.6266) \Rightarrow \frac{1}{S} = 1.7 \times 10^{-3} \text{ dist}^{-1} \Rightarrow S = 585 \text{ días}$$

$E_{\max} = \frac{a}{1}$



MOVIMIENTO APARENTE PLANETARIO (órbitas circulares:
 $\vec{r} = r \omega \cdot \hat{R}$, $\vec{F} = a \cdot \vec{F}$)



PLAETA INFERIOR

VENUS: $a = 0.723$ UA

$$\Rightarrow E_{\max} = \arccos(0.723)$$

46°

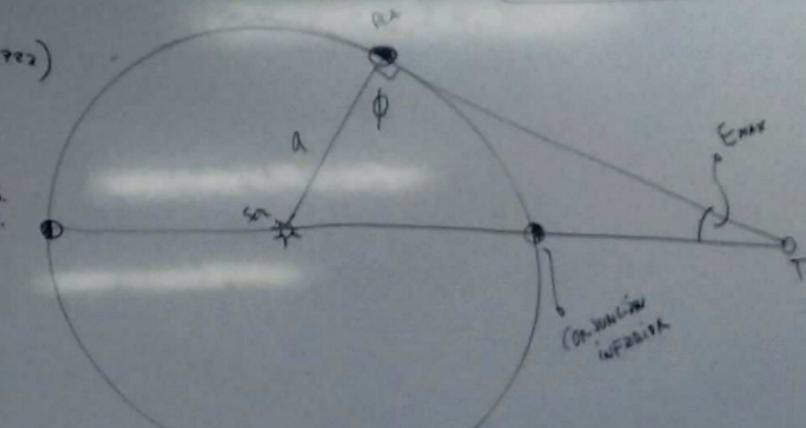
Cou.
SUP.

$$\frac{2\pi}{S} = \frac{k}{a^{3/2}} - \frac{k}{1}$$

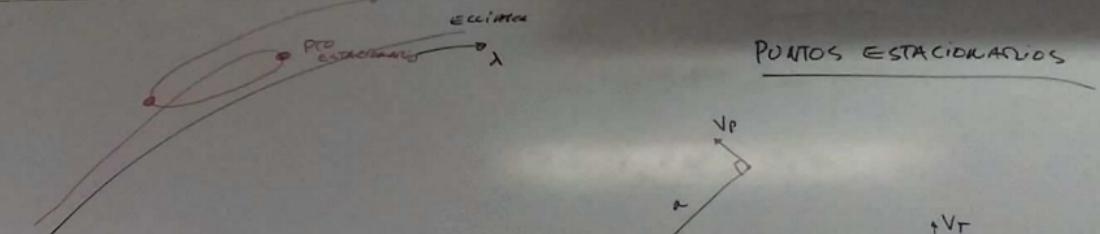
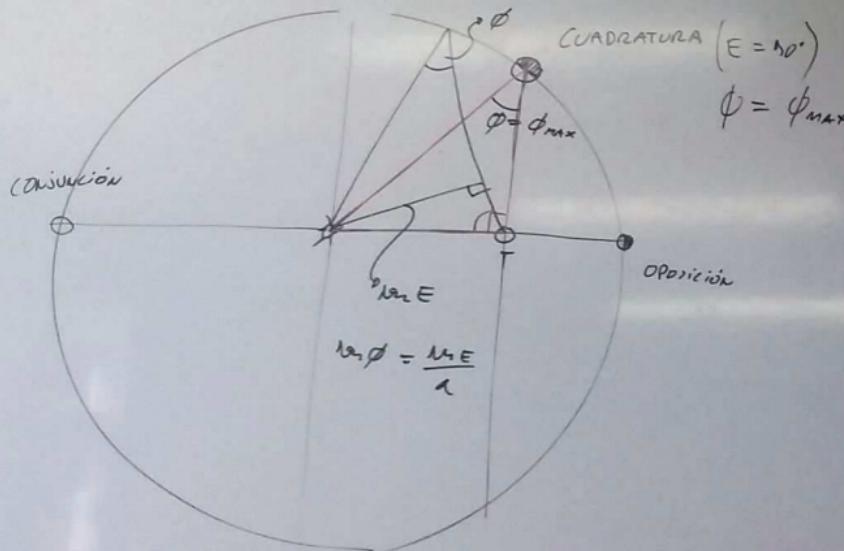
\uparrow
 v_{enus}

$$\frac{2\pi}{S} = \frac{k}{0.723^{3/2}} - k = k(0.6266) \Rightarrow \frac{1}{S} = 1.7 \times 10^{-3} \text{ días}^{-1} \Rightarrow S = 585 \text{ días}$$

AN. E_{max} = $\frac{a}{1}$



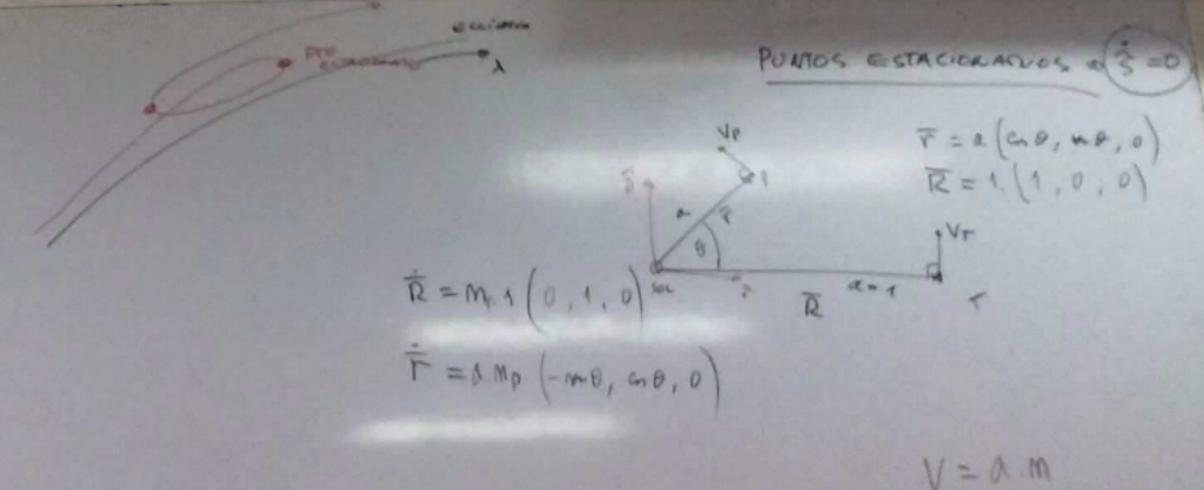
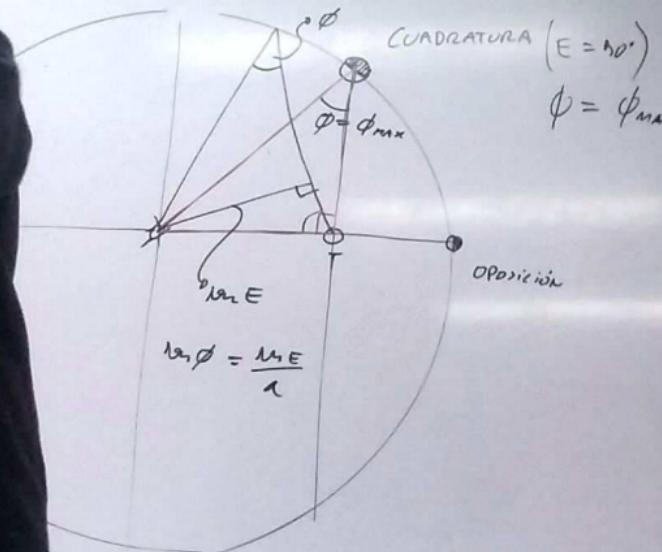
Movimiento Aparente Planetario (órbitas circulares: $\pi = 1m \cdot R$, $F = a \cdot F$)



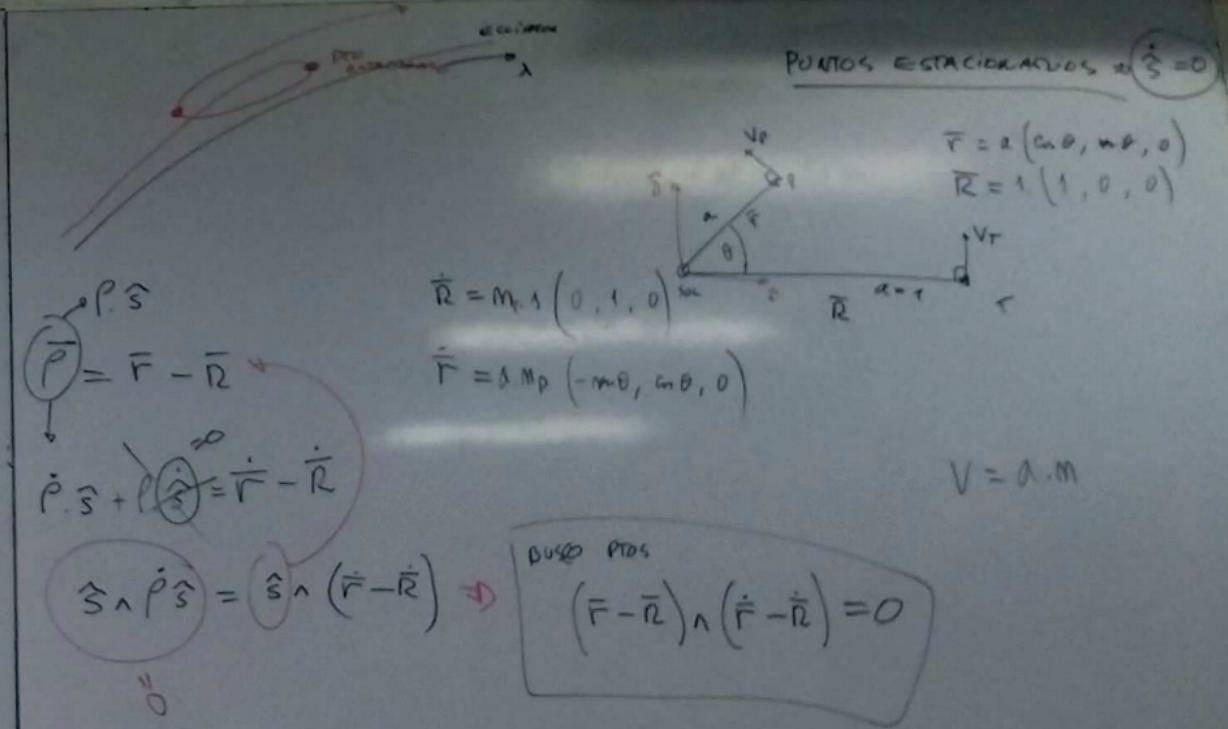
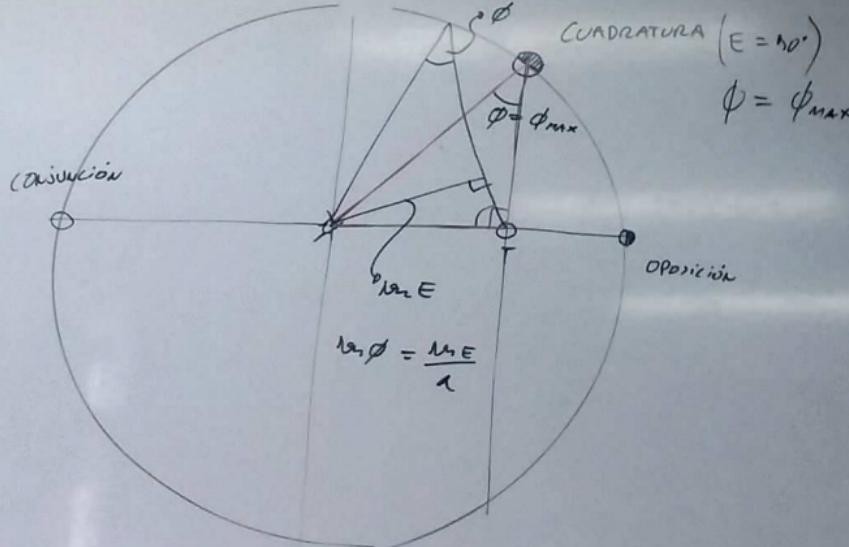
$$V = a \cdot$$

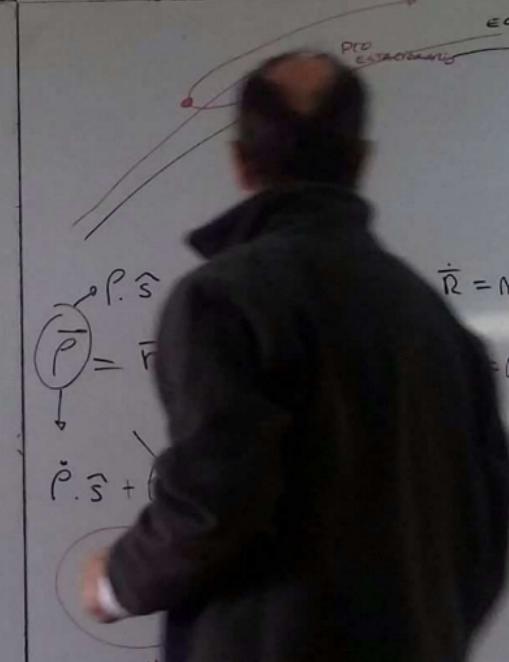
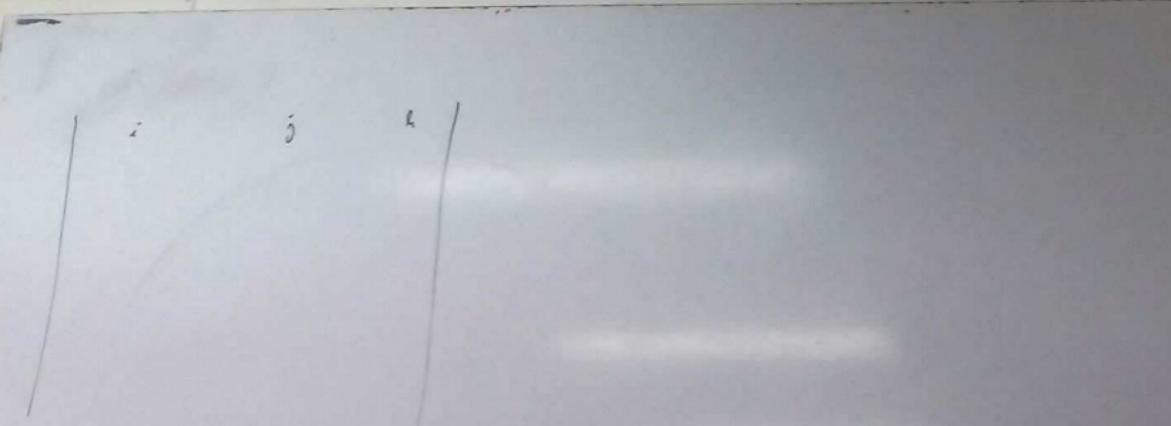


MOVIMIENTO APARENTE PLANETARIO (órbitas circulares:
 $\vec{R} = \text{un. } \hat{R}$, $\vec{F} = a \cdot \vec{r}$)



MOVIMIENTO APARENTE PLANETARIO (órbitas circulares: $\vec{R} = \text{un. } \hat{R}$, $\vec{F} = a \cdot \vec{r}$)





$$\overset{\circ}{P} = \overset{\circ}{r}$$

$$\overset{\circ}{P} +$$

$$\dot{\overset{\circ}{r}} = M_{\text{Pl}} \cdot \begin{pmatrix} 0, 1, 0 \end{pmatrix}$$

$$= \alpha \cdot M_p (-m \cos \theta, m \sin \theta, 0)$$

PUNTOS ESTACIONARIOS $\Rightarrow \dot{\overset{\circ}{r}} = 0$

$$\overset{\circ}{r} = a \begin{pmatrix} \cos \theta, \sin \theta, 0 \end{pmatrix}$$

$$\overset{\circ}{r} = 1 \cdot (1, 0, 0)$$

$$\overset{\circ}{r} = a \begin{pmatrix} 1, 0, 0 \end{pmatrix}$$

$$a = 1$$

$$\overset{\circ}{r} = a \begin{pmatrix} 1, 0, 0 \end{pmatrix}$$

$$a = 1$$

$$V = \alpha \cdot M$$

BUSCO PTOS

$$(\overset{\circ}{r} - \overset{\circ}{r}) \wedge (\dot{\overset{\circ}{r}} - \dot{\overset{\circ}{r}}) = 0$$

$$= \hat{k} \left[(M_p \cos \theta - 1) \sqrt{M_p \cos^2 \theta - M_r} + M_p v_r^2 \theta \right] = 0$$

PUNTOS ESTACIONARIOS $\dot{\zeta} = 0$

$$\vec{F} = a (\cos \theta, \sin \theta, 0)$$

$$\vec{R} = a (1, 0, 0)$$

$$\vec{F} = \alpha M_p (-\sin \theta, \cos \theta, 0)$$

$$V = \alpha \cdot M$$

$$\vec{P} \cdot \dot{\zeta} = \vec{F} - \vec{R}$$

$$\vec{P} \cdot \dot{\zeta} + \dot{\vec{P}} \cdot \dot{\zeta} = \vec{F} - \vec{R}$$

$$\dot{\zeta} \wedge \vec{P} \cdot \dot{\zeta} = \dot{\zeta} \wedge (\vec{F} - \vec{R}) \Rightarrow$$

DUSO PTOS
 $(\vec{F} - \vec{R}) \wedge (\vec{F} - \vec{R}) = 0$

$$\begin{pmatrix} i & j & \textcircled{1} \\ \alpha_{G\theta-1} & \alpha_{G\theta} & 0 \\ -\alpha_{Mp} v_{n\theta} \alpha_{\theta} & \alpha_{Mp} v_{n\theta} - M_T & 0 \end{pmatrix} = \hat{k} \left[(\alpha_{G\theta-1})(\alpha_{Mp} v_{n\theta} - M_T) + \alpha_{Mp} v_n^2 \alpha_{\theta} \right] = 0$$

$$\cancel{\alpha_{Mp} v_n^2 \alpha_{\theta}} - \cancel{\alpha_{G\theta} M_T} - \cancel{\alpha_{Mp} v_{n\theta} + M_T} + \cancel{\alpha_{Mp} v_{n\theta}^2 \alpha_{\theta}} = 0$$

$$\alpha_{Mp} - (M_T + M_p) \alpha_{G\theta} + M_T = 0$$

Diagrama de órbitas:

PUNTOS ESTACIONARIOS $\dot{s} = 0$

$$m \sqrt{r/v} = \frac{k}{v^2}$$

$$\rightarrow A \frac{k}{v^2}$$

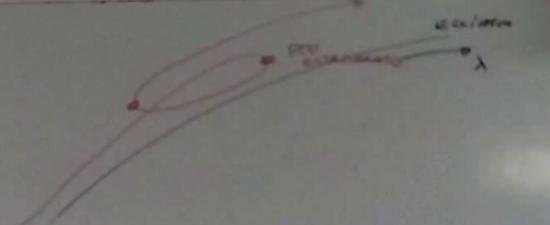


$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -M_p \sin\theta \\ M_p \cos\theta - M_r \end{pmatrix} = 0$$

$$\Rightarrow \left(1 - \left(M_p \cos\theta - M_r \right) + M_p \sin^2\theta \right) = 0$$

$$\Rightarrow M_p \cos^2\theta -$$

$$M_p \sin^2\theta = 0$$



PUNTOS ESTACIONARIOS $\dot{\theta} = 0$

$$M_p \sqrt{r/a} = \frac{\lambda}{\sin^2\theta}$$

$$\Rightarrow \lambda \frac{1}{\sin^2\theta} - \left(\frac{1}{r^2} + \frac{1}{a^2} \right) M_p \cos\theta + \frac{1}{r^2} = 0$$

$$\Rightarrow \lambda^{-1} - \left(1 + \lambda^{-1} \right) M_p \cos\theta + 1 = 0$$

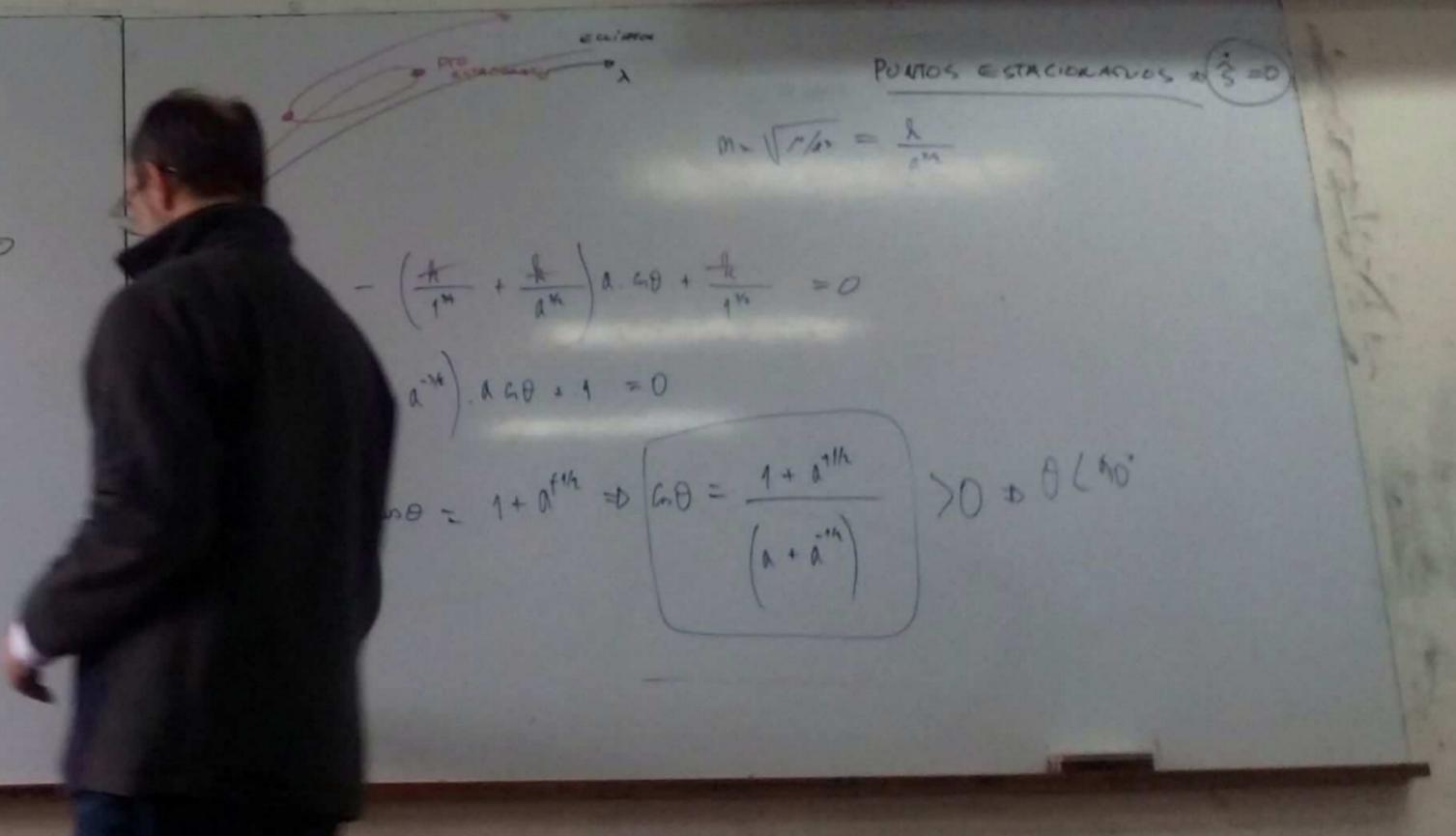
$$\Rightarrow \left(1 + \lambda^{-1} \right) M_p \cos\theta = 1 + \lambda^{-1} \Rightarrow \boxed{M_p \cos\theta = \frac{1 + \lambda^{-1}}{1 + \lambda^{-1}}}$$

$$\begin{array}{c} \text{Diagrama de momentos lineales:} \\ \text{Masa } M_p \text{ en el centro, masa } M_T \text{ en el lado izquierdo.} \\ \text{Fuerzas: } \vec{F}_T = m_T g \hat{i}, \vec{F}_{\text{centro}} = M_p g \hat{i}, \vec{F}_{\text{exterior}} = -M_p g \hat{i} \\ \text{Velocidad angular: } \omega = \dot{\theta} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -M_p g \sin \theta & M_p g \cos \theta - M_T & 0 \end{pmatrix} = \hat{k} \left[(1 - \cos \theta) (M_p g \sin \theta - M_T) + \hat{k}^2 M_p g \sin^2 \theta \right] = 0$$

$$\cancel{M_p \sin^2 \theta} - \cancel{M_p g M_T} - \cancel{M_p g \sin \theta} + M_T + \cancel{\hat{k}^2 M_p g \sin^2 \theta} = 0$$

$$\hat{k} M_p - (M_T + M_p) \cdot g \sin \theta + M_T = 0$$



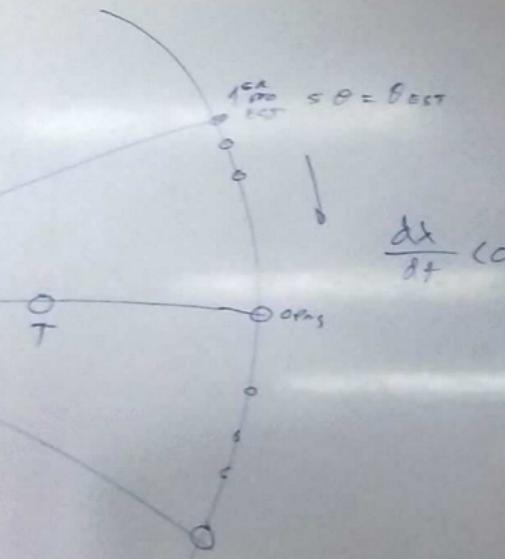


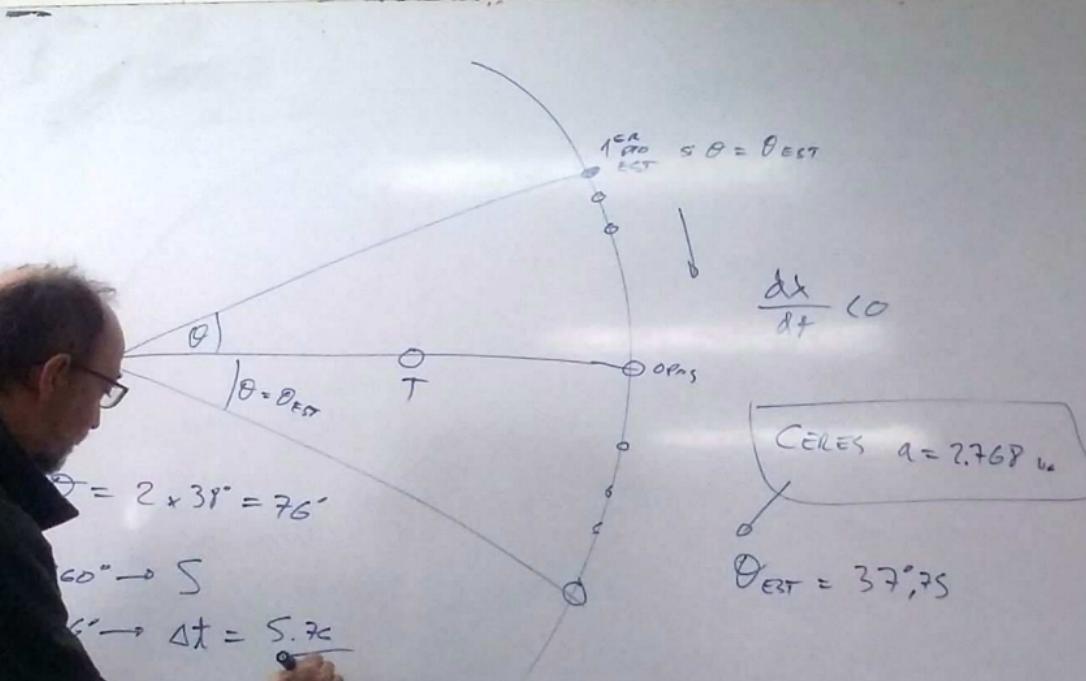
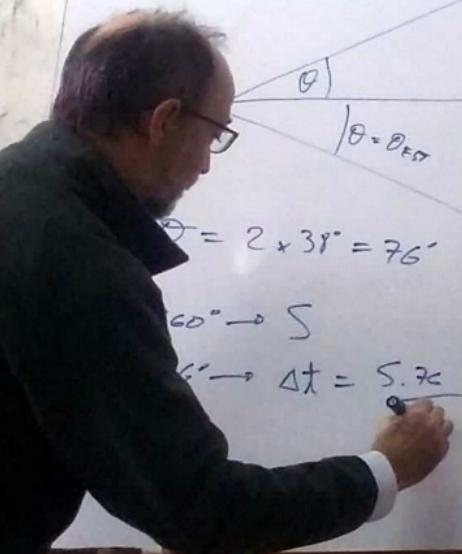
Diagram showing the elliptical orbit of a celestial body with points labeled 1^a, 2^a, 2^r, and 1^r. The Sun is at one focus. The eccentricity vector λ is shown. The text "PUNTOS ESTACIONARIOS $\Rightarrow \dot{s} = 0$ " is written above the equations.

$$m = \sqrt{r/a^3} = \frac{\lambda}{a^{3/2}}$$

$$\Rightarrow a^2 \frac{-k}{a^3 h} - \left(\frac{k}{1^{3/2}} + \frac{k}{a^{3/2}} \right) a \cdot \cos \theta + \frac{k}{1^{3/2}} = 0$$

$$\Rightarrow a^{1/2} - \left(1 + a^{-1/2} \right) \cdot a \cos \theta + 1 = 0$$

$$\Rightarrow \left(a + a^{-1/2} \right) \cdot \cos \theta = 1 + a^{1/2} \Rightarrow \boxed{\cos \theta_{\text{EST}} = \frac{1 + a^{1/2}}{\left(a + a^{-1/2} \right)}} > 0 \Rightarrow \theta < 90^\circ$$



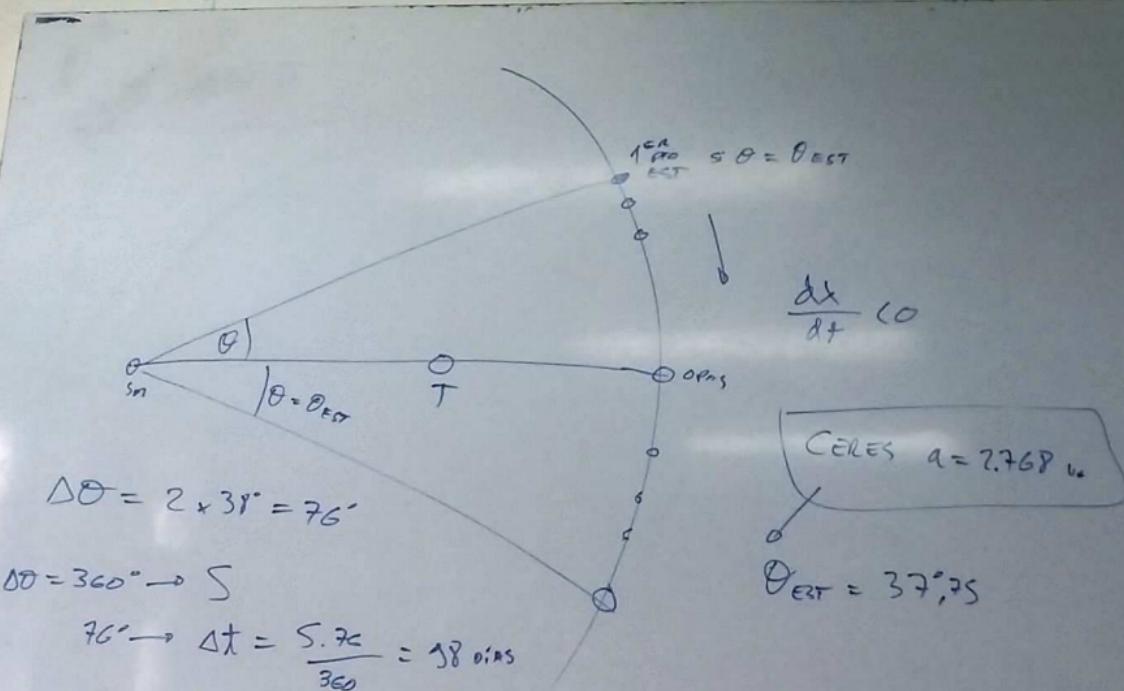
PUNTOS ESTACIONARIOS $\Rightarrow \dot{S} = 0$

$$m \sqrt{r/a} = \frac{\lambda}{a^{3/2}}$$

$$\Rightarrow a^2 \frac{\lambda}{a^{3/2}} - \left(\frac{\lambda}{a^m} + \frac{\lambda}{a^n} \right) a \cdot \omega \theta + \frac{\lambda}{a^m} = 0$$

$$\Rightarrow a^{1/2} - \left(1 + a^{-m} \right) \cdot a \omega \theta + 1 = 0$$

$$\Rightarrow \left(a + a^{-m} \right) \cdot \omega \theta = 1 + a^{1/2} \Rightarrow \boxed{\omega \theta_{EST} = \frac{1 + a^{1/2}}{\left(a + a^{-m} \right)}} > 0 \Rightarrow \theta < 90^\circ$$



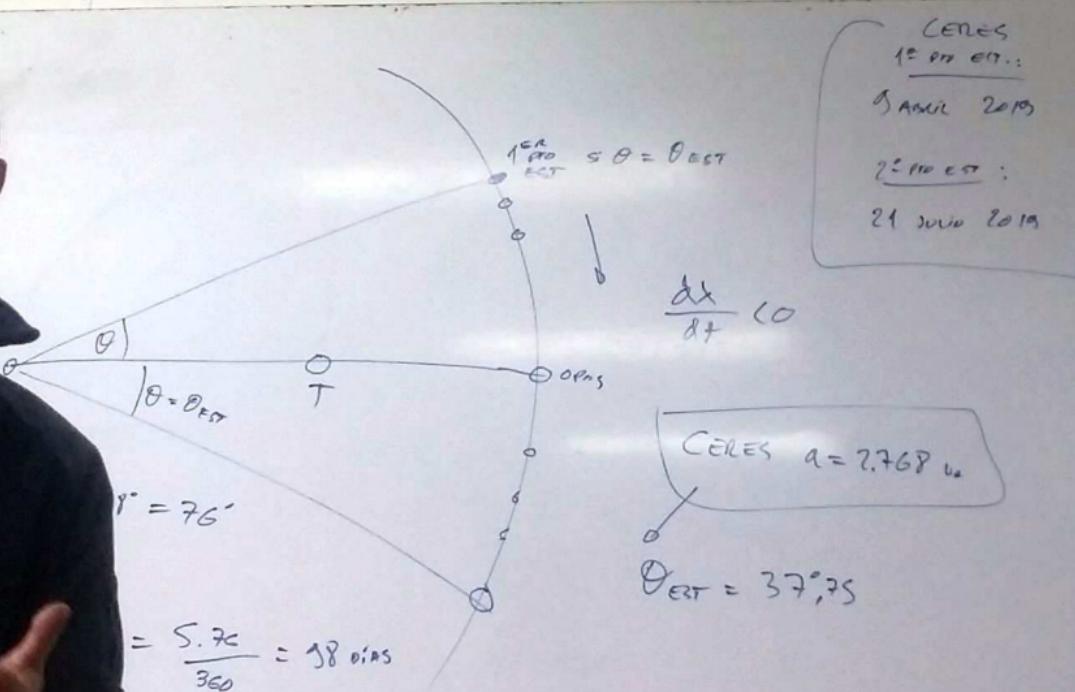
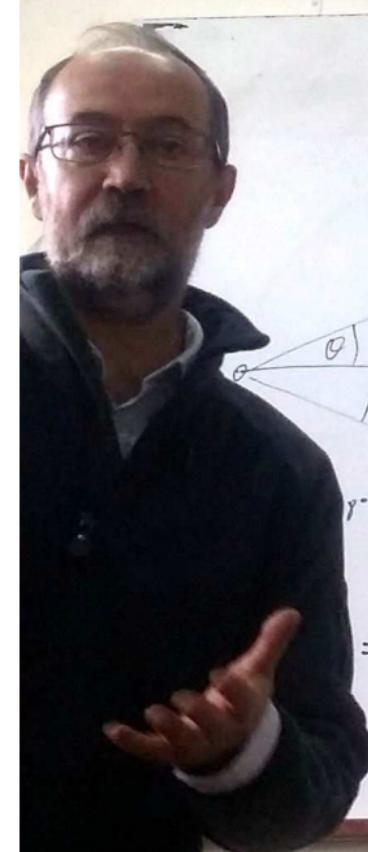
PUNTOS ESTACIONARIOS $\dot{\theta} = 0$

$$m = \sqrt{r/a} = \frac{k}{a^{3/2}}$$

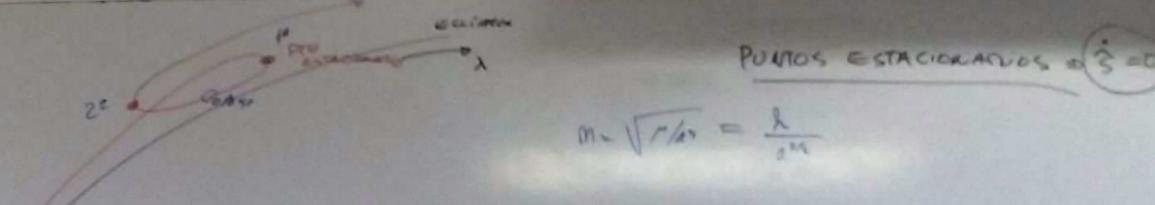
$$\Rightarrow a^2 \frac{k}{a^{3/2}} - \left(\frac{k}{a^m} + \frac{k}{a^{3/2}} \right) a \cdot \omega \theta + \frac{k}{a^{3/2}} = 0$$

$$\Rightarrow a^{1/2} - \left(1 + a^{-1/2} \right) \cdot a \omega \theta + 1 = 0$$

$$\Rightarrow \left(a + a^{-1/2} \right) \cdot \omega \theta = 1 + a^{1/2} \Rightarrow \omega \theta_{est} = \frac{1 + a^{1/2}}{\left(a + a^{-1/2} \right)} > 0 \Rightarrow \theta < 180^\circ$$



1



$$\Rightarrow \frac{a^2 - k}{a^{2k}} - \left(\frac{k}{1^m} + \frac{k}{a^m} \right) a^{-2k} + \frac{-k}{1^{m_k}} = 0$$

$$\Rightarrow a^{+k} - \left(1 + a^{-k}\right) \cdot \lambda \ln \theta + 1 = 0$$

$$\Rightarrow \left(a + a^{\frac{1}{\sin \theta}} \right) \cdot \cos \theta = 1 + a^{\frac{1}{\sin \theta}} \Rightarrow \left| \ln \theta_{\text{ext}} \right| = \frac{1+a}{\left(a + a^{\frac{1}{\sin \theta}} \right)} > 0 \Rightarrow \theta < 0$$

$$m = \sqrt{r/a} = \frac{\lambda}{\omega}$$