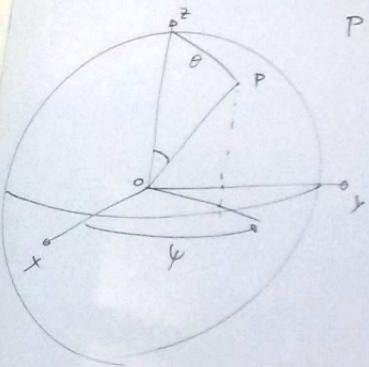
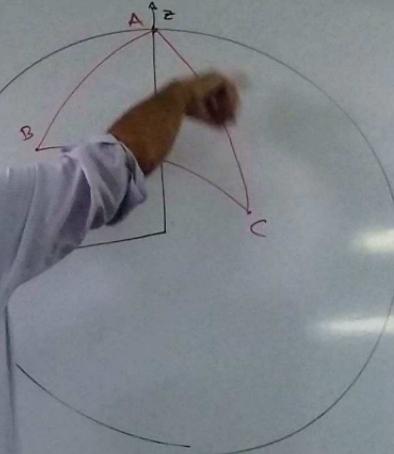
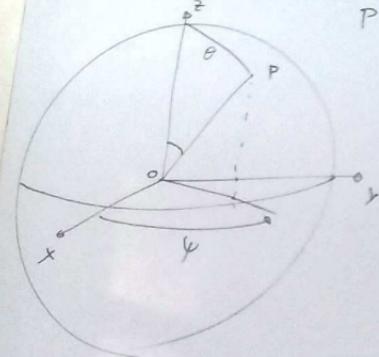


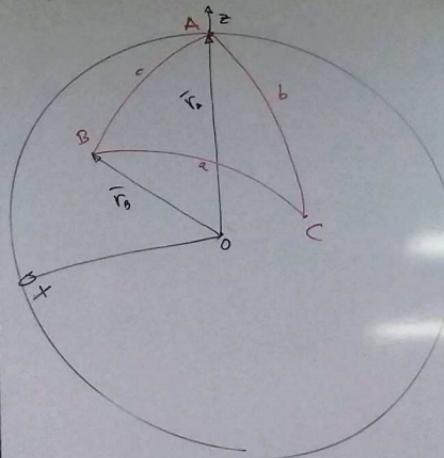
DEDUCCIÓN FÓRM. TRIG. ESFÉRICA $P(\varphi, \theta)$ 

$$\begin{cases} x = r \cos \theta \cdot \sin \varphi \\ y = r \sin \theta \cdot \sin \varphi \\ z = r \cos \varphi \end{cases}$$



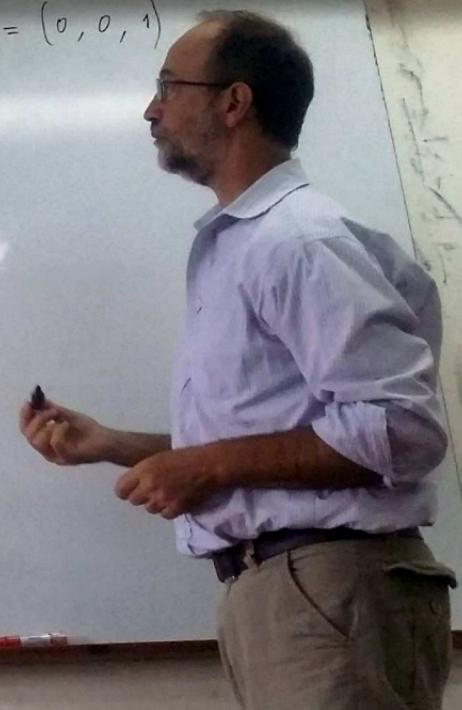
DEDUCCIÓN FÓRM. TRIG. ESFÉRICA $P(\psi, \theta)$ 

$$\begin{cases} x = r \cos \theta \cdot \cos \psi \\ y = r \cos \theta \cdot \sin \psi \\ z = r \sin \theta \end{cases}$$



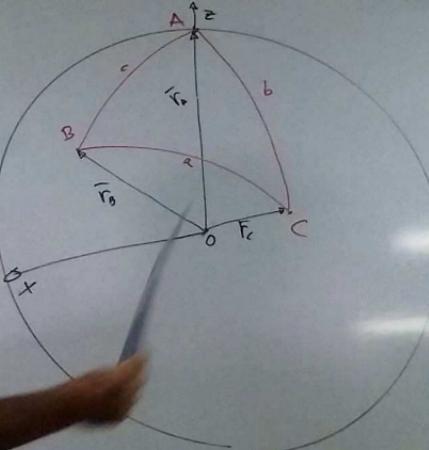
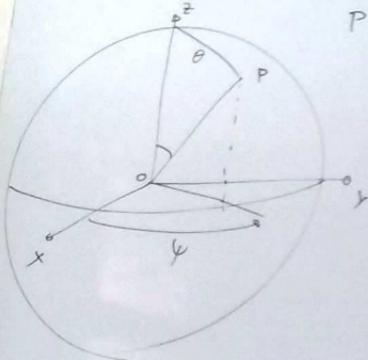
$\vec{r}_A = (i \cos \theta, \theta = 0^\circ) = (0, 0, 1)$

$\vec{r}_B = (\psi = 0^\circ, \theta =$



DEDUCCIÓN FÓRM. TRIG. ESFÉRICA $P(\psi, \theta)$ 

$$\begin{cases} x = r \cos \theta \cdot \cos \psi \\ y = r \cos \theta \cdot \sin \psi \\ z = r \sin \theta \end{cases}$$



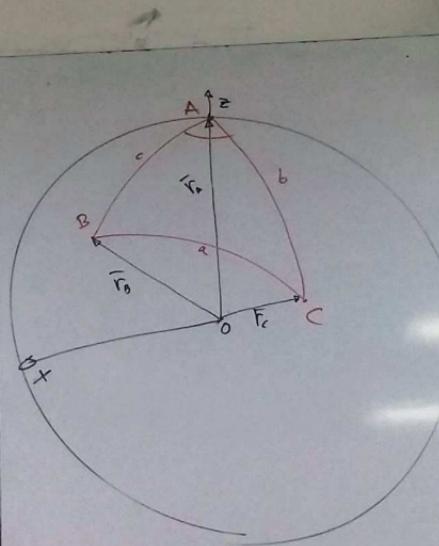
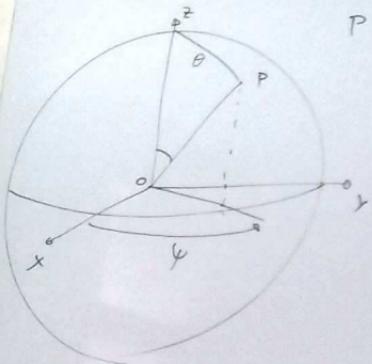
$\vec{r}_a = (\text{inc}, \theta = 0^\circ) = (0, 0, 1)$

$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (r \cos c, 0, \sin c)$

$\vec{r}_c = (\psi =$

DEDUCCIÓN FÓRM. TRIG. ESFÉRICA $P(\psi, \theta)$ 

$$\begin{cases} x = r \cos \theta \cdot \cos \psi \\ y = r \cos \theta \cdot \sin \psi \\ z = r \sin \theta \end{cases}$$



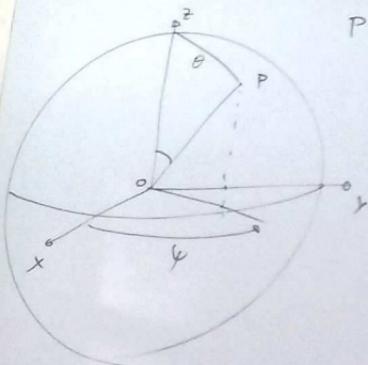
$$\vec{r}_a = (\text{umbra}, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (r \cos c, 0, \sin c)$$

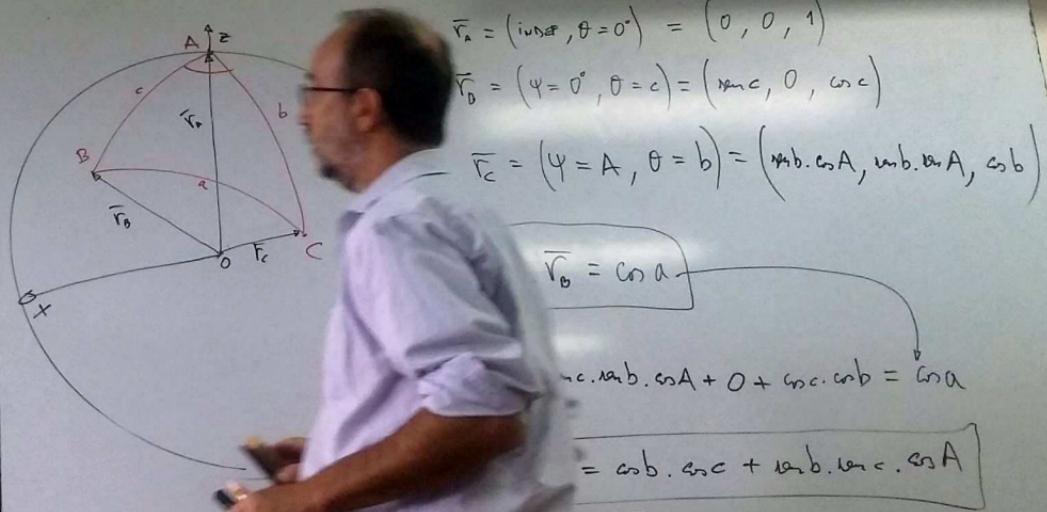
$$\vec{r}_c = (\psi = A, \theta = b) = (r \cos b \cdot \cos A, r \cos b \cdot \sin A, \sin b)$$

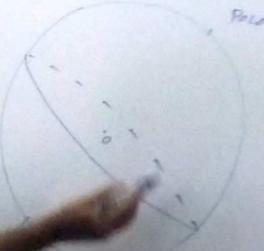
$$\vec{r}_c \cdot \vec{r}_b = \cos a$$



DEDUCCIÓN FORM. TRIG. ESFÉRICA $P(\psi, \theta)$ 

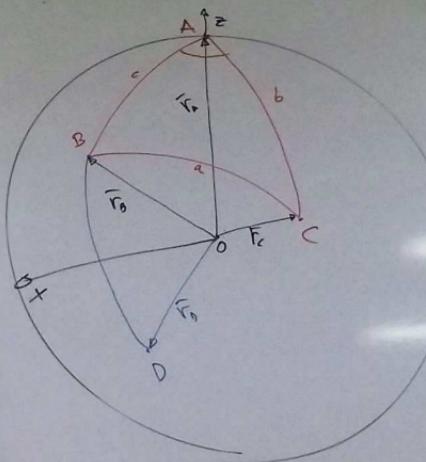
$$\begin{cases} x = r \cos \theta \cdot \cos \psi \\ y = r \cos \theta \cdot \sin \psi \\ z = r \sin \theta \end{cases}$$



DEDUCCIÓN FÓRM. TRIG. ESFÉRICAPOLO

$$\begin{cases} x = r \cos \theta \cdot \cos \psi \\ y = r \cos \theta \cdot \sin \psi \\ z = r \sin \theta \end{cases}$$

$$\bar{r}_c \wedge \bar{r}_b = \text{Ima.}$$



$$\bar{r}_a = (\text{Ima}, \theta = 0^\circ) = (0, 0, 1)$$

$$\bar{r}_b = (\psi = 0^\circ, \theta = c) = (r \cos c, 0, \sin c)$$

$$\bar{r}_c = (\psi = A, \theta = b) = (r \cos b \cdot \cos A, r \cos b \cdot \sin A, \sin b)$$

$$\bar{r}_c \cdot \bar{r}_b = \text{Ima}$$

$$\bar{r}_c \cdot \bar{r}_b = r \cos c \cdot r \cos b \cdot \cos A + 0 + \sin c \cdot \sin b = \text{Ima}$$

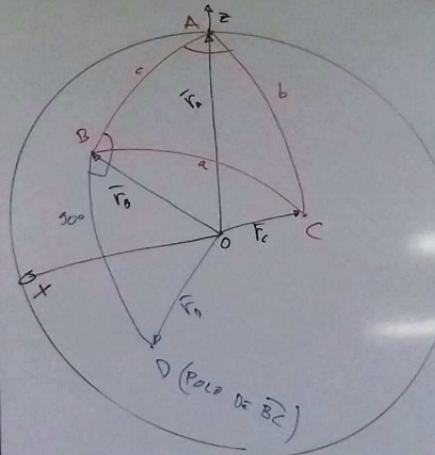
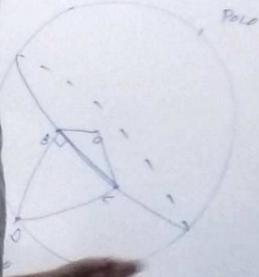
$$\text{Ima} = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

### Introducción Fórm. TRIG. ESFÉRICA

$$\begin{cases} x = \text{sen}\theta \cdot \cos\psi \\ y = \text{sen}\theta \cdot \text{sen}\psi \\ z = \cos\theta \end{cases}$$

$$\overline{r}_c \wedge \overline{r}_b = \text{sen}\alpha$$



$$\overline{r}_a = (\text{sen}\alpha, \theta = 0^\circ) = (0, 0, 1)$$

$$\overline{r}_b = (\psi = 0^\circ, \theta = c) = (\text{sen}c, 0, \cos c)$$

$$\overline{r}_c = (\psi = A, \theta = b) = (\text{sen}b \cdot \cos A, \text{sen}b \cdot \text{sen} A, \cos b)$$

$$\overline{r}_c \cdot \overline{r}_b = \cos\alpha$$

$$\overline{r}_c \cdot \overline{r}_b = \text{sen}c \cdot \text{sen}b \cdot \cos A + 0 + \cos c \cdot \cos b = \cos\alpha$$

$$\boxed{\cos\alpha = \cos b \cdot \cos c + \text{sen}b \cdot \text{sen}c \cdot \cos A}$$

F. COSENO

DEDUCCIÓN

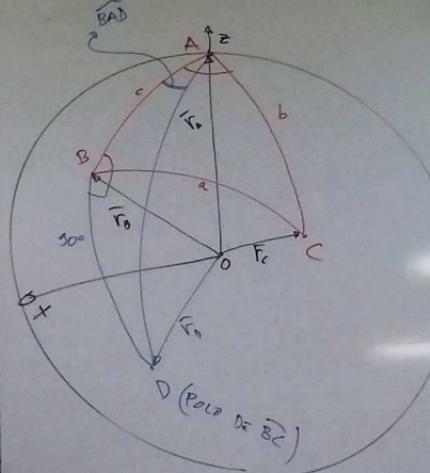
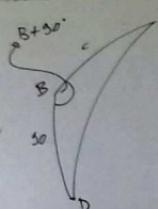
POLO

TRIG. ESFÉRICA

$$\begin{cases} x = \text{sen}\theta \cdot \text{sen}\psi \\ y = \text{sen}\theta \cdot \text{sen}\psi \\ z = \text{sen}\theta \end{cases}$$

$$\overline{r}_c \wedge \overline{r}_b = \text{sen} \alpha \cdot \overline{r}_b \quad \alpha = \widehat{BAD}$$

$$\text{sen} \widehat{AD} =$$



$$\overline{r}_a = (\text{sen}\alpha, \theta = 0) = (0, 0, 1)$$

$$\overline{r}_b = (\psi = 0^\circ, \theta = c) = (\text{sen}c, 0, \text{cos}c)$$

$$\overline{r}_c = (\psi = A, \theta = b) = (\text{sen}b \cdot \text{sen}A, \text{sen}b \cdot \text{sen}A, \text{cos}b)$$

$$\overline{r}_c \cdot \overline{r}_b = \text{cos} \alpha$$

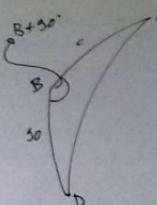
$$\overline{r}_c \cdot \overline{r}_b = \text{sen}c \cdot \text{sen}b \cdot \text{cos}A + 0 + \text{sen}c \cdot \text{sen}b = \text{cos} \alpha$$

$$\boxed{\text{cos} \alpha = \text{sen}b \cdot \text{sen}c + \text{sen}b \cdot \text{sen}c \cdot \text{cos}A}$$

F. COSENO

DEDUCCIÓN FORM. TRIG. ESFÉRICACOORDENADA Z DE  $\vec{r}_0$ 

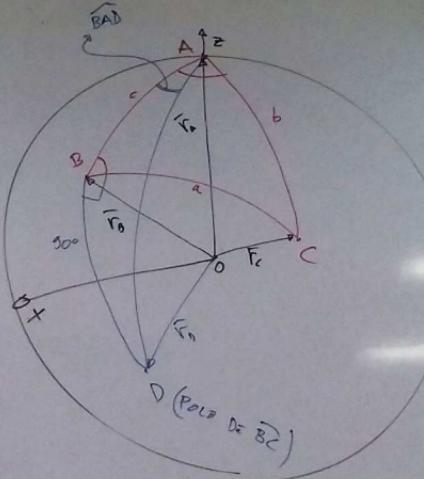
$$\begin{cases} x = \text{sen}\theta \cdot \cos\psi \\ y = \text{sen}\theta \cdot \text{sen}\psi \\ z = \cos\theta \end{cases}$$



$$\vec{r}_c \wedge \vec{r}_b = \text{sen}a \cdot \vec{r}_b \quad \psi = \widehat{BAD} \quad \theta = \widehat{AD}$$

$$\cos \widehat{AD} = \cos 50^\circ \cdot \cos c + (\text{sen} 50^\circ) \cdot \text{sen} c \cdot \cos(B+90^\circ)$$

$$\cos \widehat{AD} = -\text{sen}c \cdot \text{sen}B$$



$$\vec{r}_a = (\text{sen}\theta, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\text{sen}c, 0, \cos c)$$

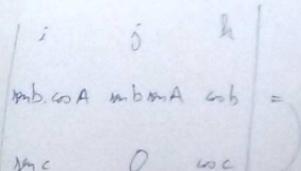
$$\vec{r}_c = (\psi = A, \theta = b) = (\text{sen}b \cdot \cos A, \text{sen}b \cdot \text{sen} A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_b = \cos a$$

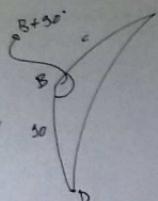
$$\vec{r}_c \cdot \vec{r}_b = \text{sen}c \cdot \text{sen}b \cdot \cos A + 0 + \text{sen}c \cdot \cos b = \cos a$$

$$\cos a = \cos b \cdot \cos c + \text{sen}b \cdot \text{sen}c \cdot \cos A$$

F. COSENO

DEDUCCIÓN FORM. TRIG. ESFERICACOORDENADA Z DE  $\vec{r}_0 = -\sin c \cos B$ 

$$\begin{cases} x = \sin \theta \cdot \cos \psi \\ y = \sin \theta \cdot \sin \psi \\ z = \cos \theta \end{cases}$$

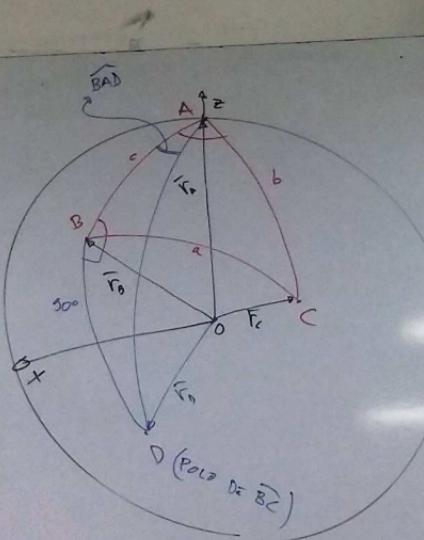


$$\vec{r}_c \wedge \vec{r}_0 = \sin \alpha \cdot \vec{r}_0 \quad \alpha = \widehat{AD}$$

$$\cos \widehat{AD} = \cos 90^\circ \cos c + (\sin 90^\circ) \sin c \cos (B + 90^\circ)$$

 $-\sin B$ 

$$\cos \widehat{AD} = -\sin c \cos B$$



$$\vec{r}_a = (\text{initial}, \theta = 0^\circ) = (0, 0, 1)$$

$$\vec{r}_b = (\psi = 0^\circ, \theta = c) = (\sin c, 0, \cos c)$$

$$\vec{r}_c = (\psi = A, \theta = b) = (\sin b \cos A, \sin b \sin A, \cos b)$$

$$\vec{r}_c \cdot \vec{r}_0 = \cos \alpha$$

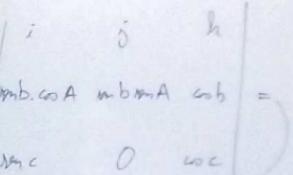
$$\vec{r}_c \cdot \vec{r}_b = \sin c \sin b \cos A + 0 + \sin c \cos b = \cos \alpha$$

$$\boxed{\cos \alpha = \cos b \cos c + \sin b \sin c \cos A}$$

F. COSENO

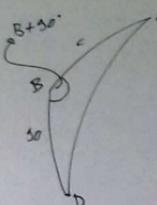
## DEDUCCIÓN FORM. TRIG. ESFÉRICA

$$\text{COORDENADA } Z \text{ DE } \bar{r}_0 = -\sin c \cos B$$



$$\begin{aligned} & \sin b \cos A \sin b \sin A \cos b = \\ & \sin c \quad O \quad \sin c \\ & (\sin b \sin A \cos c, \cos b \sin c - \sin b \cos A \cos c, \\ & -\sin b \sin A \sin c) \end{aligned}$$

$$\begin{cases} x = \sin \theta \cdot \cos \psi \\ y = \sin \theta \cdot \sin \psi \\ z = \cos \theta \end{cases}$$

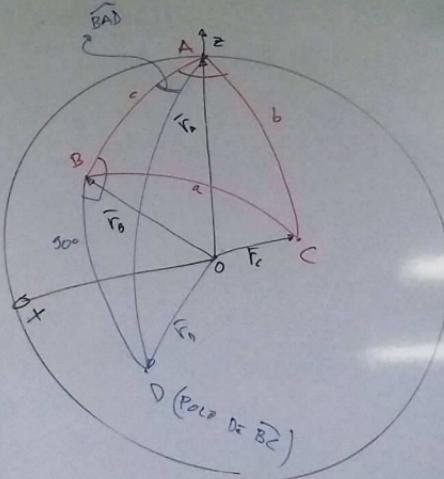


$$\bar{r}_c \wedge \bar{r}_b = \sin \alpha \cdot \bar{r}_0 \quad \psi = \widehat{BAD} \quad \theta = \widehat{AD}$$

$$\cos \widehat{AD} = \cancel{\cos 90} \cdot \cos c + \cancel{(\sin 90)} \cdot \sin c \cdot \cos (B+90)$$

$$\cos \widehat{AD} = -\sin c \cos B$$

$$-\sin B$$



$$\begin{aligned} & -\sin b \sin A \sin c = \\ & \uparrow \\ & \text{COORD. } Z \text{ DE} \\ & \bar{r}_c \wedge \bar{r}_b \end{aligned}$$

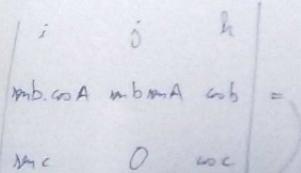
$$\begin{bmatrix} \text{Co.} \end{bmatrix}$$

$$\cos A$$



## DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

$$\text{COORDENADA } Z \text{ DE } \vec{r}_0 = -m_c m_B$$



$$\begin{cases} x = m_c \cdot \cos \psi \\ y = m_c \cdot \sin \psi \\ z = \cos \theta \end{cases}$$

$\hat{\varphi} = \widehat{BAD}$   
 $\hat{\theta} = \widehat{AD}$

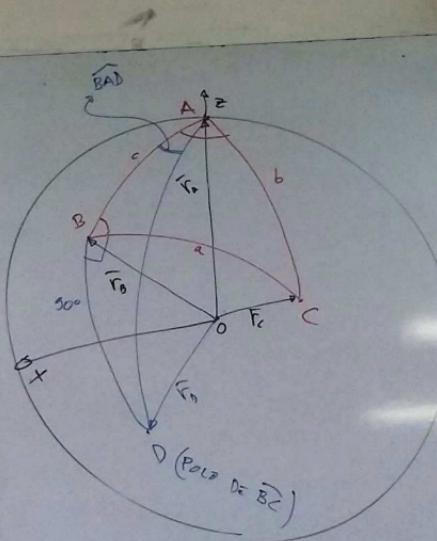
$$\vec{r}_c \wedge \vec{r}_0 = m_a \vec{r}_0$$

$$\cos \widehat{AD} = \cos 30^\circ \cos c + (\sin 30^\circ) \cdot \sin c \cdot \cos (B + 30^\circ)$$

$$-\sin B$$

$$\cos \widehat{AD} = -m_c \cdot m_B$$

$m_c \cdot \cos c - m_b \cos A - m_b \cos c$   
 $-m_b \sin A \cdot m_c$



$$f_{\text{máx}} m_A m_C = f_{\text{máx}} m_A m_C \cdot m_B$$

coord.  $\geq 0$   
 $\vec{r}_c \wedge \vec{r}_0$

$$\frac{m_a}{m_A} = \frac{m_b}{m_B} = \frac{m_c}{m_C}$$

T. SENO

$$\cos A = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

## DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

$$\text{COORDENADA } Z \text{ DE } \vec{r}_0 = -\sin c \cos B$$

$$\begin{matrix} i & j & k \\ \sin b \cos A & \sin b \sin A & \cos b \\ \hline \sin c & 0 & \cos c \end{matrix}$$

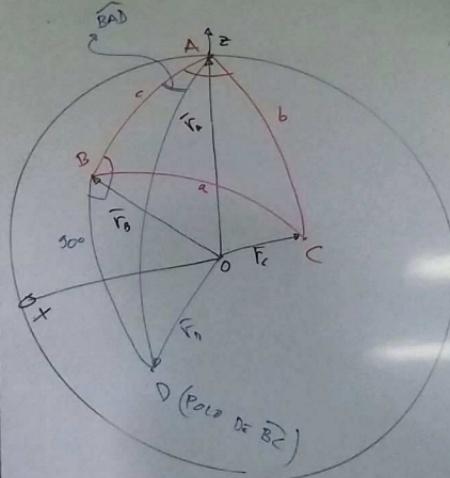
$$\begin{matrix} \sin b \sin A \cos c & \cos b \cos c - \sin b \sin A \cos c \\ \hline -\sin b \sin A \sin c & \end{matrix}$$

$$\begin{cases} x = \sin B \cdot c \\ y = \sin B \cdot b \\ z = \cos B \end{cases} \quad (*)$$

$$\vec{r}_c \wedge \vec{r}_b = \sin a \cdot \vec{r}_0$$

$$\text{COORDENADA } Z \text{ DE } \vec{r}_0$$

$$\sin \widehat{AB} \cdot \sin \widehat{BAC} = \sin(B + 90)$$



$$f_{\text{seno}} \sin A \sin C = f_{\text{seno}} \sin C \cdot \sin B$$

$$\begin{matrix} \text{COORD. } Z \text{ DE} \\ \vec{r}_c \wedge \vec{r}_b \end{matrix}$$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

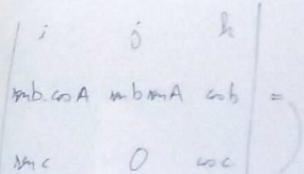
T. SENO

$$\cos A = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO

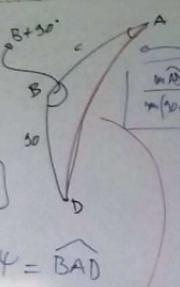
## DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

$$\text{COORDENADA } \gamma \text{ DE } \overline{r}_0 = -\sin c \cos B$$



$$\begin{cases} x = \sin \theta \cdot \cos \psi \\ y = \sin \theta \cdot \sin \psi \\ z = \cos \theta \end{cases}$$

$$(*)$$



$$\overline{r}_c \wedge \overline{r}_b = \sin \alpha \overline{r}_d$$

$$\theta = \widehat{AD}$$

$$\text{COORDENADA } \gamma \text{ DE } \overline{r}_0$$

$$\sin \widehat{AD} \cdot \sin \widehat{BAD} = \sin(B+90^\circ) \sin 90^\circ = \cos B$$

$$F. \text{ SEÑO}$$

$$-\sin \alpha \cos \beta$$

$$\text{coseno} = \sin \alpha \cos \beta$$

$$\cos b \cos c - \sin b \sin c \cos A = \sin a \cos B$$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

T. SENO

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO



### DEDUCCIÓN FÓRM. TRIG. ESFÉRICA

$$\text{COORDENADA } \Sigma \text{ DE } \bar{r}_0 = -m_c m_b$$

$$\begin{array}{c|c|c} i & j & h \\ \hline m_b \cos A & m_b m_A \cos b & = \\ m_c & 0 & m_c \end{array}$$

$$\begin{array}{c} m_b m_A \cos c \\ \text{coseno} - m_b m_A \cos c \\ - m_b m_A m_c \end{array}$$

$$\begin{cases} x = m_b \cos \psi \\ y = m_b \sin \psi \end{cases}$$

$$z = \cos \theta$$

$$\bar{r}_c \wedge \bar{r}_0 = m_a \bar{r}_0$$

$$\psi = \widehat{BAD}$$

$$\theta = \widehat{AD}$$

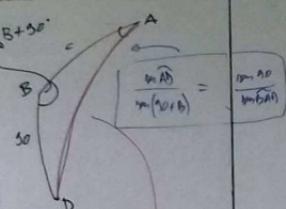
$$\text{COORDENADA } \Sigma \text{ DE } \bar{r}_0$$

$$\cos \widehat{AB} \cdot m_a \widehat{BAD} = \cos(B+90^\circ) \cdot m_a \widehat{AD} = \cos B$$

F. SENO

$$\sin B$$

$$\sin(90^\circ)$$



$$\cos b m_c - m_b \cos c \cos A = m_a \cos B$$

F. ANÁLOGA

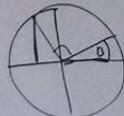
$$\frac{m_a}{m_A} = \frac{m_b}{m_B} = \frac{m_c}{m_C}$$

T. SENO

$$\begin{array}{c} \sin c = - \\ \sin b = \cos A \cos C + m_A m_C \cos B \end{array}$$

$$\cos A = \cos b \cos c + \sin b \sin c \cos A$$

F. COSENO



## DEDUCCIÓN

COORDENADA Z DE LOS VECTORES

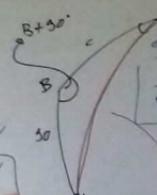
 $\cos b \cdot \cos A$  $\sin c$  $\sin A$  $-B$ 

## G. ESFÉRICA

$$x = \sin B \cdot \cos \psi$$

$$y = \sin B \cdot \sin \psi$$

$$z = \cos B$$



$$\frac{m(\overline{AB})}{m(\overline{AC})} = \frac{\sin \psi}{\sin(30^\circ)}$$

$$\cot b \cdot \sin A = \sin c \cdot \cos B + \sin B \cdot \cos c$$

F. SENO

COS B

SIN 30°

= COS B

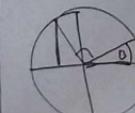
$$\cos b \cdot \sin c - \sin b \cdot \cos c \cdot \cos A = \sin a \cdot \cos B$$

## F. ANÁLOGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cot b \cdot \sin a = \sin c \cdot \cos B + \sin B \cdot \cos c$$



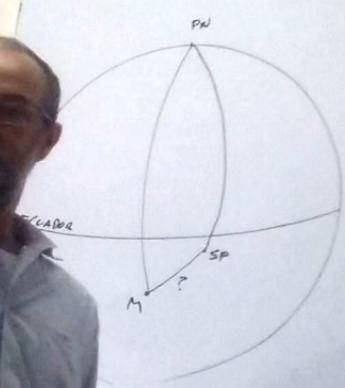
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

## T. SENO

$$\cos b = \cos A \cdot \cos C + \sin A \cdot \sin C \cdot \cos B$$

$$\cos A = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

## F. COSENO



DIST. M-SP

$$m (\lambda = -59^\circ, \phi = -35^\circ)$$

$$SP (\lambda =$$

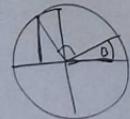
$$\cos b \cdot m c - m b \cdot \cos c \cdot \cos A = m a \cdot \cos B$$

F. ANALÓGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cot b \cdot m a = m a \cdot \cos C + m c \cdot \cot B$$



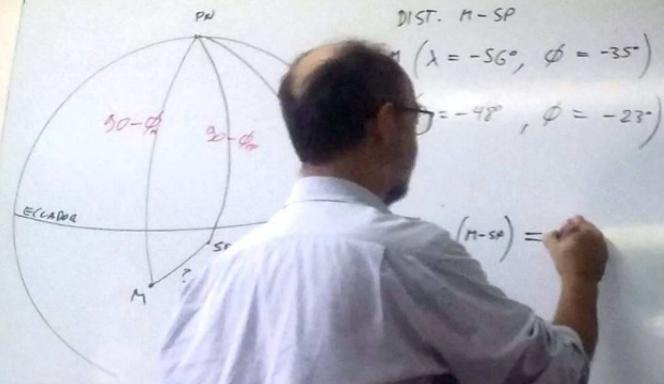
$$\frac{m a}{\sin A} = \frac{m b}{\sin B} = \frac{m c}{\sin C}$$

T. SENO

$$\begin{aligned} m c &= \dots \\ \cos b &= \cos A \cdot \cos C + m a \cdot m c \cdot \cos B \end{aligned}$$

$$\cos A = \cos b \cdot \cos c + \cos b \cdot \cos c \cdot \cos A$$

F. COSENO



$$\cot b \cdot \csc A = \csc a \cdot \cos B$$

F. ANALÓGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cot b \cdot \csc A = \csc a \cdot \cos C + \csc C \cdot \cot B$$



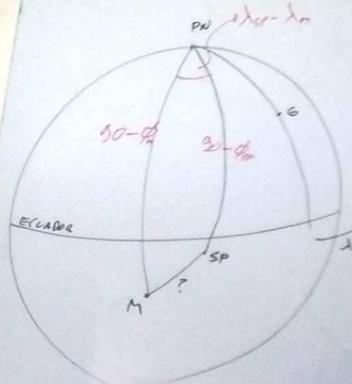
$$\frac{\csc A}{\csc A} = \frac{\csc b}{\csc B} = \frac{\csc c}{\csc C}$$

T. SENO

$$\begin{aligned} \csc c &= \dots \\ \csc b &= \csc A \cdot \csc C + \csc A \cdot \csc C \cdot \csc B \end{aligned}$$

$$\csc A = \csc b \cdot \csc C + \csc b \cdot \csc c \cdot \csc A$$

F. COSENO



DIST. M-SP

$$M(\lambda = -59^\circ, \phi = -35^\circ)$$

$$SP(\lambda = -48^\circ, \phi = -23^\circ)$$

$$\cos(M-SP) = \sin\phi_m \cdot \sin\phi_{sp} + \cos\phi_m \cdot \cos\phi_{sp} \cdot \cos(\lambda_{sp} - \lambda_m)$$

$$\widehat{M-SP} = 33^\circ \cdot \frac{\pi}{180^\circ} (\text{RAD}) \cdot R_T \rightarrow 6400 \text{ km}$$

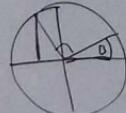
$$\cot b \cdot \cot c - \cot b \cdot \cot c \cdot \cot A = \cot a \cdot \cot B$$

F. ANALÓGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cot b \cdot \cot a = \cot a \cdot \cot c + \cot c \cdot \cot b$$



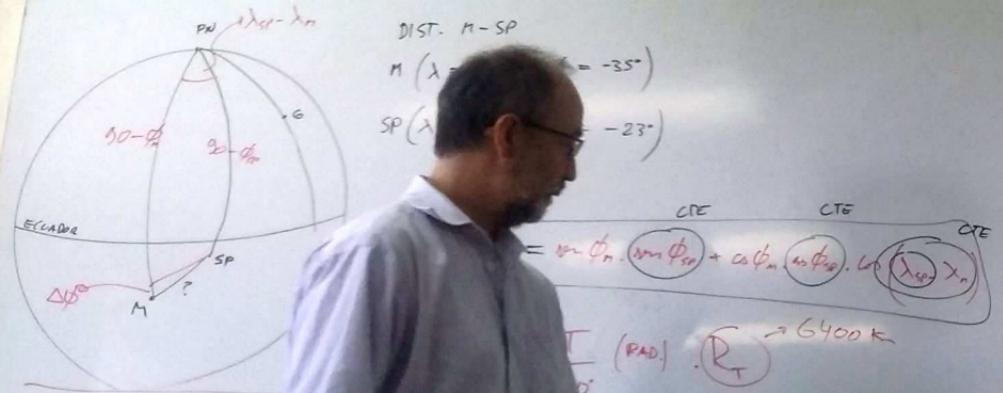
$$\frac{\cot a}{\cot A} = \frac{\cot b}{\cot B} = \frac{\cot c}{\cot C}$$

T. SENO

$$\begin{aligned} \cot c &= \dots \\ \cot b &= \cot a \cdot \cot c + \cot a \cdot \cot b \cdot \cot C \end{aligned}$$

$$\cot a = \cot b \cdot \cot c + \cot b \cdot \cot c \cdot \cot A$$

F. COSENO



DADO  $\Delta\phi_m$   
( $\phi_m = 0$ )  $\Rightarrow \Delta\phi_m$



$$\cos b \cdot \tan c - \tan b \cdot \cos c \cdot \cos A = \tan c \cdot \cos B$$

F. ANALOGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cot b \cdot \tan a = \tan a \cdot \cos C + \tan C \cdot \cot B$$



$$\tan a \cdot \Delta\phi_m = \tan \phi_m \cdot \tan \phi_m \cdot \cos(\lambda_m - \lambda_{SP}) \cdot \Delta\phi_m$$

$$\frac{\tan a}{\tan A} = \frac{\tan b}{\tan B} = \frac{\tan c}{\tan C}$$

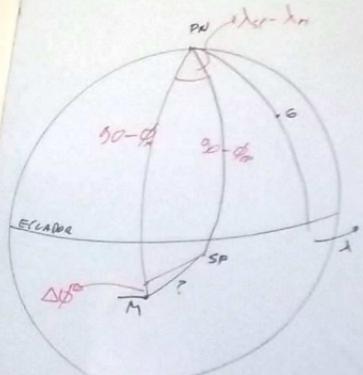
T. SENO

$$\tan c = \dots$$

$$\cot b = \cos A \cdot \cos C + \tan A \cdot \tan C \cdot \cos B$$

$$\cos a = \cos b \cdot \cos c + \tan b \cdot \tan c \cdot \cos A$$

F. COSENO



DIST. M-SP

$$M (\lambda = -50^\circ, \phi = -35^\circ)$$

$$SP (\lambda = -48^\circ, \phi = -23^\circ)$$

DADO  $\Delta\phi_n$   
( $\Delta\lambda = 0$ )  $\Rightarrow \Delta(\lambda-SP) ?$

$$\cos(\lambda-SP) = \sin\phi_n \cdot \sin\phi_{SP} + \cos\phi_n \cdot \cos\phi_{SP}$$

$$\lambda-SP = 33^\circ \cdot \frac{\pi}{180^\circ} \text{ (RAD)} \cdot R_T \rightarrow 6^\circ$$

$$-\Delta\phi_n(\lambda-SP), \Delta(\lambda-SP) = \sin\phi_{SP} \cdot \cos\phi_n \cdot \Delta\phi$$

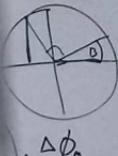
$$\cos b_m c - \sin b_m \cdot \sin c_m \cos A = m_a \cos B$$

F. ANÁLOGA

OPERANDO CON F. SENO Y F. COSENO:

F. 4 PARTES:

$$\cot b_m \cdot m_a = m_a \cdot \cos C + m_c \cdot \cot B$$



$$\frac{m_a}{m_A} = \frac{m_b}{m_B} = \frac{m_c}{m_C}$$

T. SENO

$$\begin{aligned} m_c &= - \\ \cos b &= \cos A \cdot \cos C + \sin A \cdot \sin C \cdot \cos B \end{aligned}$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

F. COSENO