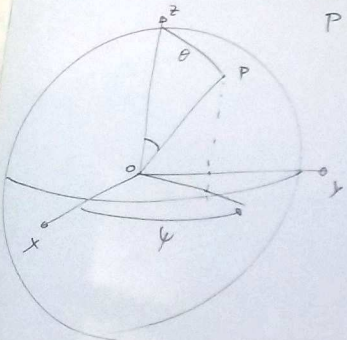
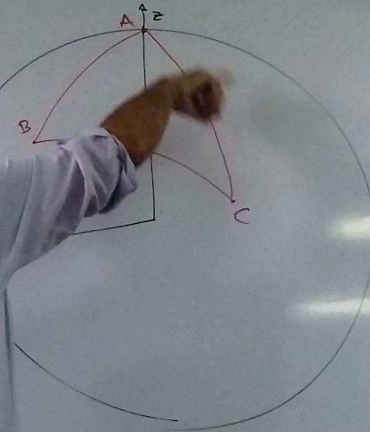


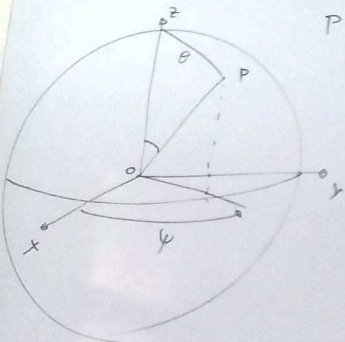
DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

 $P(\psi, \theta)$

$$\begin{cases} x = r \cdot \sin \theta \cdot \cos \psi \\ y = r \cdot \sin \theta \cdot \sin \psi \\ z = r \cdot \cos \theta \end{cases}$$

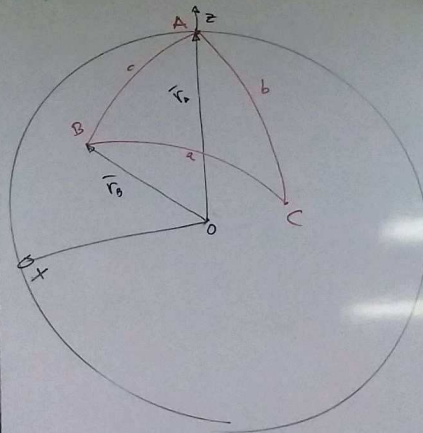


DEDUCCION FORM. TRIG. ESFERICA



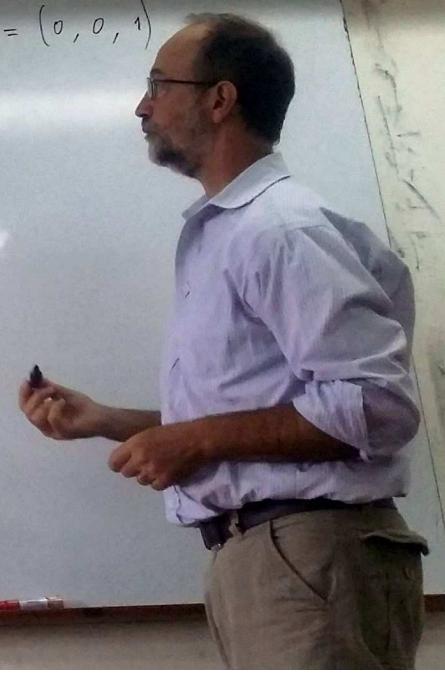
$P(\psi, \theta)$

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$

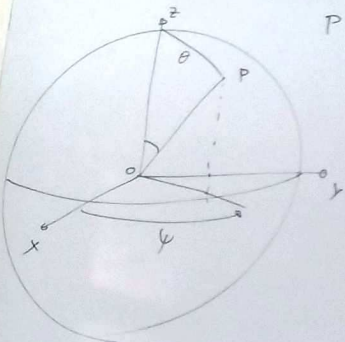


$$\vec{r}_A = (\psi = 0^\circ, \theta = 0^\circ) = (0, 0, 1)$$

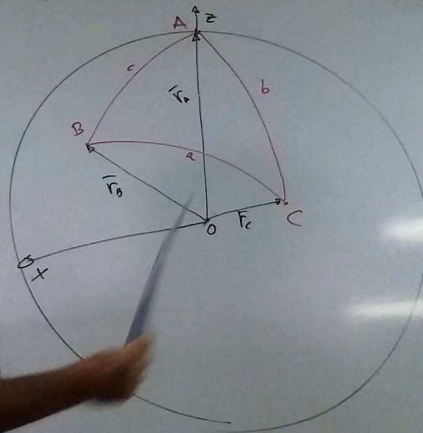
$$\vec{r}_B = (\psi = 0^\circ, \theta = \dots)$$



DEDUCCIÓN FÓRM. TRIG. ESFÉRICA


 $P(\psi, \theta)$

$$\begin{cases} x = \sin \theta \cdot \cos \psi \\ y = \sin \theta \cdot \sin \psi \\ z = \cos \theta \end{cases}$$

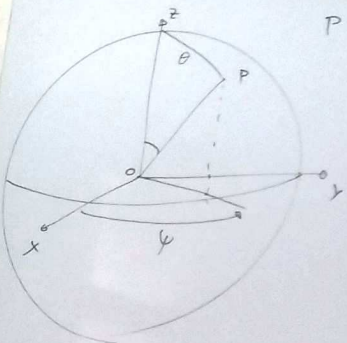


$$\vec{r}_A = (\sin \theta, \psi = 0^\circ, \theta = 0^\circ) = (0, 0, 1)$$

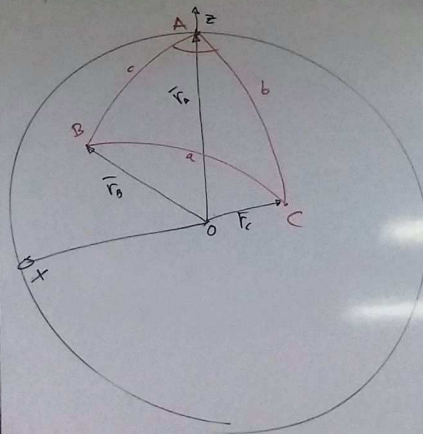
$$\vec{r}_B = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_C = (\psi =$$

DEDUCCION FÓRM. TRIG. ESFÉRICA


 $P(\psi, \theta)$

$$\begin{cases} x = \sin \theta \cdot \cos \psi \\ y = \sin \theta \cdot \sin \psi \\ z = \cos \theta \end{cases}$$



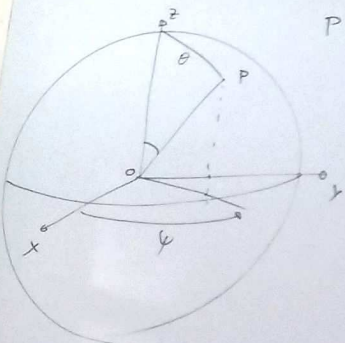
$$\vec{r}_a = (\psi=0, \theta=0) = (0, 0, 1)$$

$$\vec{r}_b = (\psi=0, \theta=c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi=A, \theta=b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

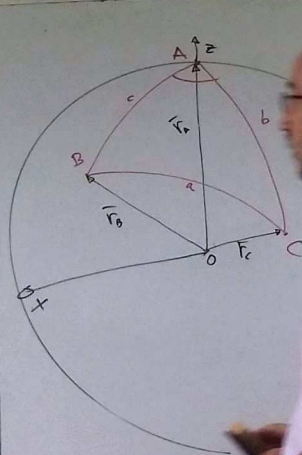
$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

DEDUCCION FORM. TRIG. ESFERICA



$P(\psi, \theta)$

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$



$$\vec{r}_A = (\psi=0^\circ, \theta=0^\circ) = (0, 0, 1)$$

$$\vec{r}_B = (\psi=0^\circ, \theta=c) = (\sin c, 0, \cos c)$$

$$\vec{r}_C = (\psi=A, \theta=b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_B = \cos a$$

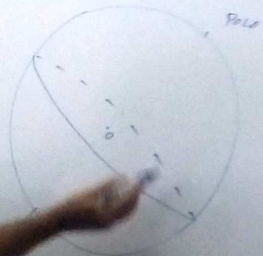
$$\sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

$$= \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

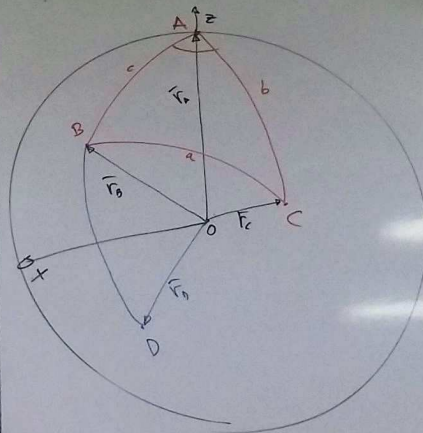
DEDUCCION FORM. TRIG. ESFERICA

POLO



$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$

$$\vec{r}_c \wedge \vec{r}_b = \sin a$$



$$\vec{r}_a = (\psi = 0^\circ, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

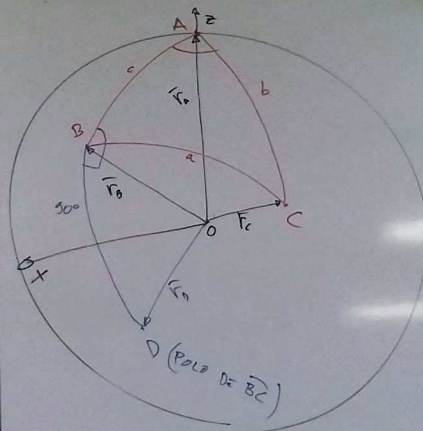
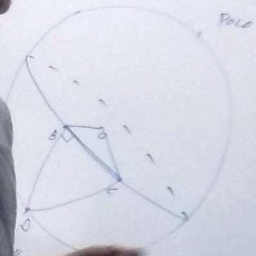
$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

DUCCIÓN FÓRM. TRIG. ESFÉRICA

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$

$$\vec{r}_c \wedge \vec{r}_b = \sin a$$



$$\vec{r}_a = (\sin \theta, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

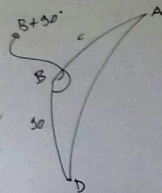
F. COSENO

DEDUCC

TRIG. ESFERICA

POLO

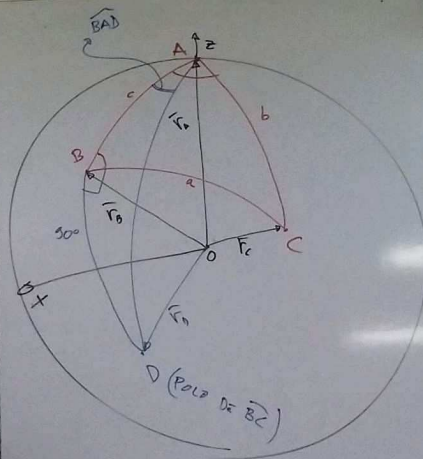
$$\begin{cases} x = r \cdot \theta \cdot \cos \psi \\ y = r \cdot \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$



$$\vec{r}_c \wedge \vec{r}_b = \sin a \cdot \vec{r}_a$$

$\psi = \widehat{BAD}$
 $\theta = \widehat{AD}$

$\cos \widehat{AD} =$



$$\vec{r}_a = (\cos \theta, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

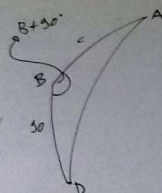
$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

COORDENADA Z DE \vec{r}_0

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$

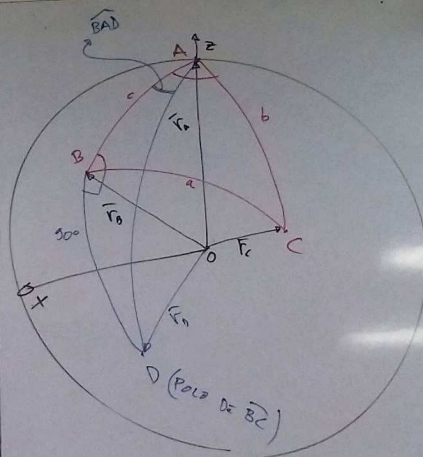


$$\vec{r}_c \wedge \vec{r}_b = \sin a \cdot \vec{r}_0$$

$\psi = \widehat{BAD}$
 $\theta = \widehat{AD}$

$$\cos \widehat{AD} = \cos 90^\circ \cdot \cos c + \sin 90^\circ \cdot \sin c \cdot \cos(B+90^\circ)$$

$$\cos \widehat{AD} = -\sin c \cdot \sin B$$



$$\vec{r}_a = (\psi = 0^\circ, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

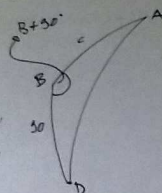
$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

DEDUCCION FORM. TRIG. ESFERICA

COORDENADA Z DE $\vec{r}_0 = -\sin c \sin B$

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$



i	j	k
$\sin b \cdot \cos A$	$\sin b \sin A$	$\cos b$
$\sin c$	0	$\cos c$

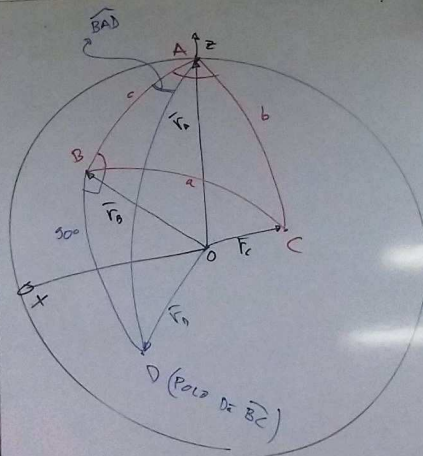
$$\vec{r}_c \wedge \vec{r}_b = \sin a \cdot \vec{r}_0$$

$\psi = \widehat{BAD}$
 $\theta = \widehat{AD}$

$$\cos \widehat{AD} = \cos 90 \cdot \cos c + \sin 90 \cdot \sin c \cdot \cos(B+90)$$

$$\cos \widehat{AD} = -\sin c \cdot \sin B$$

$$-\sin B$$



$$\vec{r}_a = (\sin \theta, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

$$\vec{r}_c \cdot \vec{r}_b = \sin c \cdot \sin b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos a$$

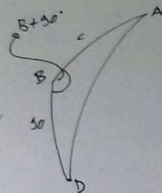
$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

COORDENADA Z DE $\vec{r}_0 = -\sin \theta \sin B$

$$\begin{cases} x = r \sin \theta \cos \psi \\ y = r \sin \theta \sin \psi \\ z = r \cos \theta \end{cases}$$



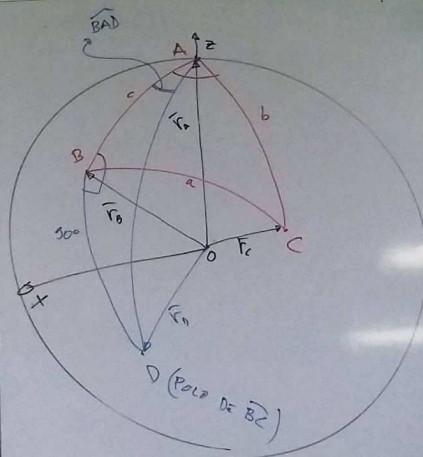
$$\begin{vmatrix} i & j & k \\ \sin b \cos A & \sin b \sin A & \cos b \\ \sin c & 0 & \cos c \end{vmatrix} =$$

$$\vec{r}_c \wedge \vec{r}_b = \sin \theta \vec{r}_0$$

$\psi = \widehat{BAD}$
 $\theta = \widehat{AD}$

$$\cos \widehat{AD} = \cancel{\cos 90^\circ \cos c} + \cancel{\sin 90^\circ \sin c \cos(B+90^\circ)}$$

$$\cos \widehat{AD} = -\sin c \sin B \quad -\sin B$$



$$-\sin b \sin A \sin c =$$

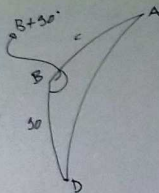
↑
COORD. Z DE
 $\vec{r}_c \wedge \vec{r}_b$

$$\cos \dots = \dots \sin A$$

DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

COORDENADA Z DE $\vec{r}_0 = -\sin \theta \sin \psi$

$$\begin{cases} x = r \sin \theta \cos \psi \\ y = r \sin \theta \sin \psi \\ z = r \cos \theta \end{cases}$$



$$\begin{vmatrix} i & j & k \\ \sin \theta \cos \psi & \sin \theta \sin \psi & \cos \theta \\ \sin \theta \cos \psi & \sin \theta \sin \psi & \cos \theta \end{vmatrix} =$$

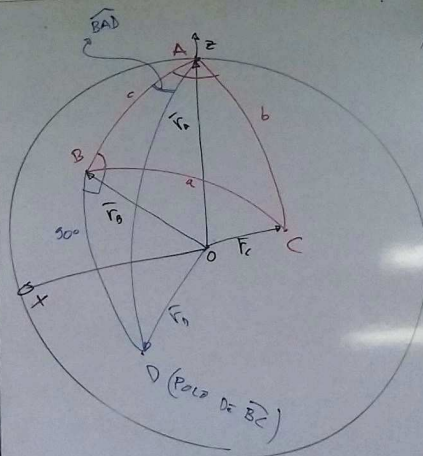
($\sin \theta \cos \psi$, $\cos \theta \sin \psi$, $-\sin \theta \sin \psi$, $-\sin \theta \cos \psi$)

$\vec{r}_c \wedge \vec{r}_b = \sin \theta \vec{r}_0$ (marked with an asterisk)

$\psi = \widehat{BAD}$
 $\theta = \widehat{AD}$

$\cos \widehat{AD} = \cos 90^\circ \cos c + \sin 90^\circ \sin c \cos (B+90^\circ)$

$\cos \widehat{AD} = -\sin c \sin B$



$\pm \sin \theta \sin \psi = \pm \sin \theta \sin \psi$ (marked with an asterisk)

Coord. z de $\vec{r}_c \wedge \vec{r}_b$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

F. COSENO

DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

COORDENADA Z DE $\vec{r}_0 = -\sin \theta \sin \phi$

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$$

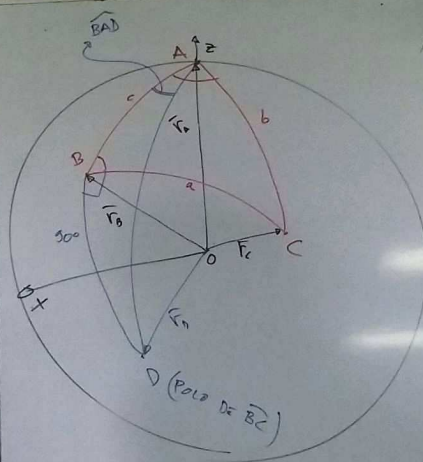
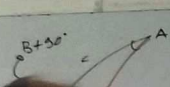
i	j	k
$\sin \theta \cos \phi$	$\sin \theta \sin \phi$	$\cos \theta$
$\sin \phi$	0	$-\cos \phi$

$$\vec{r}_c \wedge \vec{r}_b = \sin \theta \vec{r}_0$$

COORDENADA Z DE \vec{r}_0

$$\sin \theta \sin \theta \sin \theta = \sin \theta \sin \theta \sin \theta$$

$\sin \theta \sin \theta \sin \theta$
 $\sin \theta \sin \theta \sin \theta$
 $-\sin \theta \sin \theta \sin \theta$



$$\sin \theta \sin \theta \sin \theta = \sin \theta \sin \theta \sin \theta$$

COORDENADA Z DE $\vec{r}_c \wedge \vec{r}_b$

$$\frac{\sin \theta \sin \theta \sin \theta}{\sin \theta \sin \theta \sin \theta} = \frac{\sin \theta \sin \theta \sin \theta}{\sin \theta \sin \theta \sin \theta} = \frac{\sin \theta \sin \theta \sin \theta}{\sin \theta \sin \theta \sin \theta}$$

T. SEND

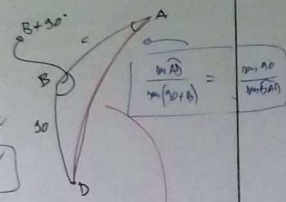
$$\cos a = \cos b \sin c + \sin b \sin c \cos A$$

F. COSENO

DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

COORDENADA \hat{z} DE $\vec{r}_0 = -\cos C \cos B$

$$\begin{cases} x = r \sin \theta \cdot \cos \psi \\ y = r \sin \theta \cdot \sin \psi \\ z = r \cos \theta \end{cases}$$



i	j	k
$\sin b \cos A$	$\sin b \sin A$	$\cos b$
$\sin c$	0	$\cos c$

$\vec{r}_c \wedge \vec{r}_b = \sin a \vec{r}_0$

$\psi = \widehat{BAD}$
 $\theta = \widehat{AD}$

COORDENADA \hat{z} DE \vec{r}_0

$\sin \widehat{AD} \cdot \sin \widehat{BAD} = \sin(B+90) \cdot \sin 90 = \cos B$

F. SENO $\cos B$ 1

$\sin b \sin A \cos c$ $\cos b \sin c - \sin b \sin A \cos c$

$-\sin b \sin A \cos c$

$\cos b \sin c - \sin b \sin A \cos c = \sin a \cos B$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

F. COSENO



DEDUCCION FORM. TRIG. ESFERICA

COORDENADA Z DE $\vec{r}_0 = -m.c.m.B$

$$\begin{cases} x = m.\theta . \cos\psi \\ y = m.\theta . \sin\psi \\ z = \cos\theta \end{cases}$$

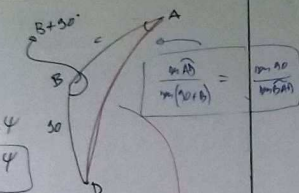
i	j	k
m.b.cosA	m.b.m.A	cosB
m.c	0	cosC

COORDENADA X DE \vec{r}_0

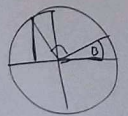
$m.\widehat{AD} . m.\widehat{BAD} = \cos(B+90) . \cos(90) = \cos B$

F. SEENO

$\vec{r}_c \wedge \vec{r}_b = m.a.\vec{r}_0$



$\psi = \widehat{BAD}$
 $\theta = \widehat{AD}$



$\cos m.c - m.b . \cos C = m.a . \cos B$

F. ANALOGA

$\frac{m.a}{m.A} = \frac{m.b}{m.B} = \frac{m.c}{m.C}$

T. SEENO

$\cos c = \dots$
 $\cos b = \cos A . \cos c + m.a . m.c . \cos B$

$\cos a = \cos b . \cos c + m.b . m.c . \cos A$

F. COSENO

$m.b.m.A \cos c$ $\cos m.c - m.b.m.A \cos c$

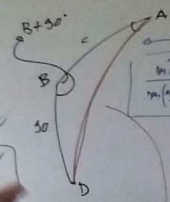
$- m.b.m.A \cos c$

DEDUCCIÓN

G. ESFÉRICA

COORDENADA \geq DE γ

$$\begin{cases} x = \rho \cdot \cos \theta \cdot \cos \psi \\ y = \rho \cdot \cos \theta \cdot \sin \psi \\ z = \rho \cdot \sin \theta \end{cases}$$



$$\frac{\widehat{m AB}}{\widehat{m (a+b)}} = \frac{\widehat{m AD}}{\widehat{m (a+b)}}$$

$$\psi = \widehat{BAD}$$

$$\theta = \widehat{AD}$$

$$\widehat{AD} \cdot \widehat{BAD} = \widehat{\sin(B+90)} \cdot \widehat{\sin 90} = \widehat{\cos B}$$

\uparrow F. SEND $\cos B$ \uparrow

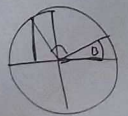
$$\widehat{\cos b \sin c} - \widehat{\sin b \cos c} \cdot \widehat{\cos A} = \widehat{\sin a \cos B}$$

F. ANALOGA

OPERANDO CON F. SEND Y F. COSEND:

F. 4 PARTES:

$$\widehat{\cot b \sin a} = \widehat{\cos a \cos C} + \widehat{\sin C \cot B}$$



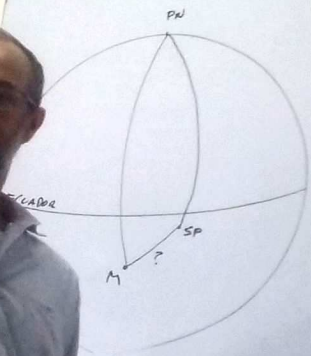
$$\frac{\widehat{\sin a}}{\widehat{\sin A}} = \frac{\widehat{\sin b}}{\widehat{\sin B}} = \frac{\widehat{\sin c}}{\widehat{\sin C}}$$

T. SEND

$$\widehat{\cos c} = \widehat{\cos a \cos c} + \widehat{\sin a \sin c \cos B}$$

$$\widehat{\cos a} = \widehat{\cos b \cos c} + \widehat{\sin b \sin c \cos A}$$

F. COSEND



DIST. M-SP
 $M (\lambda = -56^\circ, \phi = -35^\circ)$
 $SP (\lambda = ?)$

$$\cos b \sin c - \sin b \cos c \cos A = \sin a \cos B$$

F. ANÁLOGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cot b \sin a = \sin a \cos C + \sin C \cot B$$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO



$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

F. COSENO



DIST. M-SP

$$M (\lambda = -56^\circ, \phi = -35^\circ)$$

$$M (\lambda = -48^\circ, \phi = -23^\circ)$$

$$(M-SP) =$$

$$\cos m c - m b \cdot \cos A = m a \cdot \cos B$$

F. ANALOGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cot b \cdot \sin a = \cos a \cdot \cos C + \sin C \cdot \cot B$$



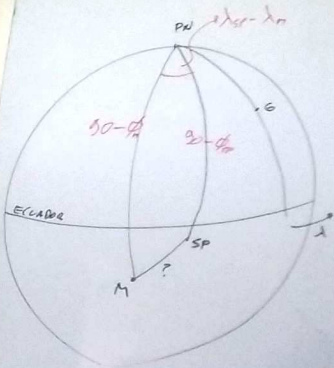
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO

$$\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos C$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO



DIST. M-SP

$$M (\lambda = -56^\circ, \phi = -35^\circ)$$

$$SP (\lambda = -48^\circ, \phi = -23^\circ)$$

$$\cos(M-SP) = \sin \phi_n \cdot \sin \phi_{sp} + \cos \phi_n \cdot \cos \phi_{sp} \cdot \cos(\lambda_{sp} - \lambda_n)$$

$$\widehat{M-SP} = 33^\circ \cdot \frac{\pi}{180^\circ} \text{ (RAD)} \cdot R_T \rightarrow 6400 \text{ km}$$

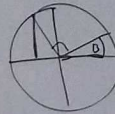
$$\cos m c - m b \cdot \cos A = m a \cdot \cos B$$

F. ANALOGA

OPERANDO CON F. SENOS Y F. COSENO:

F. 4 PARTES:

$$\cot b \cdot \sin a = \cos a \cdot \cos C + \sin C \cdot \cot B$$



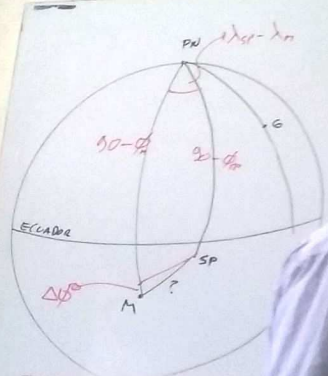
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENOS

$$\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos C$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO



DIST. M-SP
 $M (\lambda = -35^\circ)$
 $SP (\lambda = -23^\circ)$

$$= \sin \phi_m \cdot \sin \phi_{sp} + \cos \phi_m \cdot \cos \phi_{sp} \cdot \cos (\lambda_{sp} - \lambda_m)$$

$\frac{R}{R_\odot}$ (RAD) $\cdot R_\odot \rightarrow 6400 \text{ km}$

DADO $\Delta \phi_m$
 $(\Delta \lambda = 0) \Rightarrow \Delta$

$$\Delta \phi_m - \cos \phi_{sp} \cdot \sin \phi_m \cdot \cos (\lambda_{sp} - \lambda_m) \cdot \Delta \phi_m$$

$$\cos m c - m b \cdot \cos A = m a \cdot \cos B$$

F. ANALOGA

OPERANDO CON F. SENO Y F. COSENO:
 F. 4 PARTES:
 $\cos b \cdot \sin a = \sin a \cdot \cos c + \sin c \cdot \cos B$

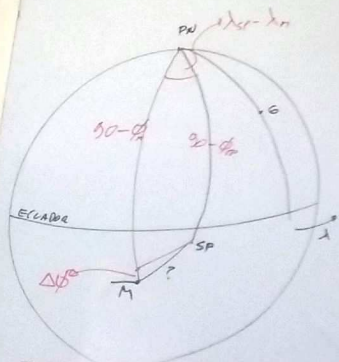
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO



$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO



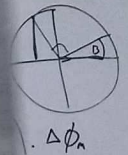
DIST. M-SP
 M ($\lambda = -56^\circ, \phi = -35^\circ$)
 SP ($\lambda = -48^\circ, \phi = -23^\circ$)

DADO Δr_M
 $(\Delta \lambda = 0) \Rightarrow \Delta(r_{M-SP}) ?$

$$\cos(\pi - \theta) = \sin \phi_m \cdot \sin \phi_{sp} + \cos \phi_m \cdot \cos \phi_{sp}$$

$$\pi - \theta = 33^\circ \cdot \frac{\pi}{180^\circ} \text{ (RAD)} \cdot \frac{R_T}{R_T} \rightarrow 6'$$

$$-\sin(\pi - \theta) \cdot \Delta(\pi - \theta) = \sin \phi_{sp} \cdot \cos \phi_m \cdot \Delta \phi$$



$$\cos m c - m b \cdot \cos c \cdot \cos A = m a \cdot \cos B$$

F. ANALOGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cos^2 b \cdot m a = \cos a \cdot \cos c + \sin c \cdot \cos^2 B$$

$$\frac{m a}{\sin A} = \frac{m b}{\sin B} = \frac{m c}{\sin C}$$

T. SENO

$$\cos b = \cos a \cdot \cos c + \sin a \cdot \sin c \cdot \cos B$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO