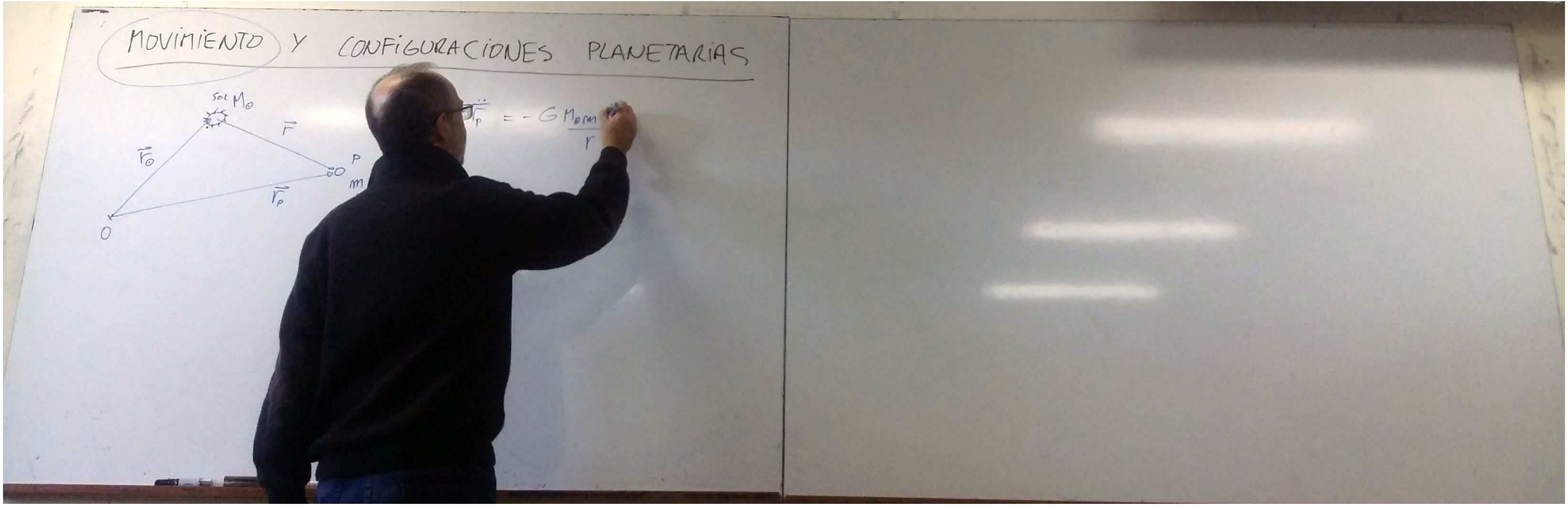
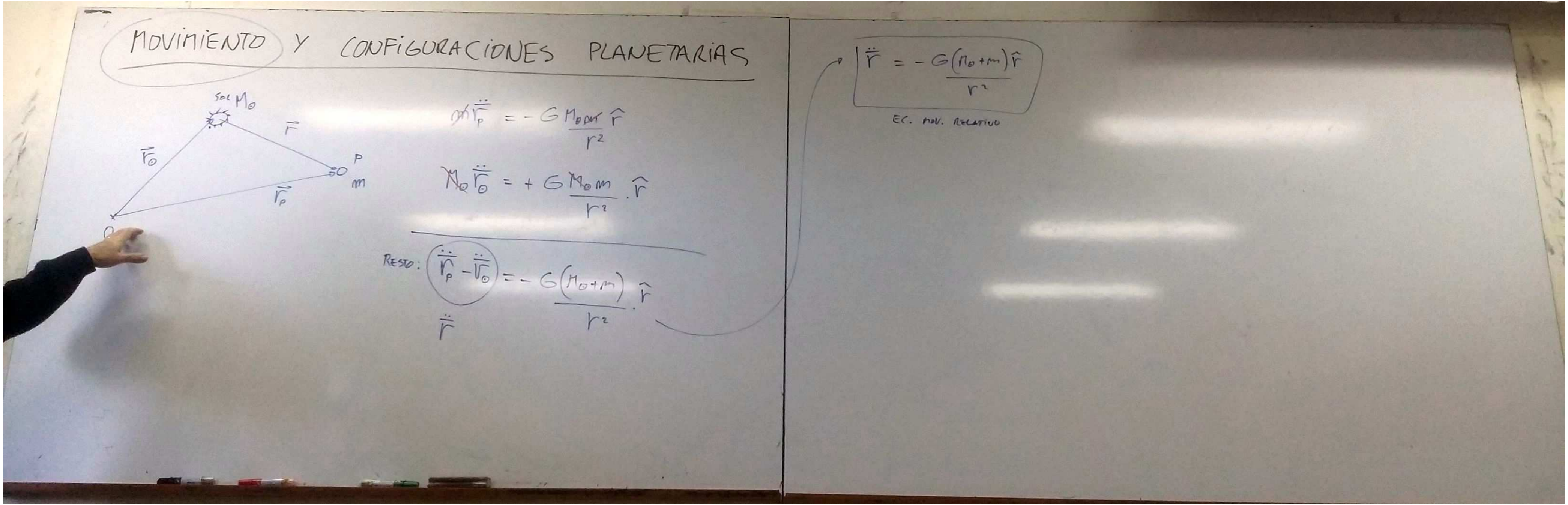
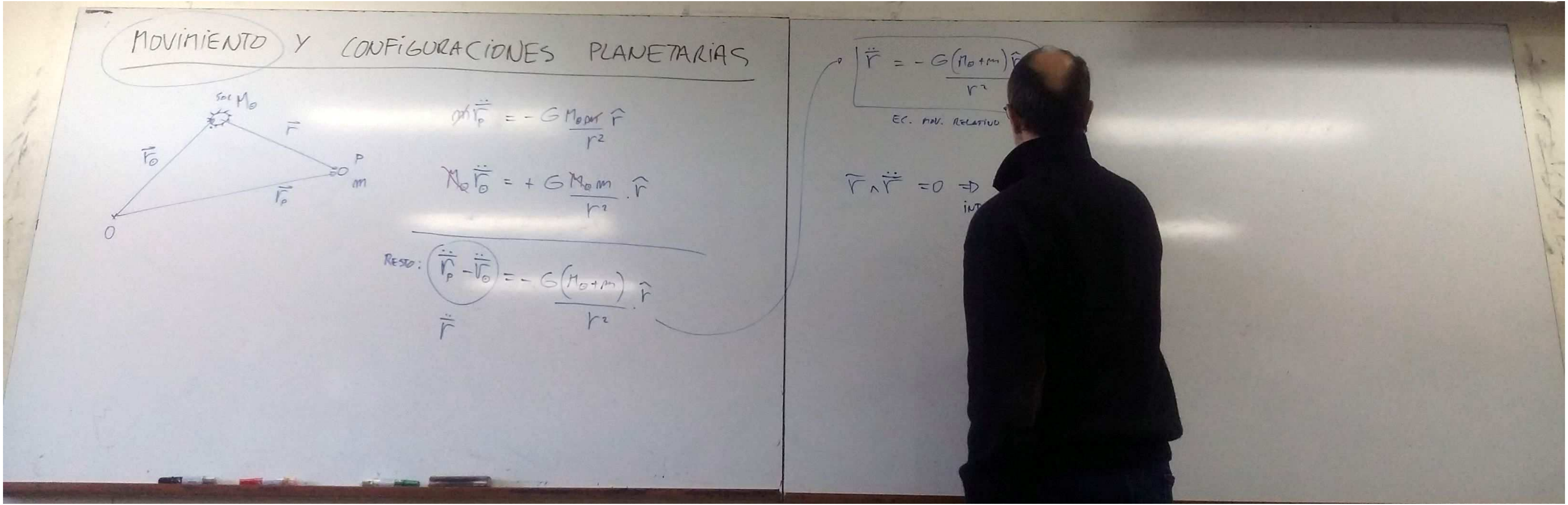
Astronomia Fundamental 2018 - clase 14

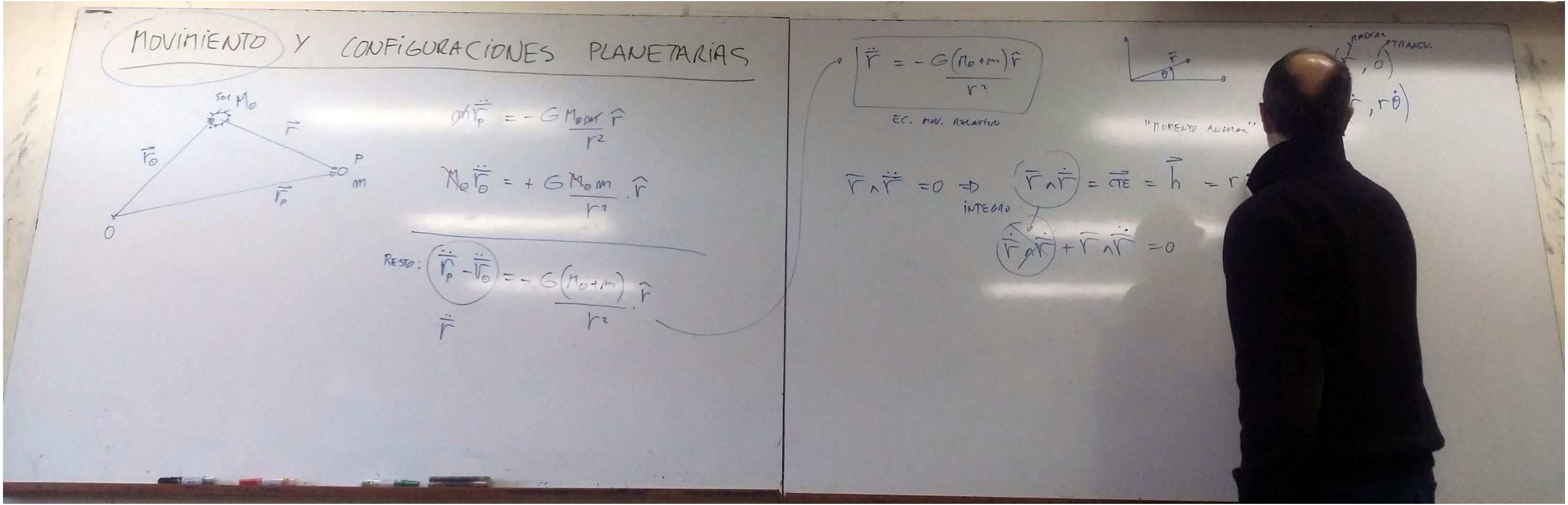



MOVIMIENTO Y CONFIGURACIONES PLANETARIAS
SOL $M_\odot$
$\vec{r}$
$\vec{r}_\odot$
P
m
$\vec{r}_p$
O
$m\ddot{\vec{r}}_p = -G\frac{M_\odot m}{r^2}\hat{r}$
$M_\odot\ddot{\vec{r}}_\odot = +G\frac{M_\odot m}{r^2}.\hat{r}$
RESTO: $\ddot{\vec{r}}_p - \ddot{\vec{r}}_\odot = -G\frac{(M_\odot+m)}{r^2}.\hat{r}$
$\ddot{\vec{r}}$
$\ddot{\vec{r}} = -G\frac{(M_\odot+m)\hat{r}}{r^2}$
EC. MOV. RELATIVO

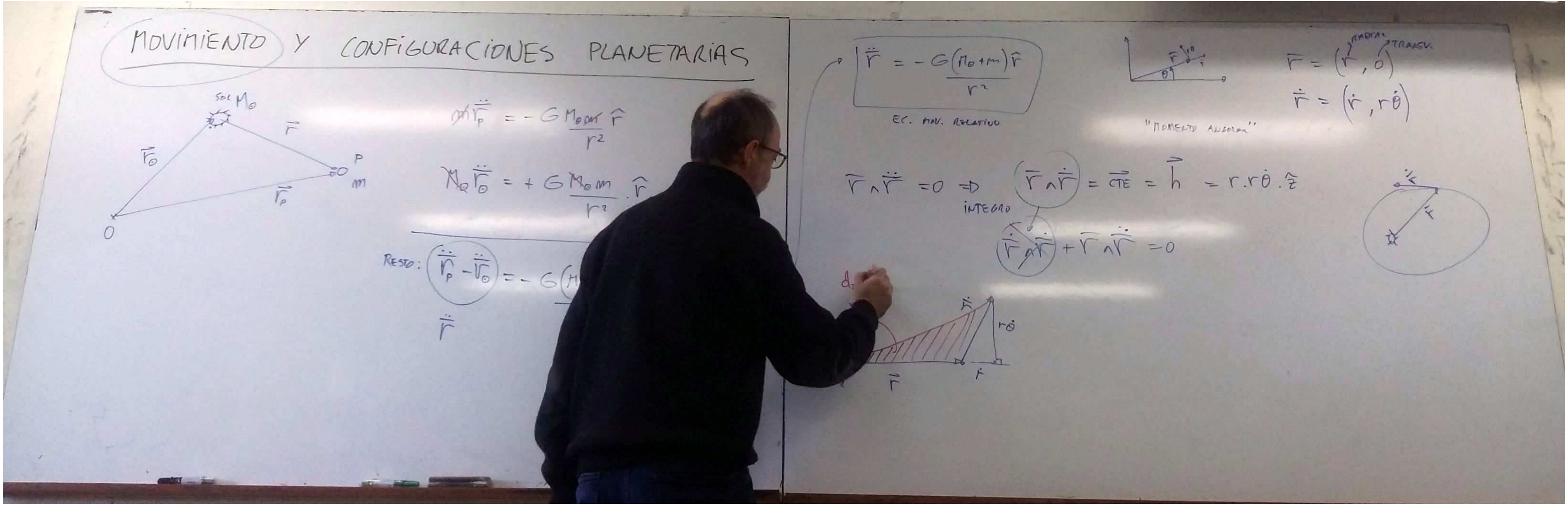
Astronomia Fundamental 2018 - clase 14



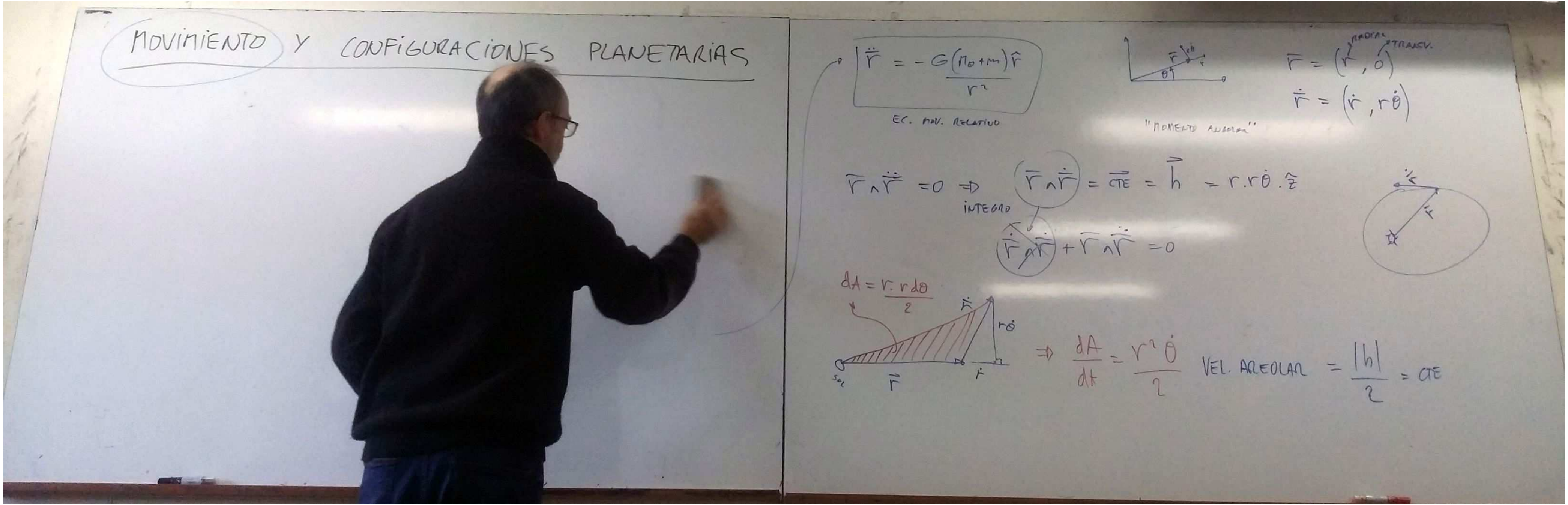
Astronomia Fundamental 2018 - clase 14



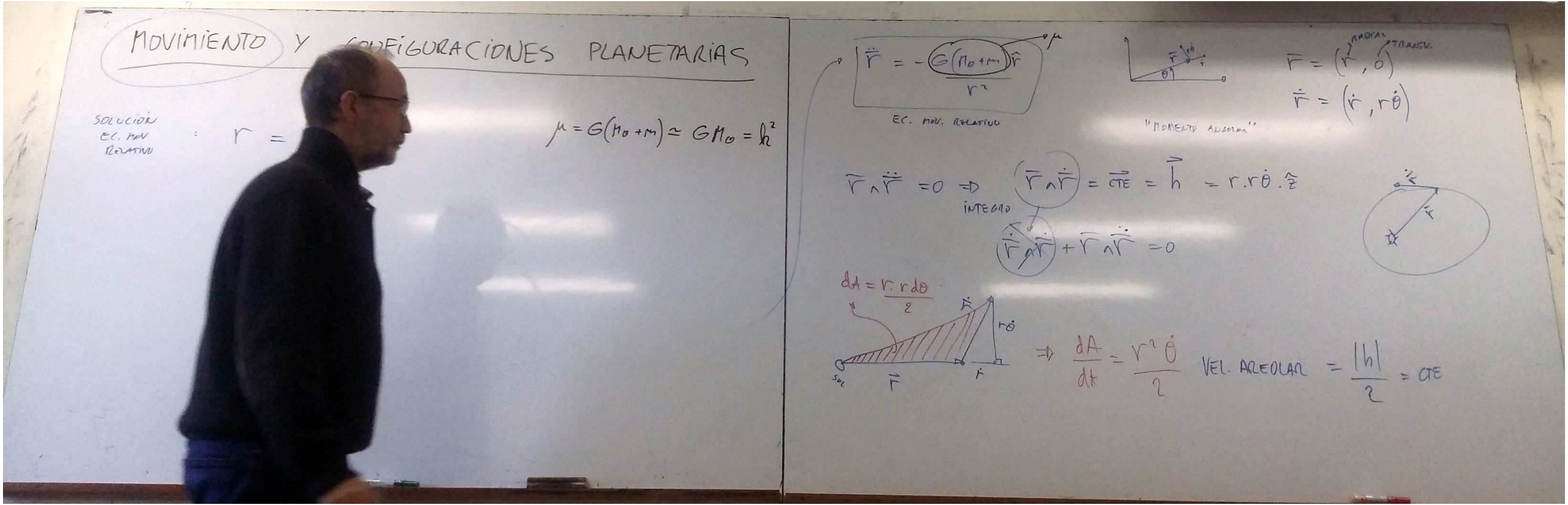
Astronomia Fundamental 2018 - clase 14



Astronomia Fundamental 2018 - clase 14



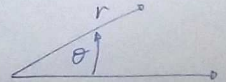
Astronomia Fundamental 2018 - clase 14



MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

SOLUCIÓN EC. MOV RELATIVO : $r = \dfrac{h^2/\mu}{1 + e\cdot\cos\theta}$

C. POLARES (r, θ)



$\mu = G(M_\odot + m) \simeq GM_\odot = k^2$

GAUSS

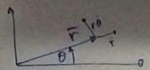
$k^2 = GM_\odot = (0.01720209895)^2$

UA, días, $M_\odot$

$$\boxed{\ddot{\vec{r}} = -\frac{G(M_\odot + m)}{r^2}\hat{r}}$$

μ

EC. MOV. RELATIVO



$\vec{r} = (r, 0)$ RADIAL, TRANSV.

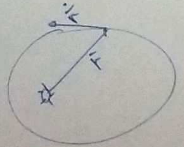
$\dot{\vec{r}} = (\dot{r}, r\dot{\theta})$

"MOMENTO ANGULAR"

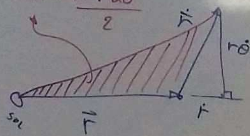
$\vec{r}\wedge\ddot{\vec{r}} = 0 \Rightarrow \vec{r}\wedge\dot{\vec{r}} = \vec{CTE} = \vec{h} = r.r\dot{\theta}.\hat{z}$

INTEGRO

$\cancel{\dot{\vec{r}}\wedge\dot{\vec{r}}} + \vec{r}\wedge\ddot{\vec{r}} = 0$

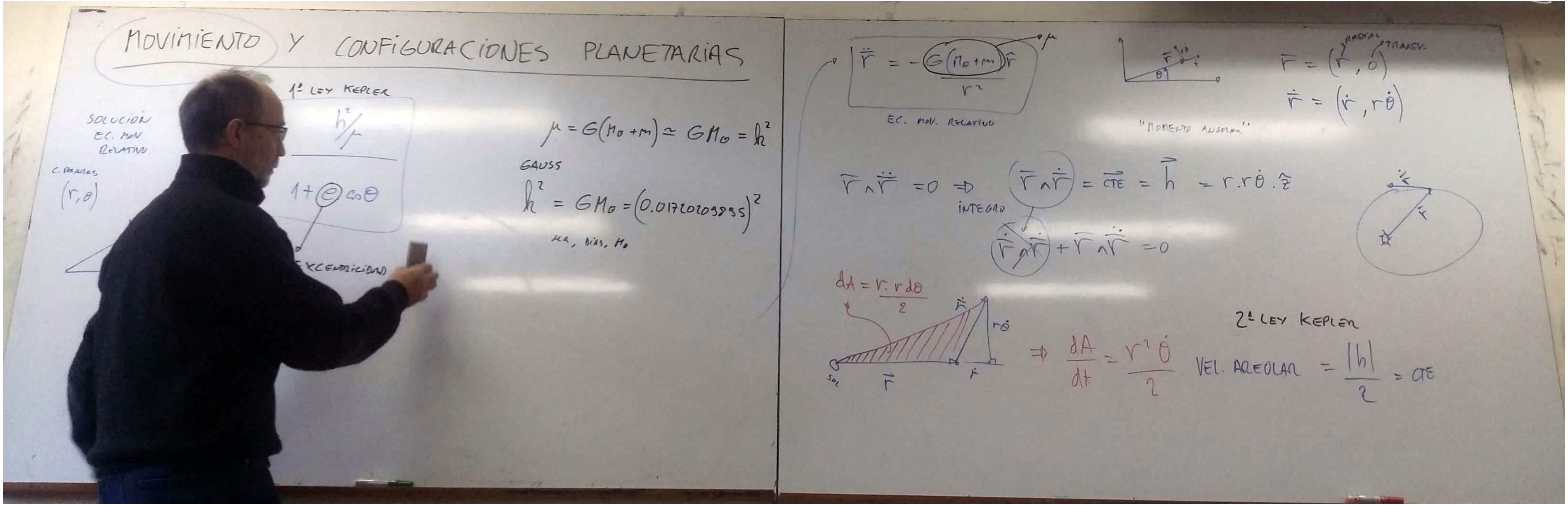


$dA = \dfrac{r \cdot r\,d\theta}{2}$

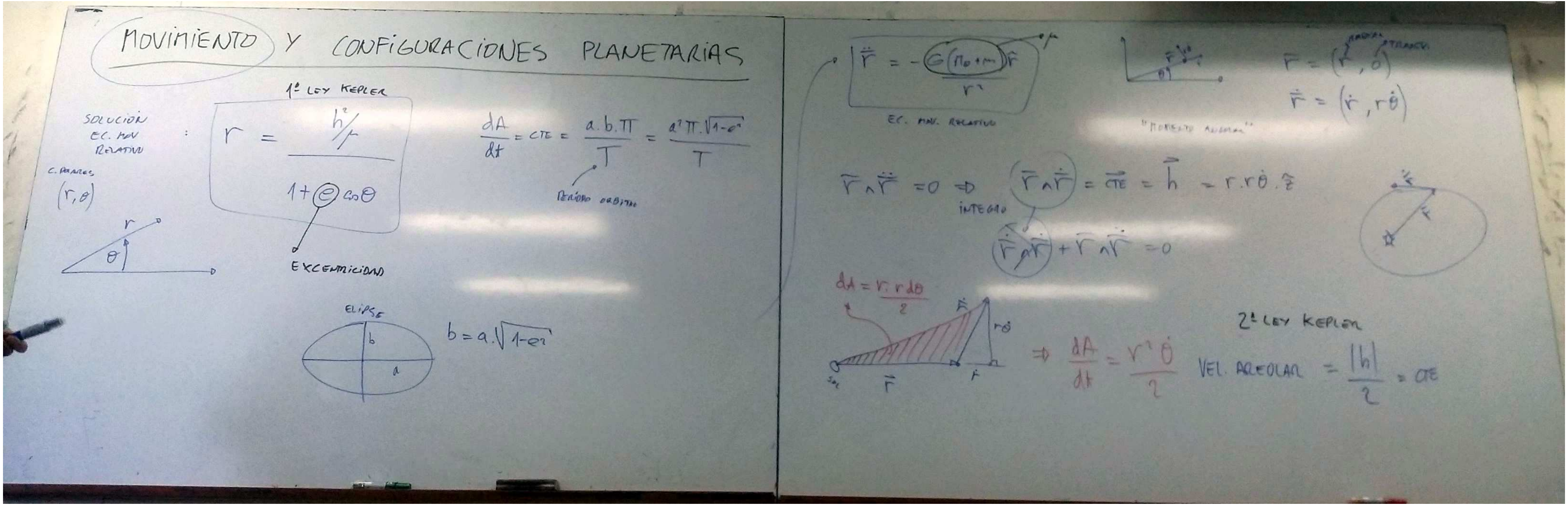


$\Rightarrow \dfrac{dA}{dt} = \dfrac{r^2\dot{\theta}}{2}$ VEL. AREOLAR $= \dfrac{|h|}{2} = $ CTE

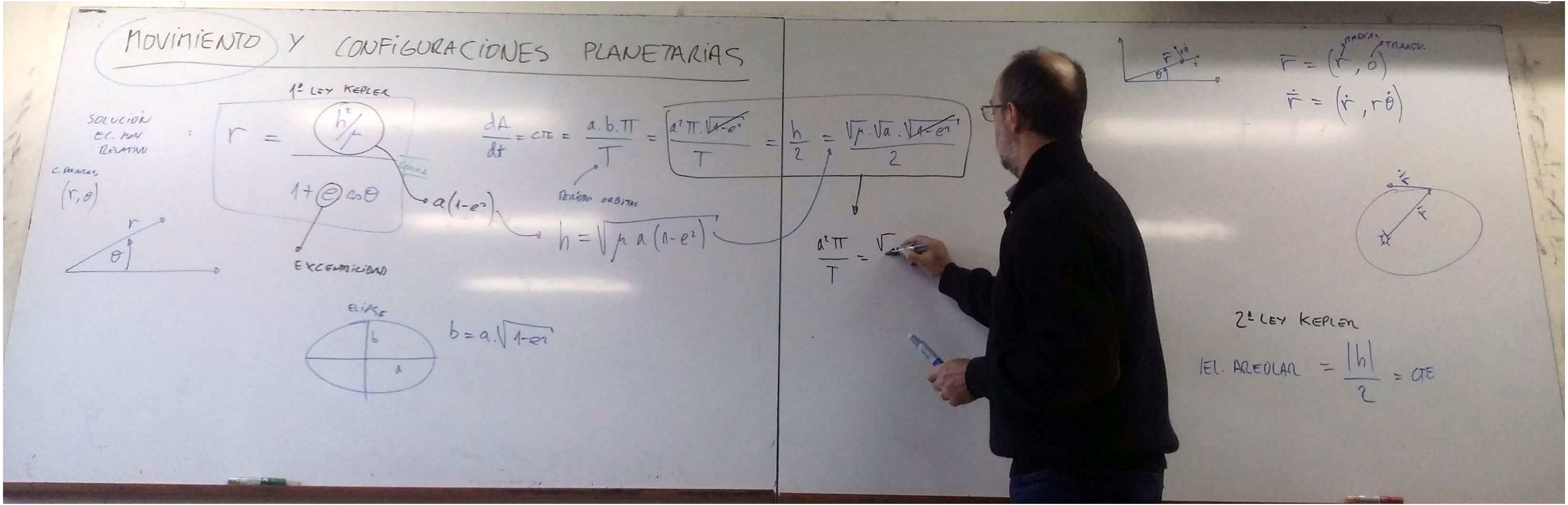
Astronomia Fundamental 2018 - clase 14



Astronomia Fundamental 2018 - clase 14



Astronomia Fundamental 2018 - clase 14



Astronomia Fundamental 2018 - clase 14

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

SOLUCIÓN EC. MOV RELATIVO :

1º LEY KEPLER

$$r = \frac{h^2/\mu}{1+e\cos\theta}$$

CÓNICA

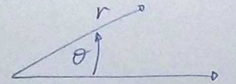
$a(1-e^2)$

$h = \sqrt{\mu\, a(1-e^2)}$

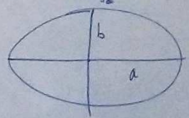
EXCENTRICIDAD

C. POLARES

(r, θ)



ELIPSE



$b = a\sqrt{1-e^2}$

ELIPSE

$$\frac{dA}{dt} = CTE = \frac{a.b.\pi}{T} = \boxed{\frac{a^2\pi\cancel{\sqrt{1-e^2}}}{T} = \frac{h}{2} = \frac{\sqrt{\mu}.\sqrt{a}.\cancel{\sqrt{1-e^2}}}{2}}$$

PERÍODO ORBITAL

$$\frac{a^2\pi}{T} = \frac{\sqrt{\mu}.\sqrt{a}}{2} \Rightarrow \frac{a^4\pi^2}{T^2} = \mu\frac{a}{4}$$

3ª LEY KEPLER

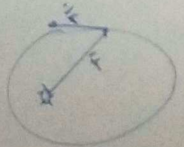
$$\frac{a^3}{T^2} = \frac{\mu}{(2\pi)^2} \Rightarrow \boxed{\frac{G(M_\odot+m)}{(2\pi)^2} = \frac{a^3}{T^2}}$$

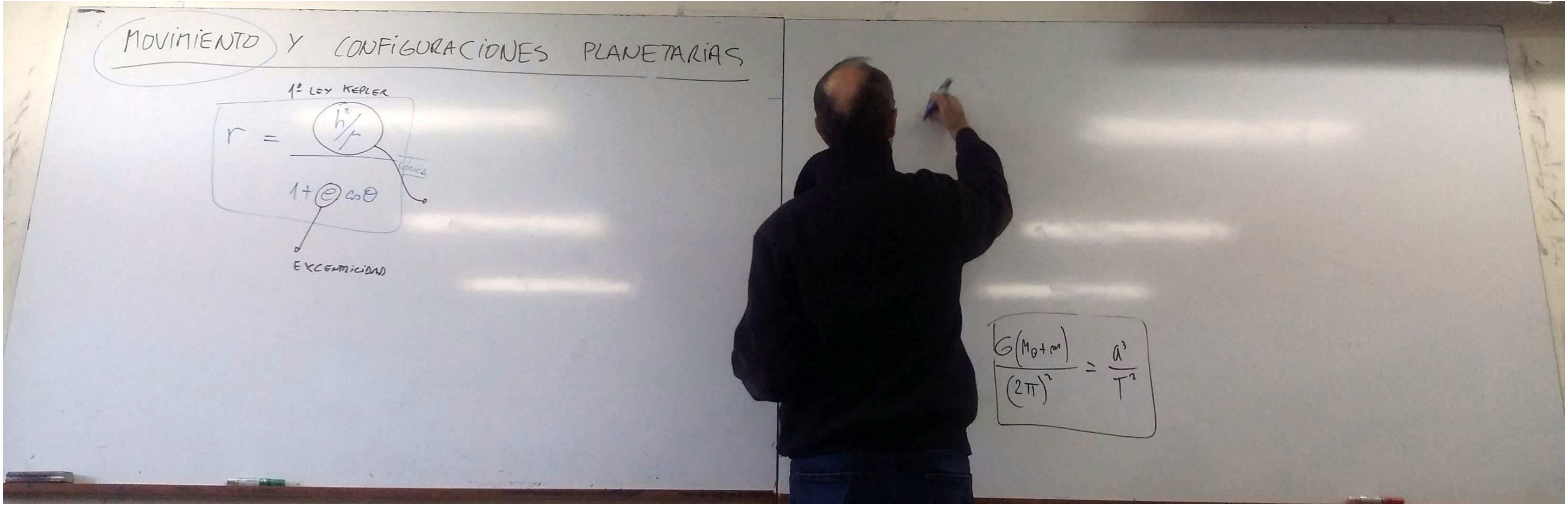
2ª LEY KEPLER

VEL. AREOLAR $= \frac{|h|}{2} = CTE$

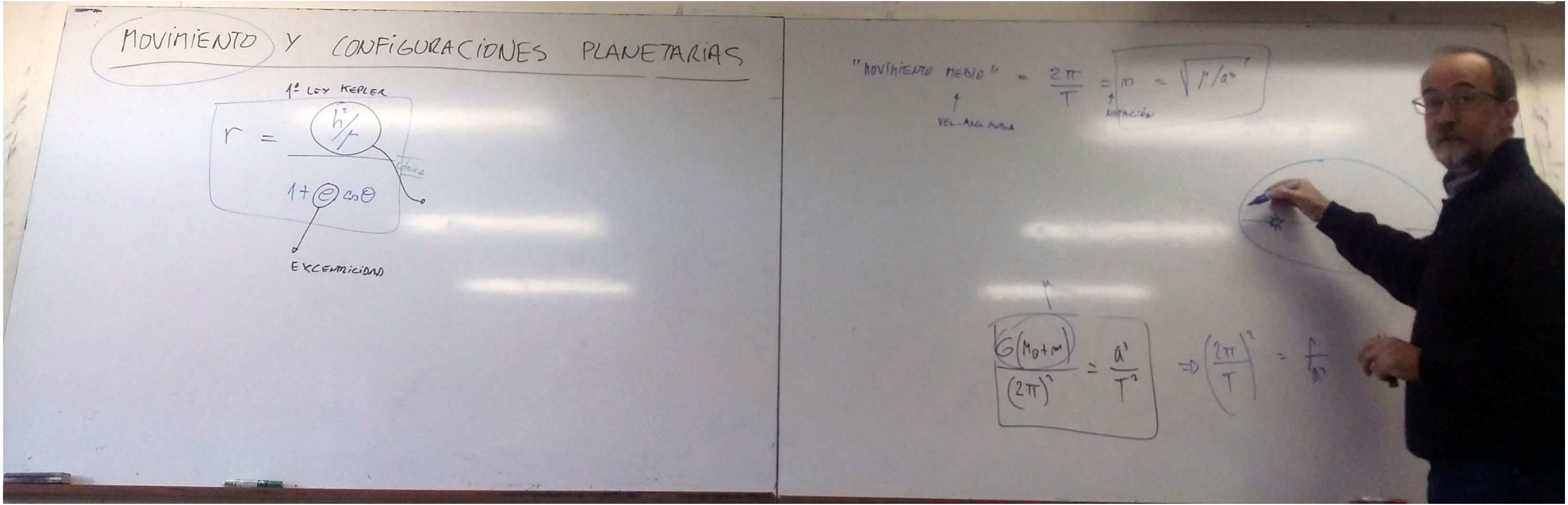
$\vec{r} = (r, 0)$ RADIAL TRANSV.

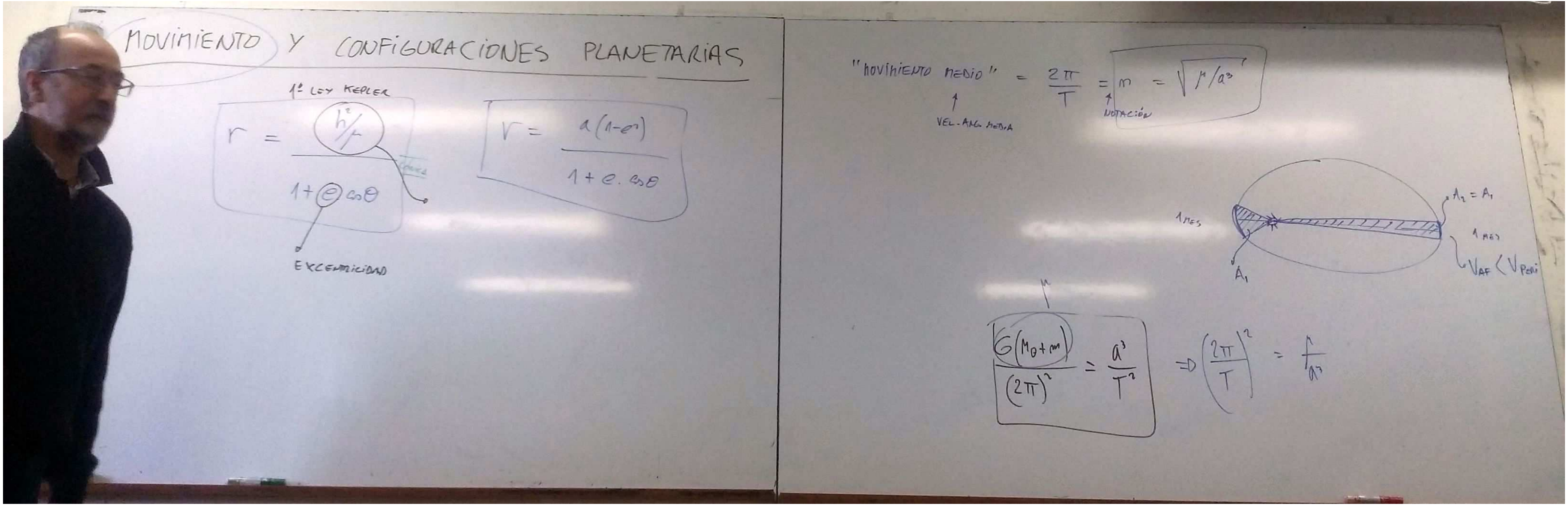
$\dot{\vec{r}} = (\dot{r}, r\dot{\theta})$



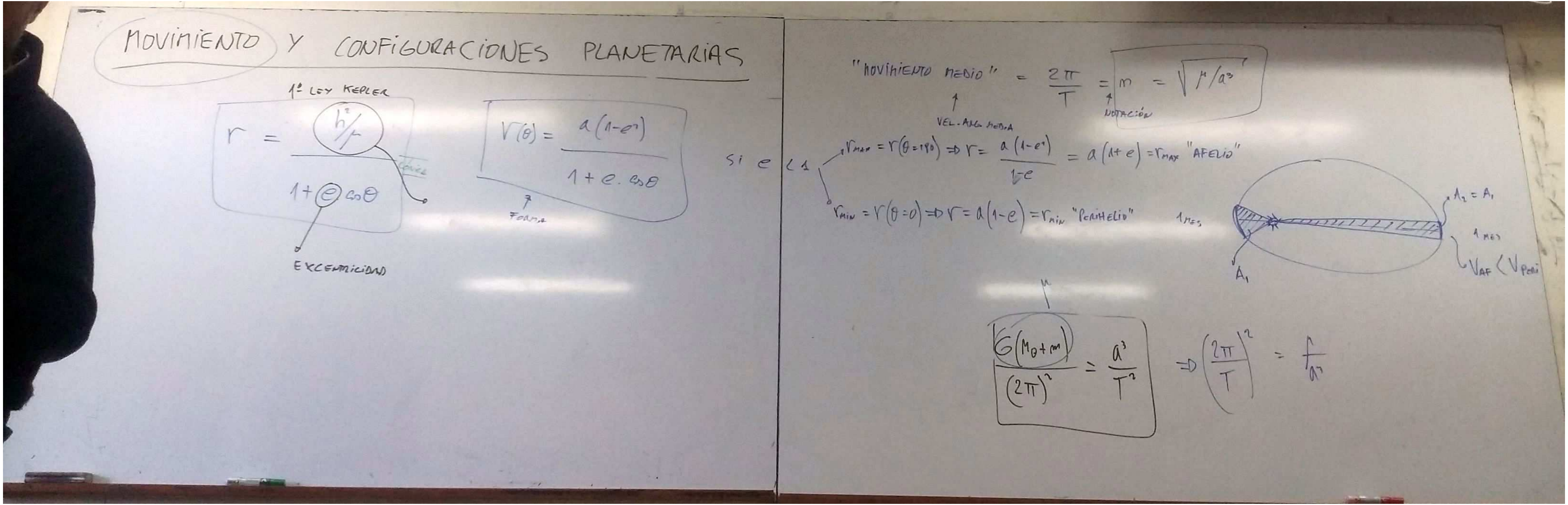

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS
1º LEY KEPLER
$r = \frac{h^2/\mu}{1 + e \cos\theta}$
EXCENTRICIDAD
$\frac{G(M_\odot + m)}{(2\pi)^2} = \frac{a^3}{T^2}$

Astronomia Fundamental 2018 - clase 14

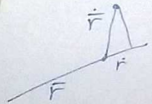



MOVIMIENTO Y CONFIGURACIONES PLANETARIAS
1° LEY KEPLER
$r = \dfrac{h^2/\mu}{1+e\cos\theta}$
EXCENTRICIDAD
$r = \dfrac{a(1-e^2)}{1+e.\cos\theta}$
"MOVIMIENTO MEDIO" $= \dfrac{2\pi}{T} = n = \sqrt{\mu/a^3}$
VEL. ANG. MEDIA
NOTACIÓN
1 MES
$A_2 = A_1$
1 MES
$V_{AF} < V_{PERI}$
A_1
μ
$\dfrac{G(M_\odot+m)}{(2\pi)^2} = \dfrac{a^3}{T^2} \Rightarrow \left(\dfrac{2\pi}{T}\right)^2 = \dfrac{\mu}{a^3}$

Astronomia Fundamental 2018 - clase 14



MOVIMIENTO Y CONFIGURACIONES PLANETARIAS



$$\ddot{\vec{r}} = -\mu \frac{\hat{r}}{r^2}$$

$$\dot{\vec{r}}\cdot\ddot{\vec{r}} = -\mu \frac{\dot{\vec{r}}\cdot\hat{r}}{r^2} \rightarrow \frac{\dot{\vec{r}}\cdot\vec{r}}{r^3} = \frac{\dot{r}\, r}{r^3} = \frac{\dot{r}}{r^2}$$

1º LEY KEPLER

$$r = \frac{h^2/\mu}{1 + e \cos\theta}$$

CÓNICA

EXCENTRICIDAD

$$r(\theta) = \frac{a(1-e^2)}{1 + e\cos\theta}$$

FORMA

si $e < 1$

$r_{max} = r(\theta = 180) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max}$ "AFELIO"

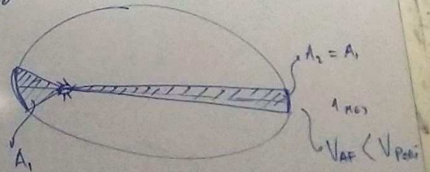
$r_{min} = r(\theta = 0) \Rightarrow r = a(1-e) = r_{min}$ "PERIHELIO"

"MOVIMIENTO MEDIO" $= \frac{2\pi}{T} = \boxed{n = \sqrt{\mu/a^3}}$

VEL. ANG. MEDIA

NOTACIÓN

1 mes



$$\Rightarrow \dot{\vec{r}}\cdot\ddot{\vec{r}} = -\mu\frac{\dot{r}}{r^2}$$

INTEG:

$$\Rightarrow \boxed{\frac{v^2}{2} = \frac{\mu}{r} + \mathcal{E}} \Rightarrow \boxed{\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}}$$

ENERGÍA ORBITAL

Astronomia Fundamental 2018 - clase 14

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

1º LEY KEPLER

$$r = \frac{h^2/\mu}{1 + e\cos\theta}$$

CÓNICA

EXCENTRICIDAD

$$r(\theta) = \frac{a(1-e^2)}{1 + e.\cos\theta}$$

FORMA

$$\ddot{\vec{r}} = -\mu \frac{\hat{r}}{r^2}$$

$$\dot{\vec{r}}.\ddot{\vec{r}} = -\mu \frac{\dot{\vec{r}}.\hat{r}}{r^2}$$

$$\frac{\dot{\vec{r}}.\vec{r}}{r^3} = \frac{\dot{r}\,r}{r^3} = \frac{\dot{r}}{r^2}$$

$$\Rightarrow \dot{\vec{r}}.\ddot{\vec{r}} = -\mu \frac{\dot{r}}{r^2}$$

INTEG:

$$\Rightarrow \boxed{\frac{v^2}{2} = \frac{\mu}{r} + \mathcal{E}}$$

ENERGÍA ORBITAL

$$\Rightarrow \mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$$

CIN POT

si $\mathcal{E} < 0 \Rightarrow r$ ACOTADO $\Rightarrow$ LIGADO $\rightarrow$ ÓRB...

si $\mathcal{E} > 0 \Rightarrow r$ NO ACOTADO $\Rightarrow$ NO LIGADO

si $e < 1$

$r_{max} = r(\theta = 180) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max}$ "AFELIO"

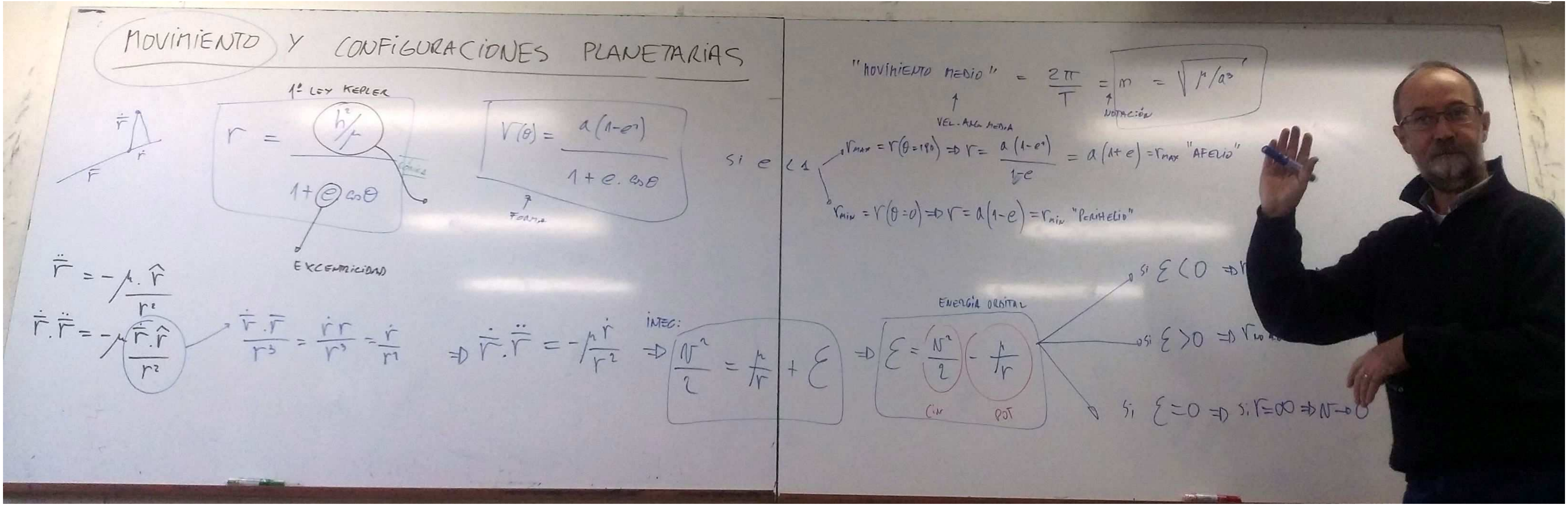
$r_{min} = r(\theta = 0) \Rightarrow r = a(1-e) = r_{min}$ "PERIHELIO"

"MOVIMIENTO MEDIO" $= \frac{2\pi}{T} = \boxed{n = \sqrt{\mu/a^3}}$

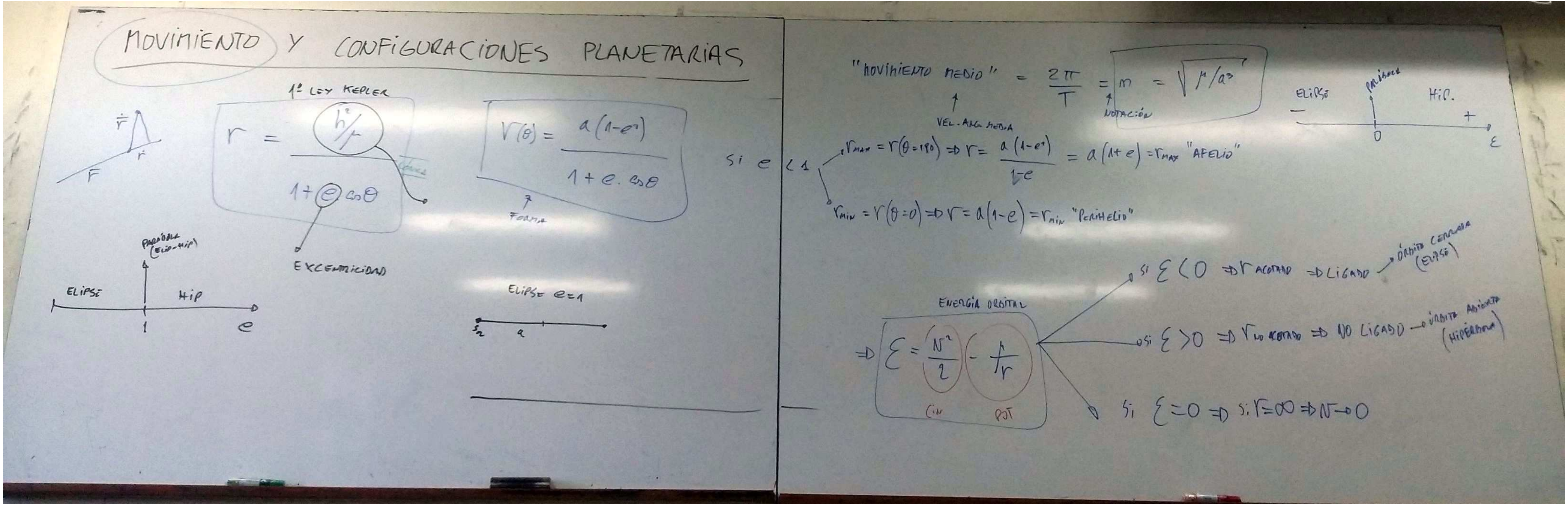
VEL. ANG. MEDIA

NOTACIÓN

Astronomia Fundamental 2018 - clase 14



Astronomia Fundamental 2018 - clase 14



MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

Suponese: $a = \frac{\Delta_{TL}}{2}$ $e=1$ L T ¿t?

$T_{CAIDA} = \frac{T}{2}$

$\frac{T^2}{a^3} = CTE = \frac{27^2}{\Delta_{TL}^3}$

$$\boxed{r(\theta) = \frac{a(1-e^2)}{1+e.\cos\theta}}$$

Forma

ELIPSE $e=1$

S, a

"MOVIMIENTO MEDIO" $= \frac{2\pi}{T} = \boxed{n = \sqrt{\mu/a^3}}$

VEL. ANG. MEDIA

NOTACIÓN

ELIPSE PARÁBOLA HIP.

$-$ 0 $+$ ε

si $e < 1$

$r_{max} = r(\theta=180) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max}$ "AFELIO"

$r_{min} = r(\theta=0) \Rightarrow r = a(1-e) = r_{min}$ "PERIHELIO"

ENERGÍA ORBITAL

$$\Rightarrow \boxed{\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}}$$

CIN POT

si $\varepsilon < 0 \Rightarrow r$ ACOTADO $\Rightarrow$ LIGADO $\rightarrow$ ÓRBITA CERRADA (ELIPSE)

si $\varepsilon > 0 \Rightarrow r$ NO ACOTADO $\Rightarrow$ NO LIGADO $\rightarrow$ ÓRBITA ABIERTA (HIPÉRBOLA)

si $\varepsilon = 0 \Rightarrow$ si $r = \infty \Rightarrow v \rightarrow 0$

MOVIMIENTO Y CONFIGURACIONES PLANETARIAS

SEMIEJE: $a = \frac{\Delta_{TL}}{2}$

$e \to 1$

L

T

¿t?

$T_{CAIDA} = \frac{T}{2}$

$$\frac{T^2}{a^3} = CTE = \frac{27^2}{\Delta_{TL}^3} = \frac{(T/2)^2}{(\Delta_{TL}/2)^3}$$

$$\boxed{r(\theta) = \frac{a(1-e^2)}{1 + e \cdot \cos\theta}}$$

CFA ELIPSE HIP

0 1 ∞

FORMA

ELIPSE $e = 1$

a

"MOVIMIENTO MEDIO" $= \frac{2\pi}{T} = \boxed{n = \sqrt{\mu/a^3}}$

VEL. ANG. MEDIA

NOTACIÓN

ELIPSE PARÁBOLA HIP.

$-$ 0 $+$ ε

SI $e < 1$

$r_{max} = r(\theta = 180) \Rightarrow r = \frac{a(1-e^2)}{1-e} = a(1+e) = r_{max}$ "AFELIO"

$r_{min} = r(\theta = 0) \Rightarrow r = a(1-e) = r_{min}$ "PERIHELIO"

ENERGÍA ORBITAL

$$\Rightarrow \boxed{\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}}$$

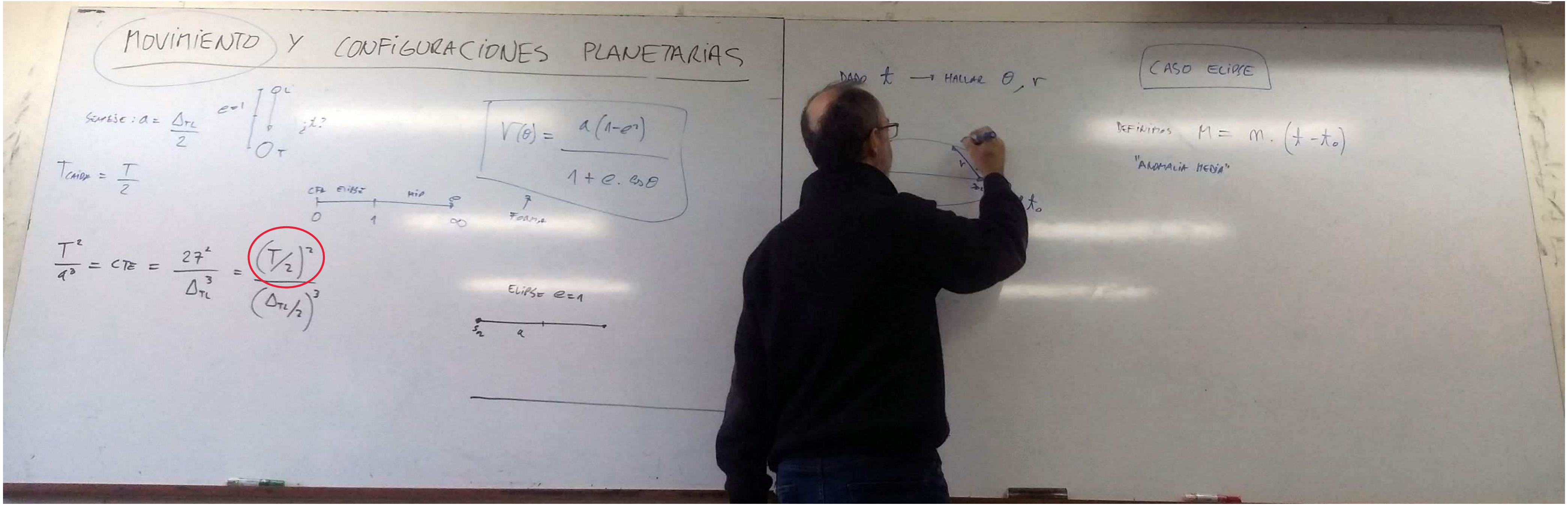
CIN POT

SI $\varepsilon < 0 \Rightarrow r$ ACOTADO $\Rightarrow$ LIGADO $\to$ ÓRBITA CERRADA (ELIPSE)

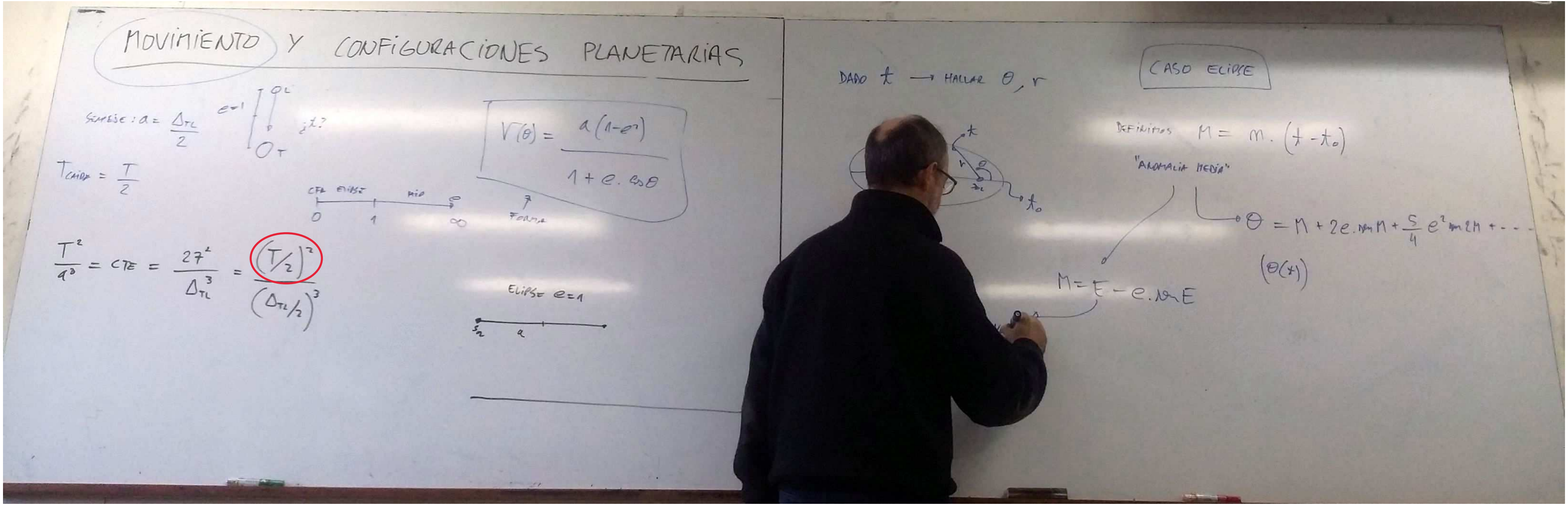
SI $\varepsilon > 0 \Rightarrow r$ NO ACOTADO $\Rightarrow$ NO LIGADO $\to$ ÓRBITA ABIERTA (HIPÉRBOLA)

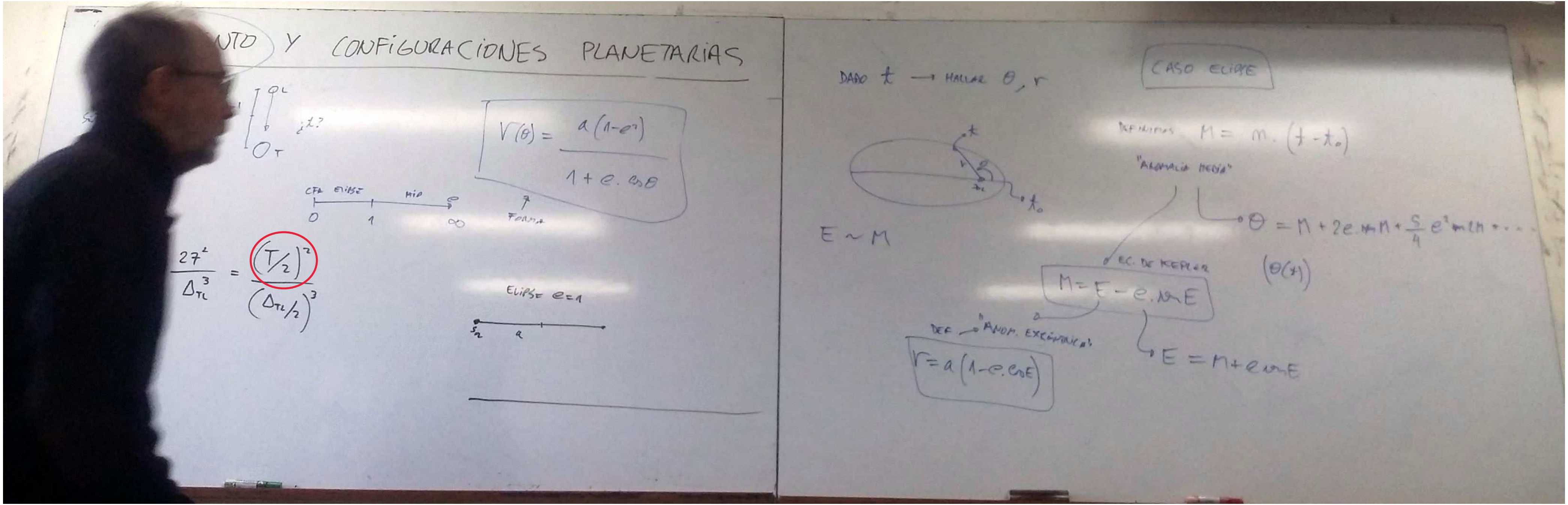
SI $\varepsilon = 0 \Rightarrow$ SI $r = \infty \Rightarrow v \to 0$

Astronomia Fundamental 2018 - clase 14



Astronomia Fundamental 2018 - clase 14




Y CONFIGURACIONES PLANETARIAS
DADO t → HALLAR θ, r
CASO ELIPSE
"ANOMALIA MEDIA"
E ~ M
EC. DE KEPLER
ELIPSE e=1

Astronomia Fundamental 2018 - clase 14