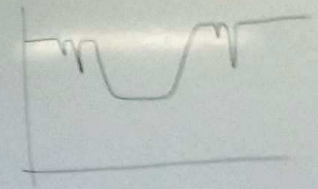
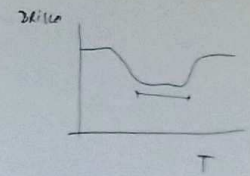
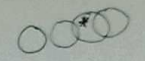
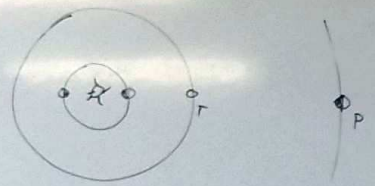
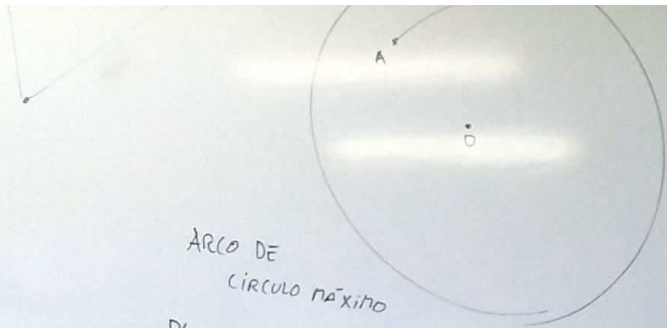


MIER. 10-11:30 → PRÁCTICO

VIERNES 10-12

..... DEPTO/AFY6/B





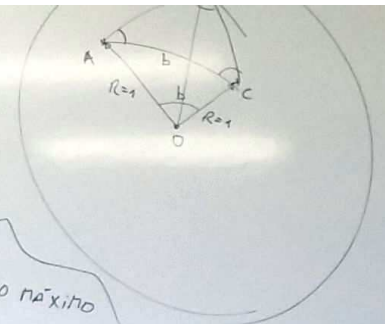
ARCO DE
CIRCULO MÁXIMO
PLANO POR O \cap ESFERA



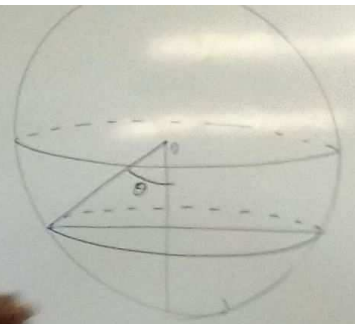


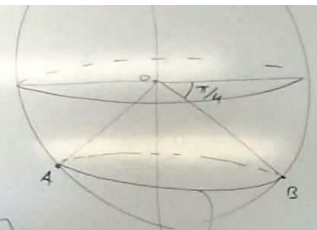
máximo
ESFERA





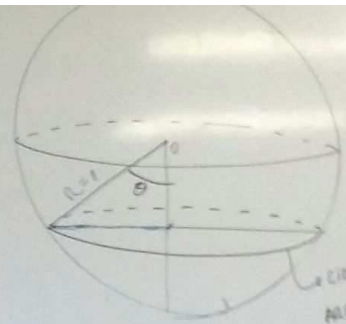
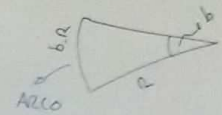
ARCO DE
CIRCULO MÁXIMO
PLANO POR O \cap ESFERA



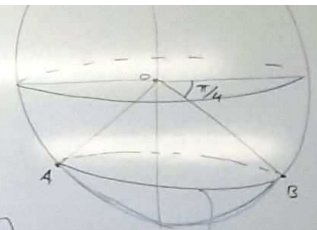
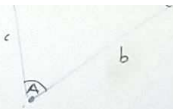


ARCO DE
CIRCULO MÁXIMO
ANO POR 5. N ESFERA

R
PARALELO
NOROCCIDENTAL
 $\widehat{AB} = \pi \cdot \sin \frac{\pi}{4}$



CIRCULO MENOR
ARCO TOTAL = $2\pi R \cdot \sin \theta$

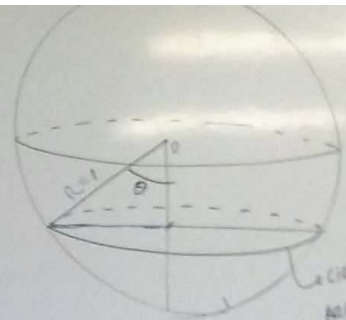
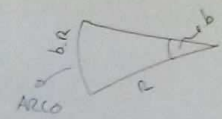


ARCO DE
CIRCULO MÁXIMO
PLANO POR O ∩ ESFERA

R PARALELO
CIRCULAR

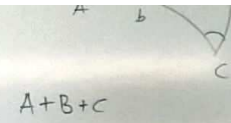
$\widehat{AB} = \pi \cdot \left(\frac{\pi}{4}\right)$ DOS

$\widehat{AB} (C_{MÁX}) = \pi/2$

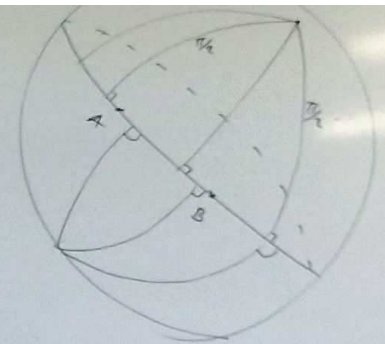


CIRCULO MENOR

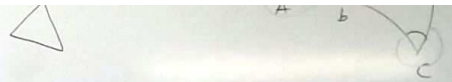
ARCO TOTAL = $2\pi R \cdot \cos \theta$



$$A+B+C$$



Polo

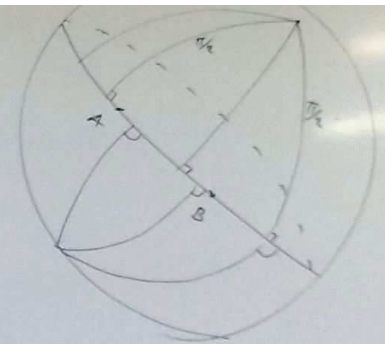


$$\pi < A+B+C < 3\pi$$

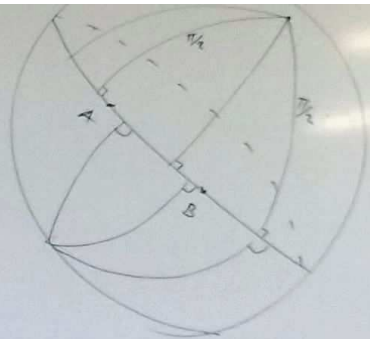
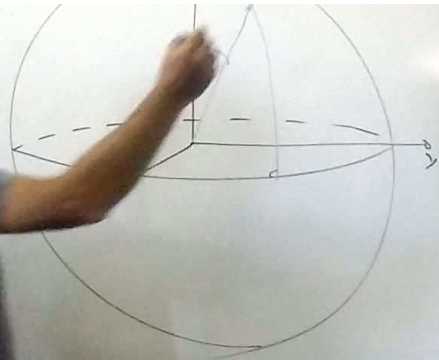
ESD ESFÉRICO: $E = A+B+C - \pi$

$$A = E \cdot R^2$$

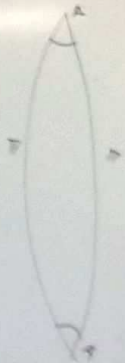
SRADT (T. ...)



Polos



Polos



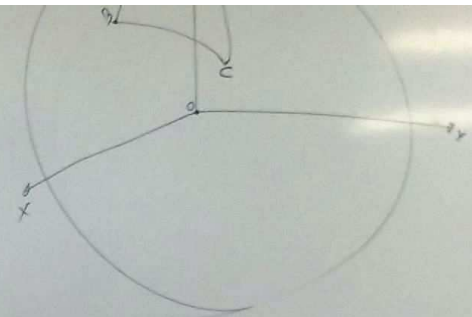
CAJO

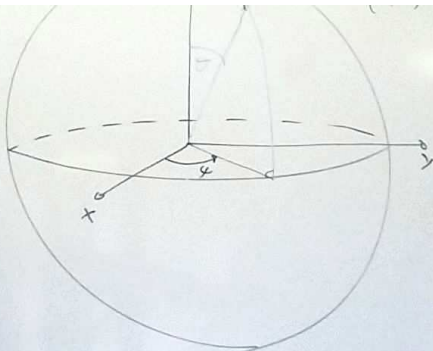


$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

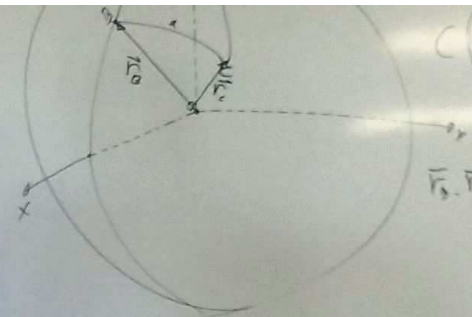
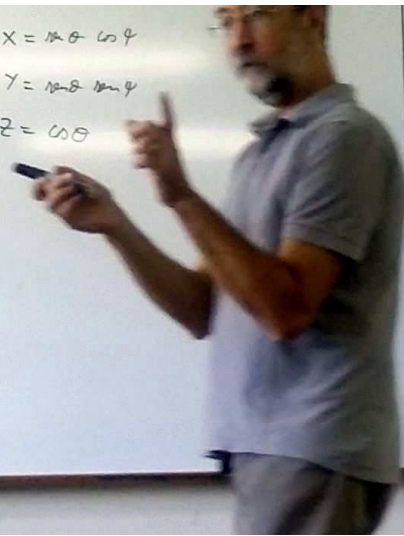




$$x = r \sin \theta \cos \phi$$

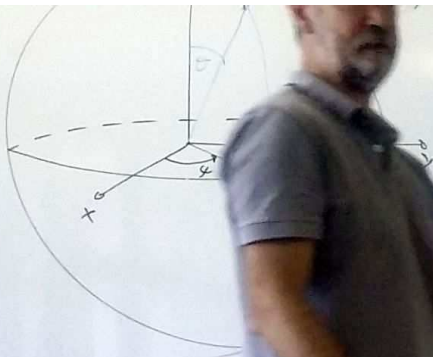
$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$



$$C(\theta = \theta_0, \phi = \phi_0) = (r \sin \theta_0 \cos \phi_0, r \sin \theta_0 \sin \phi_0, r \cos \theta_0)$$

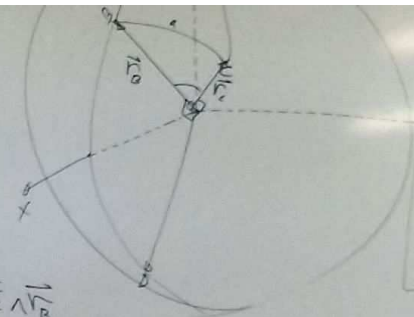
$$\vec{r}_1 \cdot \vec{r}_2 =$$



$$x = r \sin \theta \cos \varphi$$

$$y = r \sin \theta \sin \varphi$$

$$z = r \cos \theta$$



$$C(\theta=b, \varphi=A) = (r \sin b \cos A, r \sin b \sin A, r \cos b)$$

$$\vec{r}_b \cdot \vec{r}_c = r^2 \cos \alpha$$

$$\vec{r}_b \cdot \vec{r}_c = r^2 (\sin b \cos A \cos c + \sin b \sin A \sin c + \cos b \cos c)$$

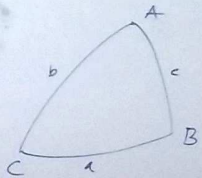
$$\vec{r}_c \wedge \vec{r}_b$$

F. SCLD

$$\Rightarrow \cos \alpha = \cos b \cos c + \sin b \sin c \cos A$$

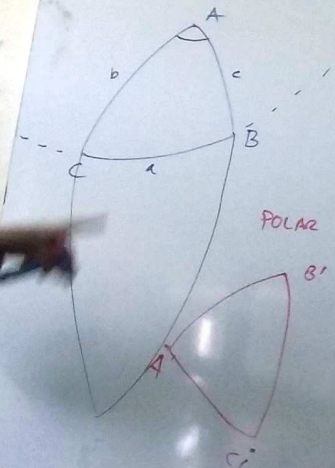
F. cos α = ...

△ ESFÉRICOS



△ ESFÉRICOS

$$\text{ÁREA} = E = A + B + C - \pi$$

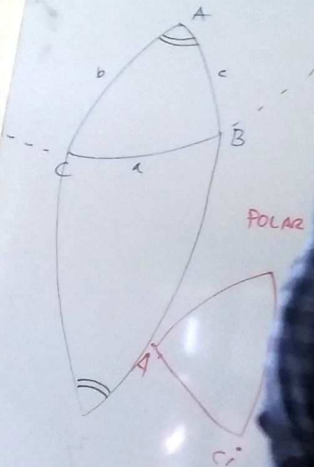


$$a' = \pi - A$$

$$A' = \pi - a$$

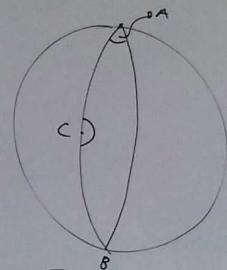
$$b' = \pi - B$$

Δ ESFÉRICOS



$$\text{ÁREA} = E = A + B + C - \pi$$

ÁREA GAIJO: 2A

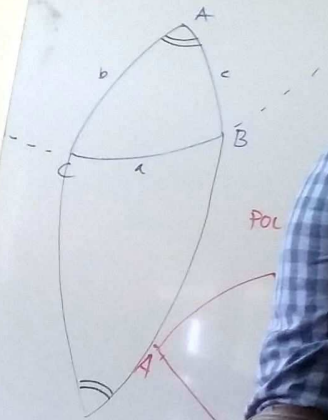


ÂNGULO	ÁREA GAIJO
$2\pi \rightarrow$	4π
$A \rightarrow$	$2A$

$$\text{ÁREA} = A + B + C - \pi$$

$$2A + \pi - \pi = 2A$$

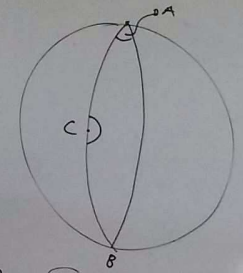
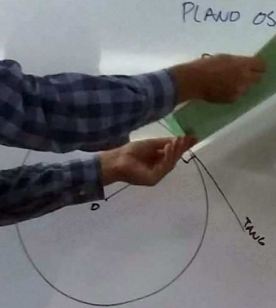
Δ ESFÉRICOS



$$\text{ÁREA} = E = A + B + C - \pi$$

$$\text{ÁREA GAIJO: } 2A$$

GEODÉSICA → PROPIEDAD:
 PLANO OSCULANTE ⊥
 TANGENTE

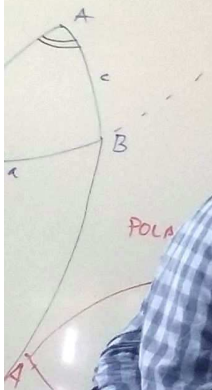


ÁNGULO	ÁREA GAIJO
2π	4π
A	$2A$

$$\text{ÁREA} = A + B + C - \pi$$

$$2A + \pi - \pi = 2A$$

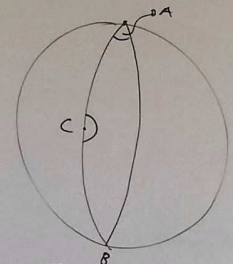
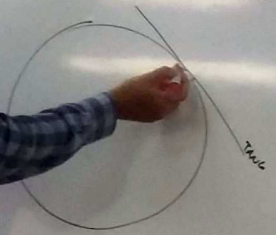
△ ESFÉRICOS



$$\text{ÁREA} = E = A + B + C - \pi$$

ÁREA GAJO: $2A$

GEODÉSICA → PROPIEDAD:
 PASA POR O → PLANO OSCULANTE ⊥
 PLANO TANGENTE

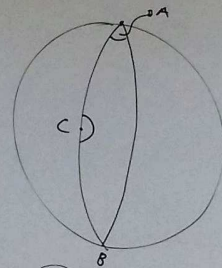
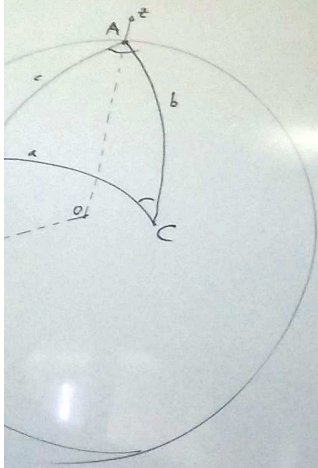


$$\text{ÁREA} = A + B + C - \pi$$

$$2A + \pi - \pi = 2A$$

ÁNGULO	ÁREA GAJO
2π	4π
A	$2A$

	$\frac{\pi}{6}$
180°	$\rightarrow \pi$
$30'$	$\rightarrow \frac{\pi}{6}$



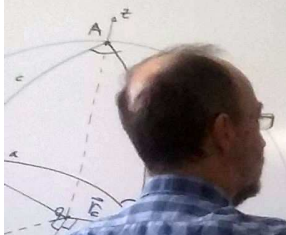
ÂNGULO	ÁREA GAISD
2π	$\rightarrow 4\pi$
A	$\rightarrow 2A$

$$\text{ÁREA} = A + D + C - \pi$$

$$2A + \pi - \pi = 2A$$

$$180^\circ \rightarrow \pi$$

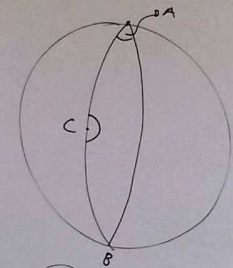
$$30' \rightarrow \pi/6$$



$$\vec{r}_c \cdot \vec{r}_B \rightarrow r_c \cdot r_B \cdot \cos \theta$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

$$\vec{r}_c \wedge \vec{r}_B = \vec{r}_0$$



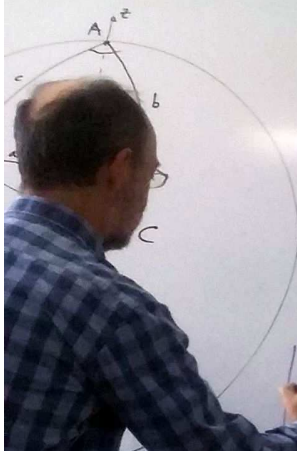
$$\text{ÁREA} = A + B + C - \pi$$

$$2A + \pi - \pi = 2A$$

ÂNGULO	ÁREA GAU
2π	$\rightarrow 4\pi$
A	$\rightarrow 2A$

$$180^\circ \rightarrow \pi$$

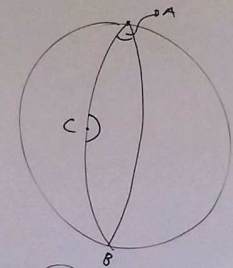
$$30' \rightarrow \pi/6$$



$\vec{r}_c \cdot \vec{r}_b \rightarrow r \cdot \cos \theta$
 $\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$

$\vec{r}_c \wedge \vec{r}_b = \vec{r}_0 \cdot \sin a$
 $(\sin c \cdot 0, \cos c)$
 $(\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$

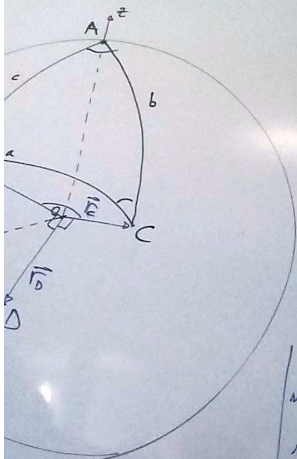
i | j | k



ÁREA = $A + C - \pi$
 $2A + \pi - \pi = 2A$

ÂNGULO	ÁREA GAÚD
2π	$\rightarrow 4\pi$
A	$\rightarrow 2A$

	$\frac{\pi}{6}$
180°	$\rightarrow \pi$
$30'$	$\rightarrow \pi/6$



$$\vec{r}_C \cdot \vec{r}_B \rightarrow F \cdot \cos \theta$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

$$\vec{r}_C + \vec{r}_B = \vec{r}_D \quad \sin a$$

$$(\sin c, 0, \cos c)$$

$$(\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$$

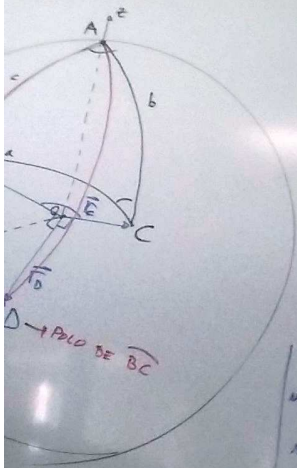
$$\begin{vmatrix} i & j & k \\ \sin b \cos A & \sin b \sin A & \cos b \\ \sin c & 0 & \cos c \end{vmatrix} = (\sin b \sin A \cos c, -\sin b \cos A \cos c + \sin b \sin c, -\sin b \sin A \sin c)$$

ÂNGULO	ARCO GRAD
2π	$\rightarrow 4\pi$
A	$\rightarrow 2A$

$$\frac{\pi}{6}$$

$$180^\circ \rightarrow \pi$$

$$30' \rightarrow \pi/6$$

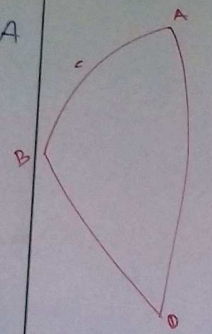


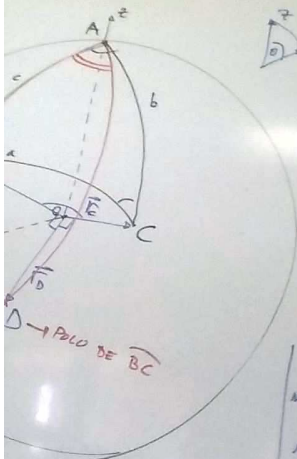
$\vec{r}_c \cdot \vec{r}_b \rightarrow r \cdot r \cdot \cos \alpha$
 $\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$

$\vec{r}_c \cdot \vec{r}_b = \vec{r}_a \cdot \sin a$
 $(\sin c, 0, \cos c)$
 $(\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$

$x = \sin a \sin \alpha$
 $y = \sin a \cos \alpha$
 $z = \cos a$

$$\begin{vmatrix} i & j & k \\ \sin b \cos A & \sin b \sin A & \cos b \\ \sin c & 0 & \cos c \end{vmatrix} = (\sin b \sin A \cos c, -\sin b \cos A \cos c + \sin b \sin A \sin c)$$





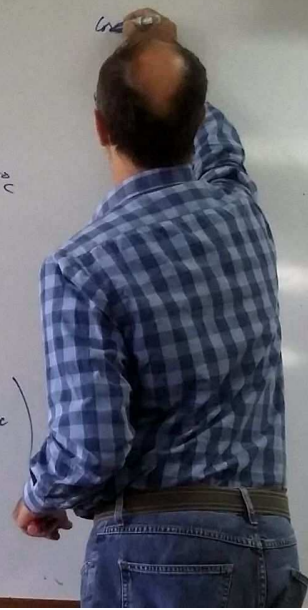
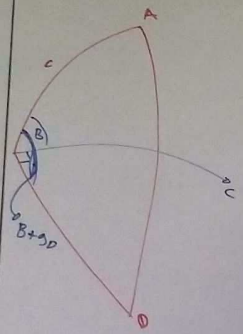
$\vec{r}_c \cdot \vec{r}_b \rightarrow F \cdot \cos \alpha$
 $\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$

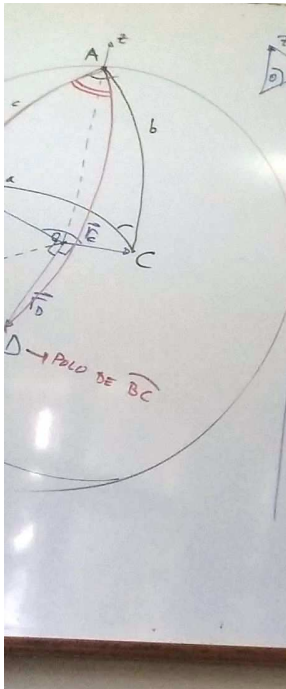
$\vec{r}_c \cdot \vec{r}_b = \vec{r}_D \cdot \vec{r}_A$ $\sin a$
 $(\sin c, 0, \cos c)$
 $(\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$

$X = \sin AD \cdot \cos \widehat{BAD}$
 $Y = \sin AD \cdot \sin \widehat{BAD}$
 $Z = \cos AD$

POLAR δ
 ACIMUT ψ
 POLAR δ

i	j	k	
$\sin b \cos A$	$\sin b \sin A$	$\cos b$	$= (\sin b \sin A \cos c, -\sin b \cos A \cos c + \cos b \sin c, -\sin b \sin A \sin c)$
$\sin c$	0	$\cos c$	



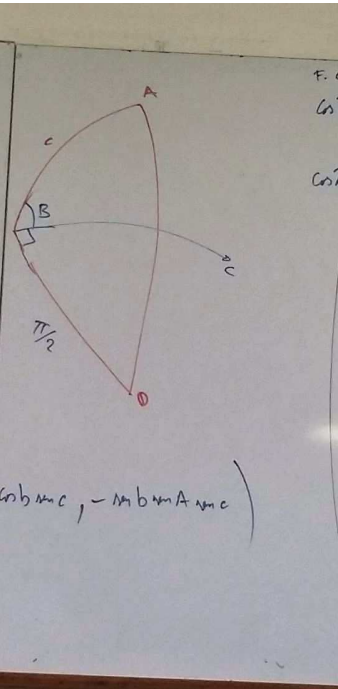


$\vec{r}_C \cdot \vec{r}_B \rightarrow F. \cos \alpha$
 $\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$

$\vec{r}_C \cdot \vec{r}_D = \vec{r}_D \sin a$
 $(\sin c, 0, \cos c)$
 $(\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$

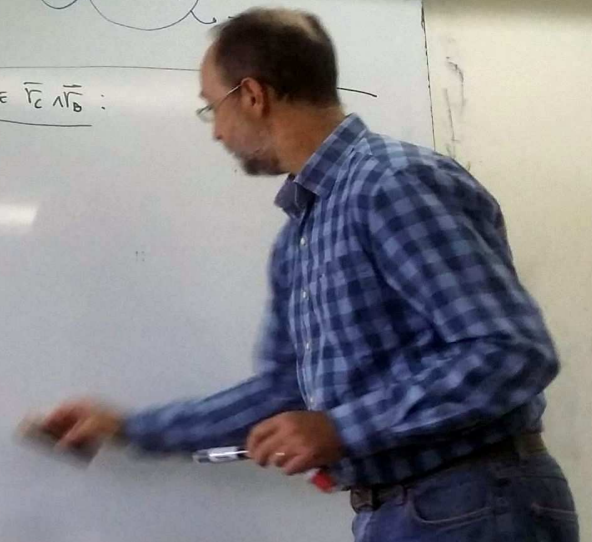
$X = \sin a \cdot \cos \text{BAD}$
 $Y = \sin a \cdot \sin \text{BAD}$
 $Z = \cos \text{AD}$

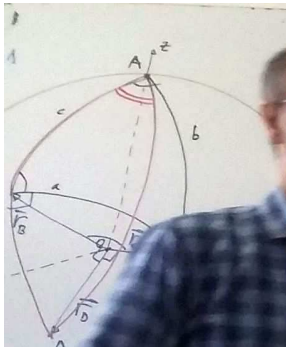
$\begin{vmatrix} i & j & k \\ \sin b \cos A & \sin b \sin A & \cos b \\ \sin c & 0 & \cos c \end{vmatrix} = (\sin b \sin A \cos c, -\sin b \cos A \cos c + \cos b \sin c, -\sin b \sin A \sin c)$



$F. \cos \alpha$
 $\cos \widehat{AD} = \cos c \cdot \cos \widehat{BD} + \sin c \cdot \sin \widehat{BD} \cdot \cos (B + 90^\circ)$
 $\cos \widehat{AD} = -\sin c \cdot \sin B$

COORDENADA Z DE \vec{r}_C E \vec{r}_D :





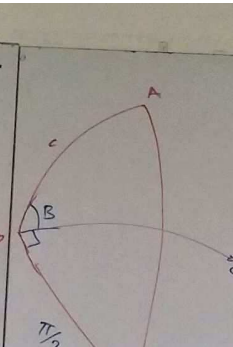
$\vec{r}_c \cdot \vec{r}_a \rightarrow F. \cos \alpha$
 $\cos \alpha = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$

$\vec{r}_c = \begin{pmatrix} \cos \alpha \\ 0 \\ \cos c \end{pmatrix}$

$\vec{r}_a = \begin{pmatrix} \cos \alpha \cos A \\ \sin \alpha \cos A \\ \cos c \end{pmatrix}$

$\vec{r}_b = \begin{pmatrix} \cos \alpha \sin A \\ \sin \alpha \sin A \\ \cos c \end{pmatrix}$

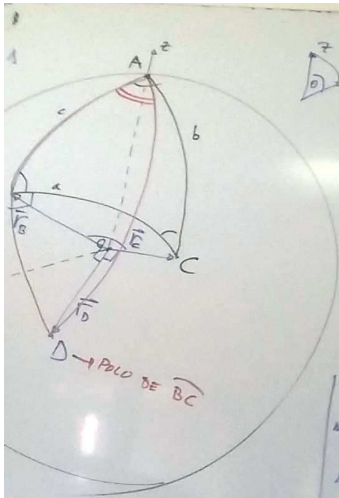
$\vec{r}_c \cdot \vec{r}_b = \cos \alpha \sin A \cos c + \sin \alpha \sin A \cos c = \cos \alpha \sin A \cos c + \sin \alpha \sin A \cos c$



$F. \cos \alpha$
 $\cos \widehat{AD} = \cos c \cdot \cos \widehat{BD} + \sin c \cdot \sin \widehat{BD} \cdot \cos (B + 90^\circ)$
 $\cos \widehat{AD} = -\sin c \cdot \sin B$

COORDENADA Z DE \vec{r}_c \wedge \vec{r}_b :
 $\sin b \cdot \sin A \cdot \sin c = \sin a \cdot (\sin c \cdot \sin B)$
 $\Rightarrow \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}$

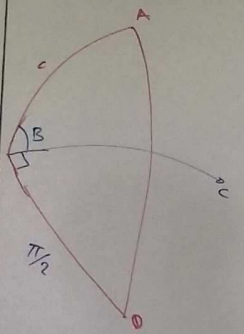
$\cos b \sin c, -\sin b \sin A \sin c$



$\vec{r}_c \cdot \vec{r}_b \rightarrow F. \cos \alpha$
 $\cos \alpha = \dots$
 $\vec{r}_c \cdot \vec{r}_b = \vec{r}_a \sin \alpha$
 $(\sin c, 0, \cos c)$
 $(\sin b \cos A, \sin b \sin A)$

i	j	k
$\sin b \cos A$	$\sin b \sin A$	$\cos b$
$\sin c$	0	$\cos c$

- $\cos \widehat{BAD}$
- $\sin \widehat{BAD}$



$\cos b \sin c$, $-\sin b \sin A \sin c$

F. cos α
 $\cos \widehat{AD} = \cos c \cdot \cos \widehat{BD} + \sin c \cdot \sin \widehat{BD} \cdot \cos (B + 90^\circ)$
 $\cos \widehat{AD} = -\sin c \cdot \sin B$

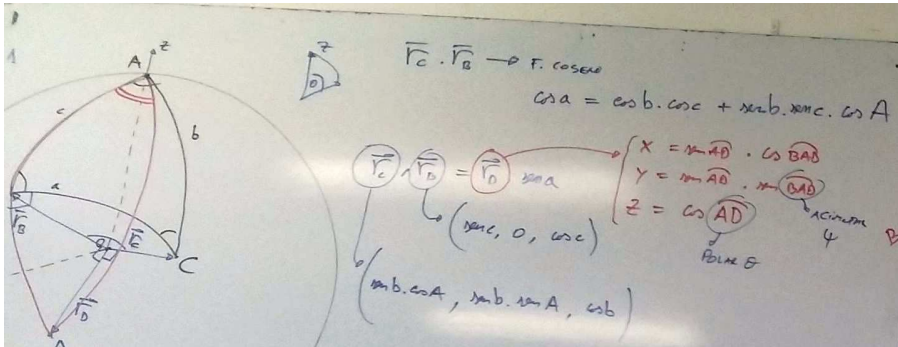
COORDENADA Z DE $\vec{r}_c \wedge \vec{r}_b$:

$\sin b \cdot \sin A \cdot \sin c = \sin a \cdot (\sin c \cdot \sin B)$

$\Rightarrow \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$ F. SENO

COORD. Y DE $\vec{r}_c \wedge \vec{r}_b$

$\square = \sin a \cdot \sin \widehat{AD} \cdot \sin \widehat{BAD}$



$\vec{r}_C \cdot \vec{r}_B \rightarrow$ F. coseno

$$\cos A = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

$\vec{r}_C \cdot \vec{r}_B = \vec{r}_D$ (small triangle)

$$\begin{cases} x = \sin AD \cdot \cos BAD \\ y = \sin AD \cdot \sin BAD \\ z = \cos AD \end{cases}$$

Polos B

$(\sin b \cdot \cos A, \sin b \cdot \sin A, \cos b)$

i	j	k
$\sin b \cos A$	$\sin b \sin A$	$\cos b$
$\sin c$	0	$\sin c$

$= (\sin b \sin A \cos c, -\sin b \cos A \cos c + \cos b \sin c, -\sin b \sin A \sin c)$

F. coseno

$$\cos \widehat{AD} = \cos c \cdot \cos \widehat{BD} + \sin c \cdot \sin \widehat{BD} \cdot \cos (B + 90^\circ)$$

$\cos \widehat{AD} = -\sin c \cdot \sin B$

COORDENADA Z DE $\vec{r}_C \wedge \vec{r}_B$:

$\sin b \cdot \sin A \cdot \sin c = \sin a \cdot (\sin c \cdot \sin B)$

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \quad \text{F. SEND}$$

COORD. Y DE $\vec{r}_C \wedge \vec{r}_B$

$\square = \sin a \cdot \sin \widehat{AD} \cdot \sin \widehat{BAD}$

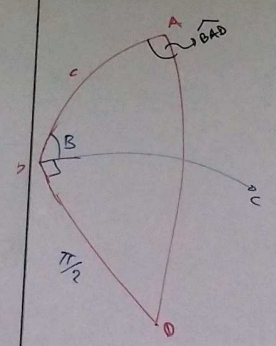
$$\frac{\sin \widehat{BAD}}{\sin \frac{\pi}{2}} = \frac{\sin (B + 90^\circ)}{\sin AD}$$

$\sin \widehat{BAD} \cdot \sin \widehat{AD} = \cos B$



$a \sin c - mb \cos c \cos A = ma \cos B$

F. ANALOGA



F. cosBAD
 $\cos \widehat{AD} = \cos c \cdot \cos \widehat{BD} + m c \cdot \sin \widehat{BD} \cdot \cos (B + 90^\circ)$
 $\cos \widehat{AD} = -m c \cdot \sin B$

COORDENADA Z DE $\vec{r}_C \wedge \vec{r}_B$:

$m b \cdot \sin A \cdot \sin c = m a \cdot (\sin c \cdot \sin B)$

$\Rightarrow \frac{m a}{m A} = \frac{m b}{m B} = \frac{m c}{m C}$ F. SEND

$(m b \sin A \cos c, -m b \cos A \cos c + \cos b \sin c, -m b \sin A \sin c)$

COORD. Y DE $\vec{r}_C \wedge \vec{r}_B$

$\square = m a \cdot \sin \widehat{AD} \cdot \sin \widehat{BAD}$

$\frac{\sin \widehat{BAD}}{\sin \pi/2} = \frac{\sin (B + 90^\circ)}{\sin \widehat{AD}}$
 $\sin \widehat{BAD} \cdot \sin \widehat{AD} = \cos B$

$$a \cos c - mb \cos A = ma \cos B$$

F. ANALOGA

$$\cot b \cdot \sin A = \cos A \cdot \cos C + \sin C \cdot \cot B$$

F. 4 PARTES



$$\left(\sin b \sin A \cos c, -mb \cos A \cos c + \right)$$



F. COSEAD

$$\cos \widehat{AD} = \cos c \cdot \cos \widehat{BD} + \sin c \cdot \sin \widehat{BD} \cdot \cos (B+90^\circ)$$

$$\cos \widehat{AD} = -\sin c \cdot \sin B$$

COORDENADA Z DE $\vec{r}_C \wedge \vec{r}_B$:

$$\sin b \cdot \sin A \cdot \sin c = \sin a \cdot (\sin c \cdot \sin B)$$

$$\Rightarrow \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \quad \text{F. SENO}$$

COORD. Y DE $\vec{r}_C \wedge \vec{r}_B$

$$\square = \sin a \cdot \sin \widehat{AD} \cdot \sin \widehat{BAD}$$

$$\frac{\sin \widehat{BAD}}{\sin \frac{1}{2}C} = \frac{\sin (B+90^\circ)}{\sin AD}$$

$$\sin \widehat{BAD} \cdot \sin \widehat{AD} = \cos B$$

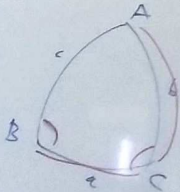
$$\cos b \sin c - \sin b \cos c \cos A = \sin a \cos B$$

F. ANÁLOGA

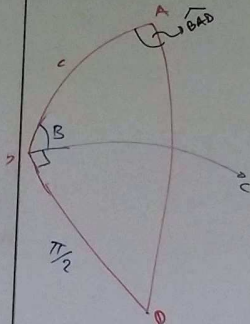
F. COSENO, F. SENO \Rightarrow D

$$\cos b \sin a = \cos a \cos c + \sin c \cot B$$

F. 4 PARTES



$$\left(\sin b \sin A \cos c, -\sin b \cos A \cos c + \cos b \sin c, -\sin b \sin A \sin c \right)$$



F. COSENO

$$\cos \widehat{AD} = \cos c \cdot \overset{=0}{\cos \widehat{BD}} + \sin c \cdot \overset{=1}{\sin \widehat{BD}} \cdot \cos(B+90^\circ) \rightarrow -\sin B$$

$$\cos \widehat{AD} = -\sin B$$

COORDENADA Z DE $\vec{r}_A \wedge \vec{r}_B$:

$$\sin b \sin A \sin c = \sin a \cdot (\sin c \sin B)$$

$$\Rightarrow \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \quad \text{F. SENO}$$

COORD. Y DE $\vec{r}_A \wedge \vec{r}_B$

$$\square = \sin a \cdot \sin \widehat{AD} \cdot \sin \widehat{BAD}$$

$$\frac{\sin \widehat{BAD}}{\sin \frac{\pi}{2}} = \frac{\sin(B+90^\circ)}{\sin \widehat{AD}}$$

$$\sin \widehat{BAD} \cdot \sin \widehat{AD} = \cos B$$

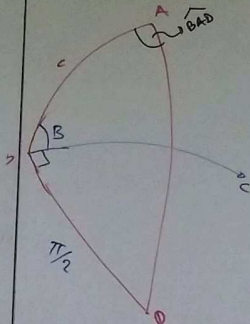
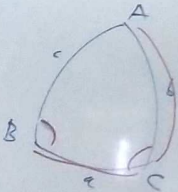
$$\cos b \cdot \sin c - \sin b \cdot \cos c \cdot \cos A = \sin a \cdot \cos B$$

F. ANALOGA

F. COSENO, F. SECO \Rightarrow

$$\cot b \cdot \sin a = \cos a \cdot \cos c + \sin c \cdot \cot B$$

F. 4 PARTES



F. COSENO

$$\cos \widehat{AD} = \cos c \cdot \cos \widehat{BD} + \sin c \cdot \sin \widehat{BD} \cdot \cos (B + 90^\circ)$$

$$\cos \widehat{AD} = -\sin c \cdot \sin B$$

COORDENADA Z DE $\vec{r}_A \wedge \vec{r}_B$:

$$\sin b \cdot \sin A \cdot \sin c = \sin a \cdot (\sin c \cdot \sin B)$$

$$\Rightarrow \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \quad \text{F. SECO}$$

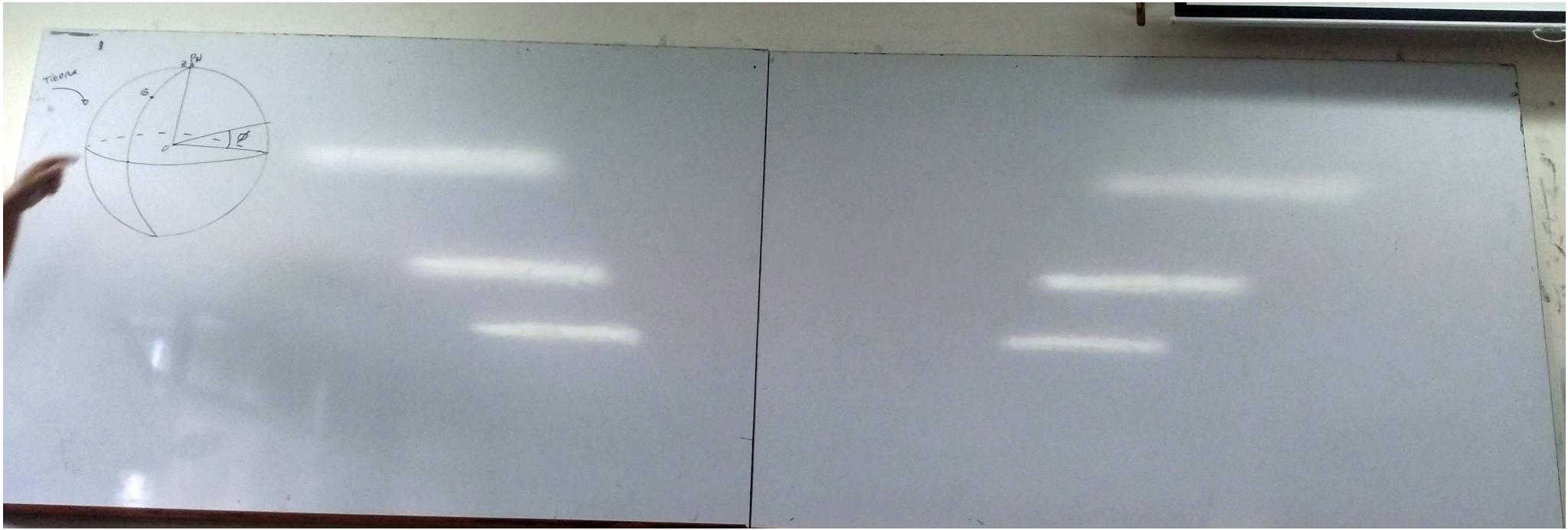
$$\left(\sin b \sin A \cos c, -\sin b \sin A \sin c, \cos b \sin c, -\sin b \sin A \sin c \right)$$

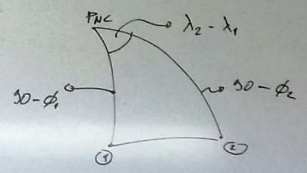
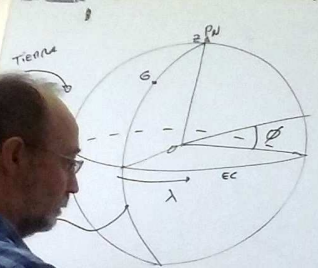
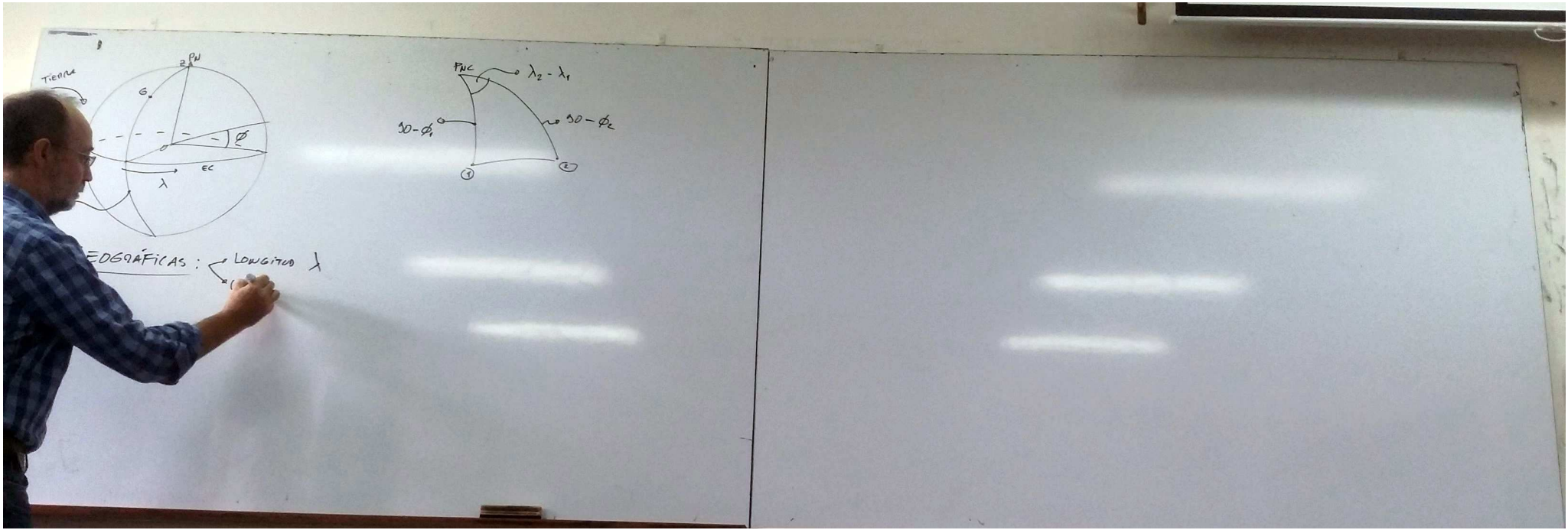
COORD. Y DE $\vec{r}_A \wedge \vec{r}_B$

$$\square = \sin a \cdot \sin \widehat{AD} \cdot \sin \widehat{BD}$$

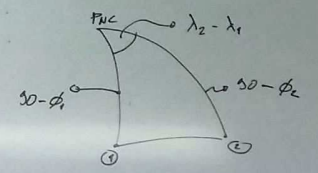
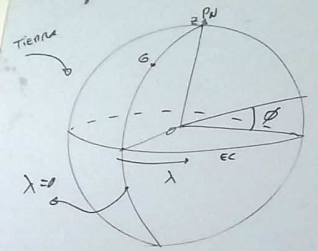
$$\frac{\sin \widehat{BD}}{\sin \frac{\pi}{2}} = \frac{\sin (B + 90^\circ)}{\sin \widehat{AD}}$$

$$\sin \widehat{BD} \cdot \sin \widehat{AD} = \cos B$$

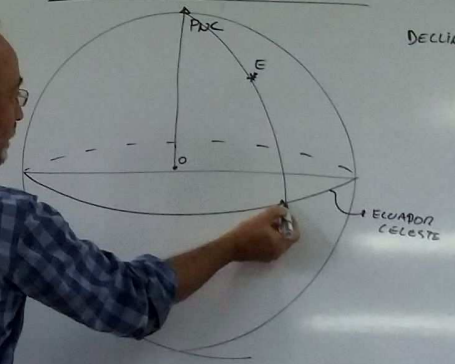




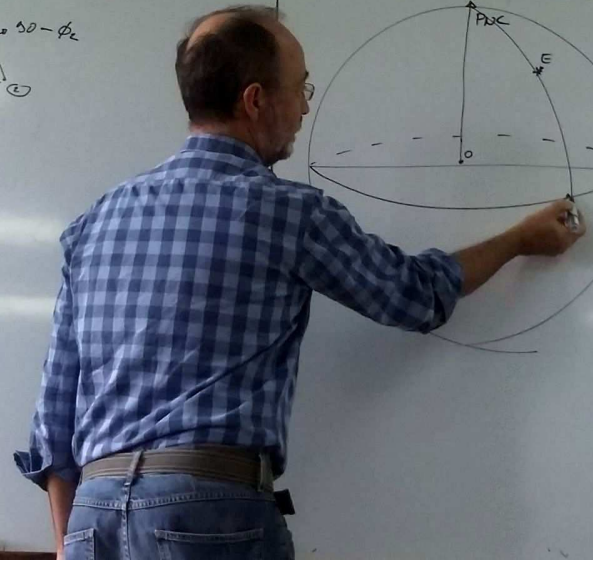
EDGAFICAS: → LONGITUD λ

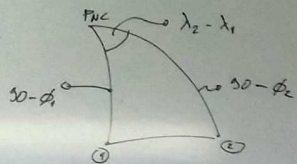
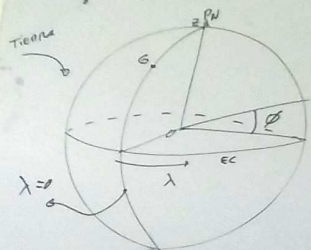


S. COORD. ECUATORIALES CELESTES



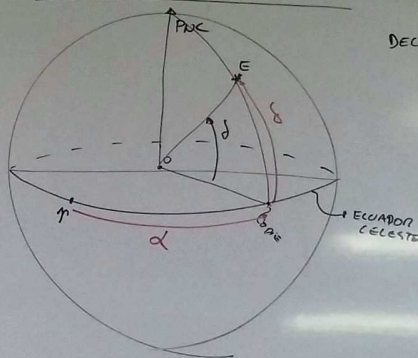
C. GEOGRAFICAS :
- LONGITUD λ (0, 360)
- LATITUD ϕ : (-90, +90)





C. GEOGRÁFICAS:
LONGITUD λ (0, 360)
LATITUD ϕ : (-90, +90)

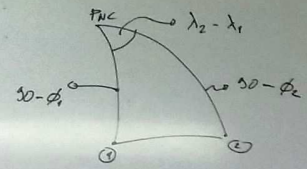
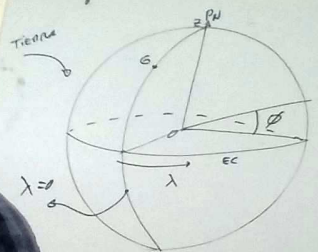
S. COORD. ECUATORIALES CELESTES



DECLINACION δ (-90°, +90°)

ASCENSION RECTA α (0)

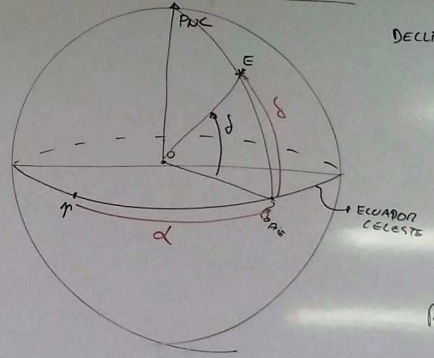




GEOGRÁFICAS:

- LONGITUD λ $(0, 360)$
- LATITUD ϕ $(-90, +90)$

S. COORD. ECUATORIALES CELESTES



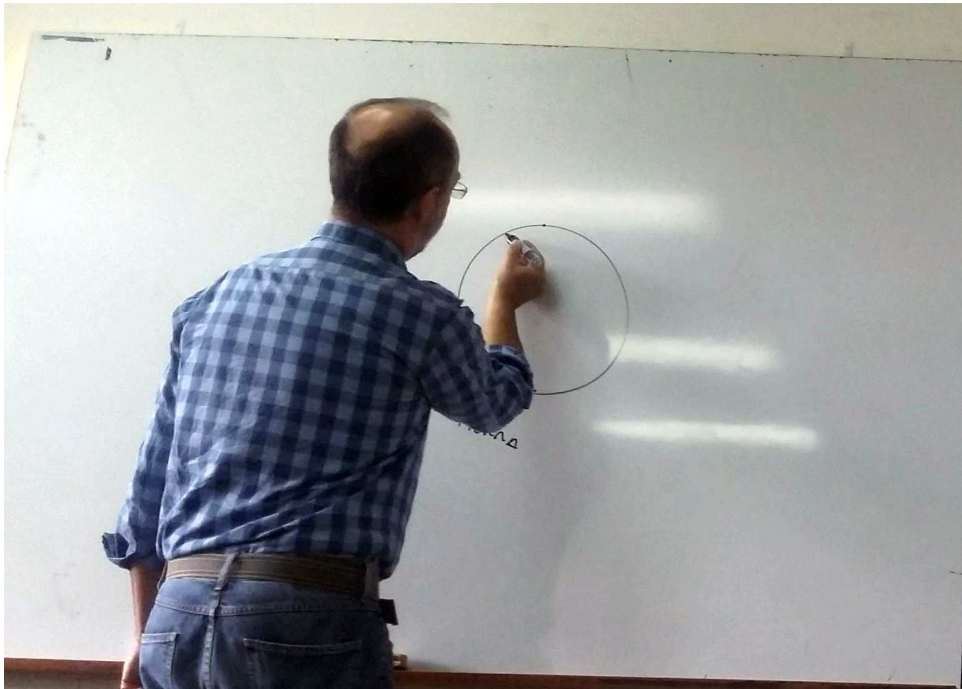
DECLINACIÓN δ $(-90^\circ, +90^\circ)$

ASCENSION RECTA α $(0^h, 24^h)$

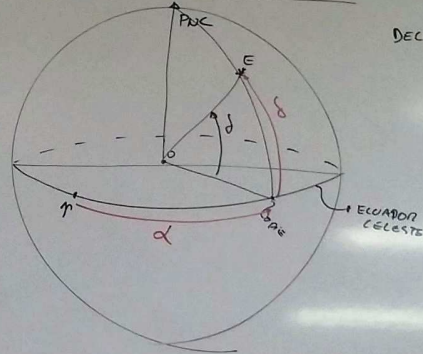
$1^h \rightarrow 15^\circ$

RECT:

$$\begin{cases} x = m \cdot \cos \delta \\ y = m \cdot \sin \delta \\ z = m \delta \end{cases}$$



S. COORD. ECUATORIALES CELESTES



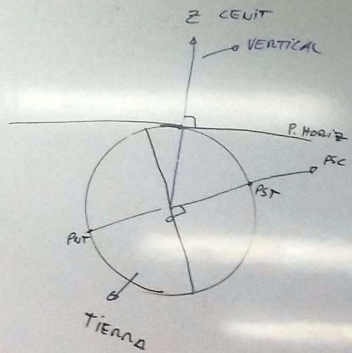
DECLINACION δ ($-90^\circ, +90^\circ$)

ASCENSION RECTA α ($0^h, 24^h$)

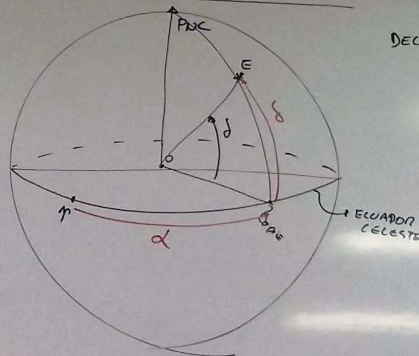
$$1^h \rightarrow 15^\circ$$

RECT :

$$\begin{cases} X = \cos \delta \cdot \cos \alpha \\ Y = \cos \delta \cdot \sin \alpha \\ Z = \sin \delta \end{cases}$$



S. COORD. ECUATORIALES CELESTES



DECLINACION δ ($-90^\circ, +90^\circ$)

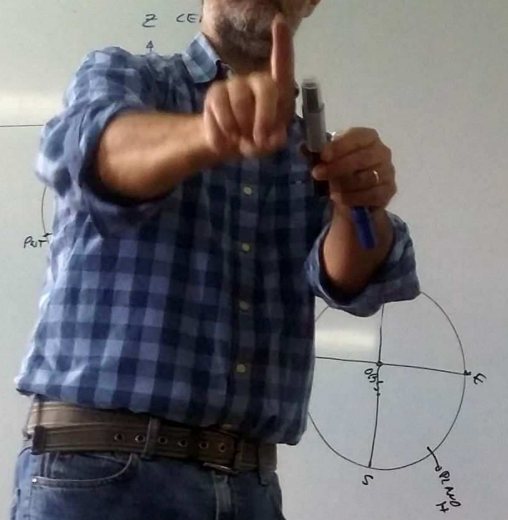
ASCENSION RECTA α ($0^h, 24^h$)

$1^h \rightarrow 15^\circ$

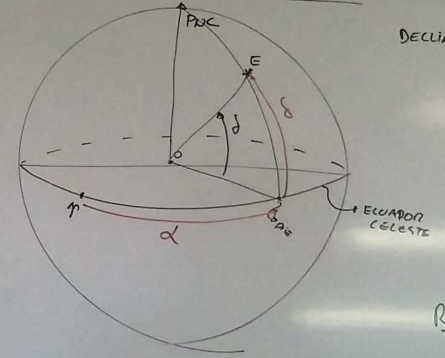
RECT:
$$\begin{cases} x = m \delta \cdot \cos \delta \\ y = m \delta \cdot \sin \delta \\ z = m \delta \end{cases}$$

$\lambda_{\text{hor}} \approx -56^\circ$

RECTA MERIDIANA : P. HOR () P. MERIDIANO



S. COORD. ECUATORIALES CELESTES



DECLINACION δ ($-90^\circ, +90^\circ$)

ASCENSION RECTA α ($0^h, 24^h$)

$1^h \rightarrow 15^\circ$

RECT :

$$\begin{cases} x = \cos \delta \cdot \cos \alpha \\ y = \cos \delta \cdot \sin \alpha \\ z = \sin \delta \end{cases}$$

$\lambda_{\text{hor}} \approx -56^\circ$

RECTA MERIDIANA : P. HOR () P. MERIDIANO

TEOREMA DE LA LATITUD

ALTURA DE PNC = ϕ

Z CENIT
VER

S. COORD. ECUATORIALES CELESTES

DECLINACION δ ($-90^\circ, +90^\circ$)

ASCENSION RECTA α ($0^h, 24^h$)

$1^h \rightarrow 15^\circ$

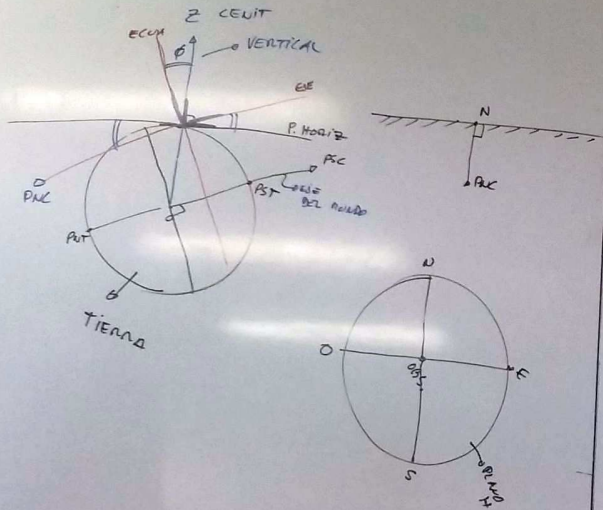
RECT:
$$\begin{cases} x = \cos \delta \cdot \cos \alpha \\ y = \cos \delta \cdot \sin \alpha \\ z = \sin \delta \end{cases}$$



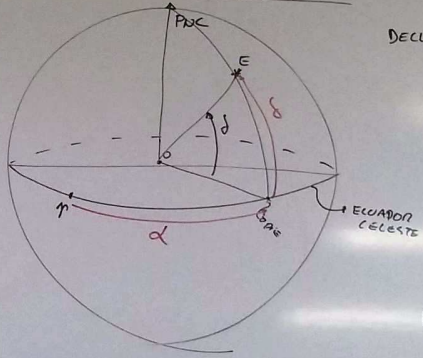
$\approx -56^\circ$

P. MERIDIANO

A
= ϕ



S. COORD. ECUATORIALES CELESTES



DECLINACION δ ($-90^\circ, +90^\circ$)

ASCENSION RECTA α ($0^h, 24^h$)

$1^h \rightarrow 15^\circ$

RECT:
$$\begin{cases} X = \cos \delta \cdot \cos \alpha \\ Y = \cos \delta \cdot \sin \alpha \\ Z = \sin \delta \end{cases}$$

$\lambda_{\text{Polar}} \approx -56^\circ$

RECTA MERIDIANA : P. HOR. () P. MERIDIAN.

TEOREMA DE LA LATITUD

ALTURA DE PNC = ϕ

The diagram on the left illustrates the geometry of the sky as seen from a specific location. It shows a celestial sphere with the Zenith (Z) at the top. A vertical circle is drawn through Z. The point where the vertical circle intersects the horizon is labeled P. HOR. (Point of Horizon). The point where the vertical circle intersects the celestial meridian is labeled P. MERIDIAN. Other points shown include PNC (Pole of the Celestial Sphere), P. C. (Celestial Pole), and P. S. T. (Point of the Sky). A smaller diagram below shows a circle representing the horizon with a vertical line from the center to the top (Z) and a point on the horizon labeled PNC. The angle between the vertical line and the line to PNC is labeled phi (φ).

S. COORD. ECUATORIALES CELESTES

DECLINACION δ ($-90^\circ, +90^\circ$)

ASCENSION RECTA α ($0^h, 24^h$)

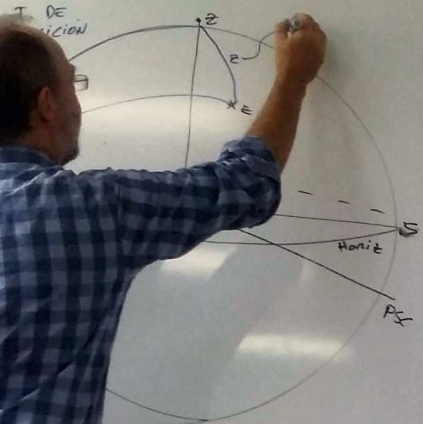
$1^h \rightarrow 15^\circ$

RECT :
$$\begin{cases} X = m \delta \cdot \cos \delta \\ Y = m \delta \cdot \sin \delta \\ Z = m \delta \end{cases}$$

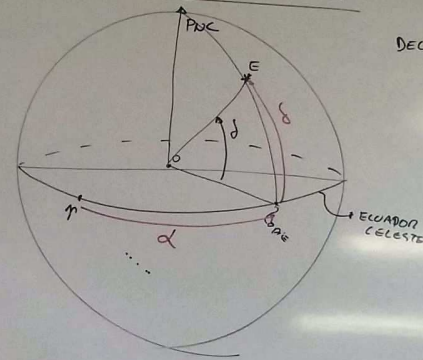
The diagram on the right shows a celestial sphere with the Celestial Equator (EQUADOR CELESTE) as a horizontal line. A point E is marked on the sphere. The angle between the Celestial Equator and the line from the center to E is labeled delta (δ), representing Declination. The angle measured along the Celestial Equator from a reference point to the projection of E is labeled alpha (α), representing Right Ascension. The point PNC is at the top pole. A smaller diagram below shows a circle representing the equator with a point on the circumference labeled E. The angle between the horizontal line and the line to E is labeled delta (δ). The angle measured along the circumference from a reference point to the projection of E is labeled alpha (α).

$\lambda_{\text{polar}} \approx -56^\circ$
 : P. Horiz \cap P. Meridiano

A DE LA
 ITUD
 PNC = ϕ



S. COORD. ECUATORIALES CELESTES



DECLINACION δ ($-90^\circ, +90^\circ$)
 ASCENSION RECTA α ($0^h, 24^h$)

$1^h \rightarrow 15^\circ$

RECT: $\begin{cases} X = \cos \delta \cdot \cos \alpha \\ Y = \cos \delta \cdot \sin \alpha \\ Z = \sin \delta \end{cases}$

$\lambda_{\text{hor}} \approx -56^\circ$

: P. Hor () P. Meridiano

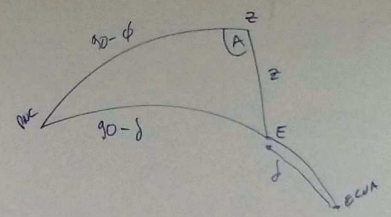
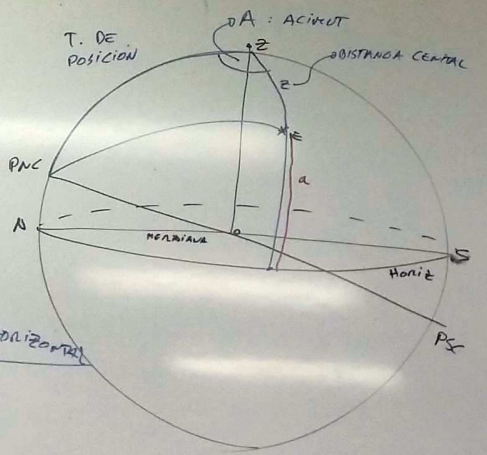
DE LA
ITUD

$PNC = \phi$

SIST. HORIZONTAL

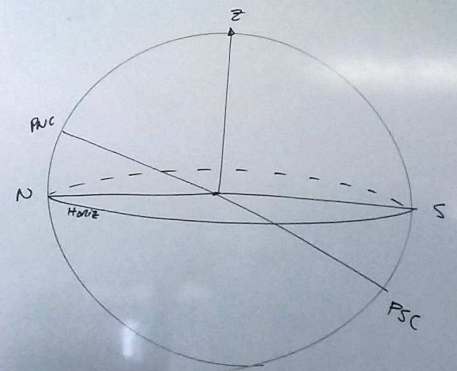
(A, z)

NOSE

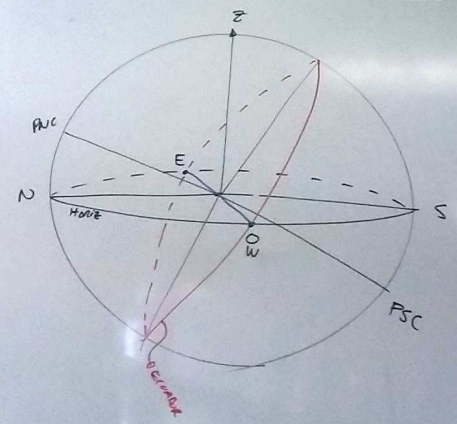


TRIÁNGULO DE POSICIÓN

ACIMUT : A

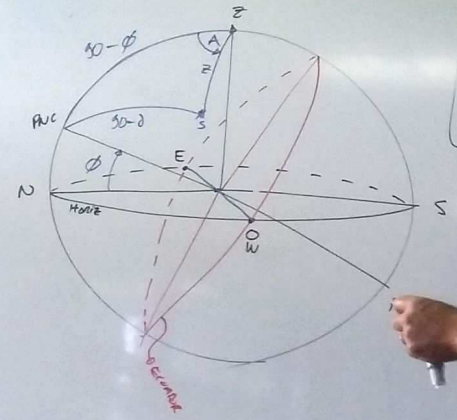


TRIÁNGULO DE POSICIÓN

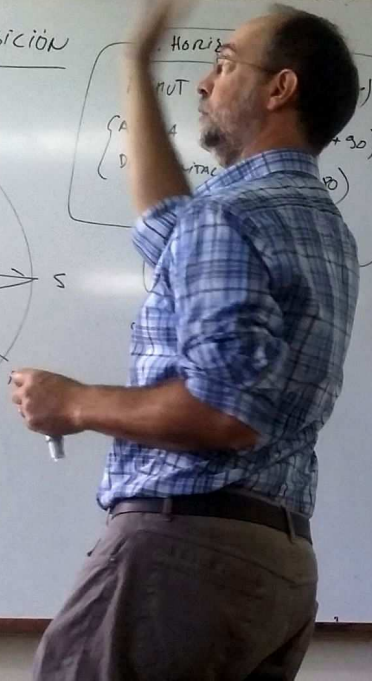


ACIMUT : A $(0, 360^\circ)$
ALURA : a $(-90, +90)$
DIST. CENTAL : Z $(0, 180)$

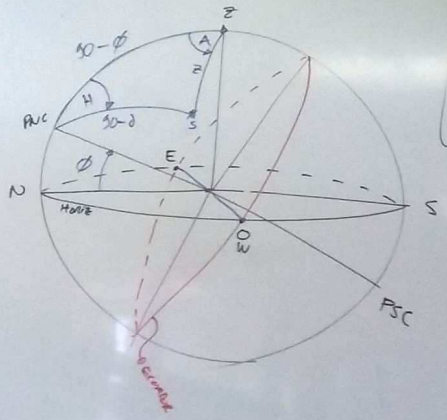
TRIÁNGULO DE POSICIÓN



Horiz
MUT
(A
D
TAC
(+30)
(0)



TRIÁNGULO DE POSICIÓN



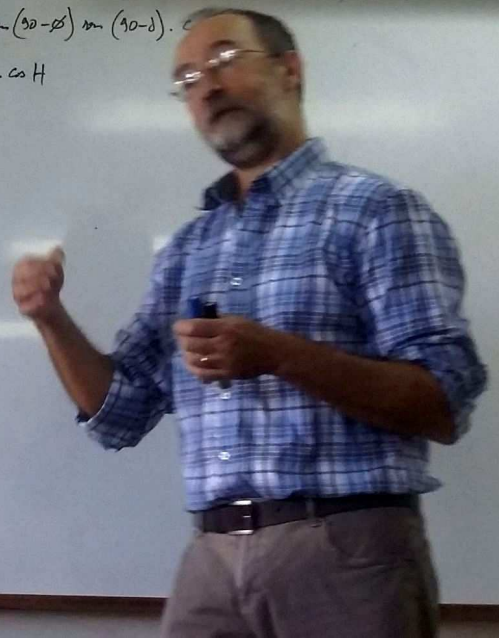
C. HORIZONTALES
 ACIMUT : A (0, 360°)
 ALTURA : a (-90, +90)
 DIST. CENTAL : z (0, 180)

C. ECLIPTIQUES
 alpha (0, 24h)
 delta (-90, +90)

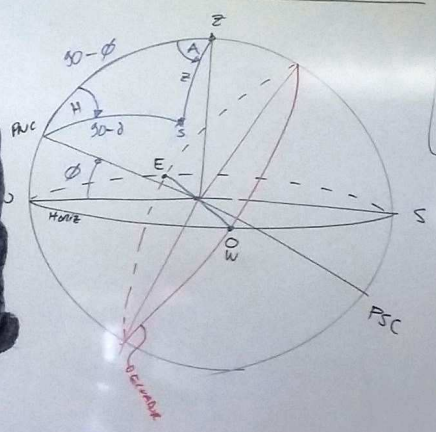
H : ANGOLO HORARIO (0h, 24h)

$$\cos z = \cos(90-\phi) \cdot \cos(90-d) + \sin(90-\phi) \sin(90-d) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin d + \cos \phi \cdot \cos \delta \cdot \cos H$$



TRIÁNGULO DE POSICIÓN



C. HORIZONTALES

ACIMUT : $A \quad (0, 360^\circ)$

ALTIMA : $a \quad (-90, +90)$

DIST. CENTRAL : $z \quad (0, 180)$

C. ECCATORIALES

$\alpha \quad (0, 24^h)$

$\delta \quad (-90, +90)$

H : ANGLULO HORARIO $(0^h, 24^h)$

$$\cos z = \cos(90-\phi) \cdot \cos(90-\delta) + \sin(90-\phi) \sin(90-\delta) \cdot \cos H$$

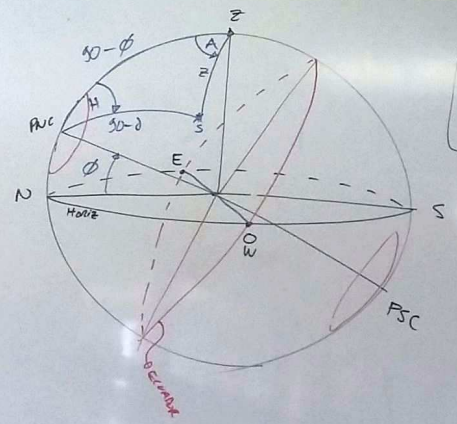
$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : $z = 90 \Rightarrow \cos H_{\text{sal}} = -\tan \phi \tan \delta$

APP

- MATHSAPP

TRIÁNGULO DE POSICIÓN



C. HORIZONTALES

ACIMUT : A $(0, 360^\circ)$

ALTURA : a $(-90, +90)$

DIST. CENTRAL : z $(0, 180)$

C. ECLIPTICALES

α $(0, 24^h)$

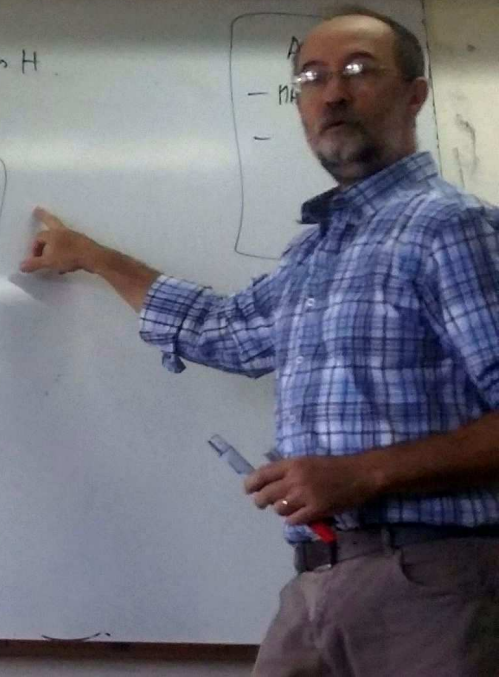
δ $(-90, +90)$

H : ANGULO HORARIO $(0^h, 24^h)$

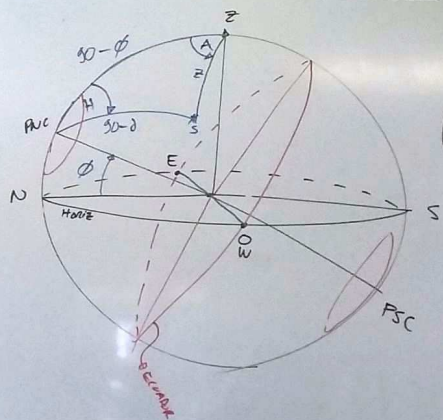
$$\cos z = \cos(90 - \phi) \cdot \cos(90 - \delta) + \sin(90 - \phi) \sin(90 - \delta) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : $z = 90 \Rightarrow \cos H_{sal/puesta} = -\tan \phi \tan \delta$



TRIÁNGULO DE POSICIÓN



C. HORIZONTALES

- ACIMUT : $A \quad (0, 360^\circ)$
- ALTURA : $a \quad (-90, +90)$
- DIST. CENTRAL : $z \quad (0, 180)$

C. ECATORIALES

- $\alpha \quad (0, 24^h)$
- $\delta \quad (-90, +90)$

H : ANGULO HORARIO $(0^h, 24^h)$

$$\cos z = \cos(90-\phi) \cdot \cos(90-\delta) + \sin(90-\phi) \sin(90-\delta) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : $z = 90 \Rightarrow \cos H_{sal/puesta} = -\frac{\sin \phi \sin \delta}{\cos \phi}$ 0,12

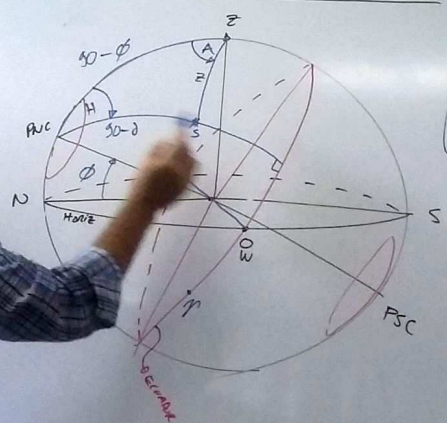
$H > 0 ?$

$H < 0 ?$

APP

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TRIÁNGULO DE POSICIÓN



C. HORIZONTALES
 ACIMUT : A (0, 360°)
 ALTURA : a (-90, +90)
 DIST. CENTRAL : z (0, 180)

C. ECUATORIALES
 α (0, 24h)
 δ (-90, +90)

H : ANGULO HORARIO (0h, 24h)

$$\cos z = \cos(90-\phi) \cdot \cos(90-\delta) + \sin(90-\phi) \sin(90-\delta) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : z = 90

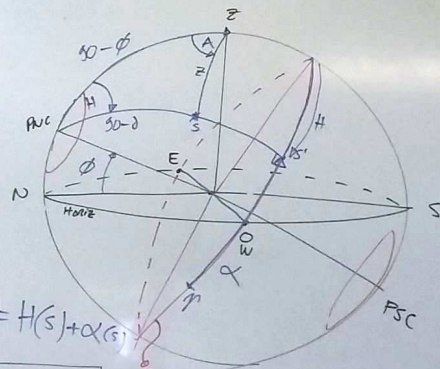
$$\cos H_{salida/puesta} = -\tan \phi \tan \delta \quad 0, 12$$

H > 0 ?

H < 0 ?

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TRIÁNGULO DE POSICIÓN



$H(\eta) = H(\zeta) + \alpha(\zeta)$

$TSL = H + \alpha$

C. HORIZONTALES
 ACIMUT : A (0, 360°)
 ALTURA : a (-90, +90)
 DIST. CENTAL : z (0, 180)

C. ECUATORIALES
 α (0, 24h)
 δ (-90, +90)

H : ANGULO HORARIO (0h, 24h)

$\cos(90-\phi) \cdot \cos(90-\delta) + \sin(90-\phi) \sin(90-\delta) \cdot \cos H$
 $\sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$

ESTR : z = 90 ⇒

$\cos H_{\text{max}} = -\tan \phi \tan \delta$

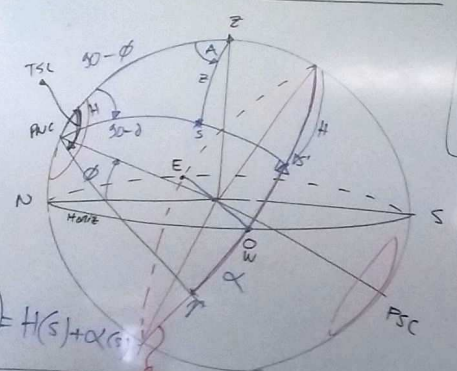
0, 12

H > 0 ?

H < 0 ?

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TRIÁNGULO DE POSICIÓN



C. HORIZONTALES
 ACIMUT : A (0, 360°)
 ALTURA : a (-90, +90)
 DIST. CENTRAL : z (0, 180)

C. ECLIPTIQUES
 alpha (0, 24h)
 delta (-90, +90)

H : ANGULO HORARIO (0h, 24h)

$H(\eta) = H(S) + \alpha(S)$
 $H + \alpha$
 Reloj

$$\cos z = \cos(90-\phi) \cdot \cos(90-\delta) + \sin(90-\phi) \sin(90-\delta) \cdot \cos H$$

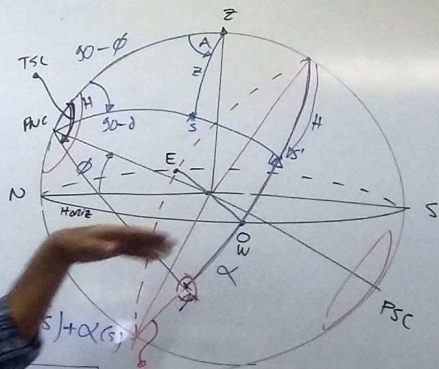
$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : z = 90 \Rightarrow $\cos H_{sal/puesta} = -\tan \phi \tan \delta$ 0, 12

H > 0 ?
 H < 0 ?

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TRIÁNGULO DE POSICIÓN



$TSL = H + \alpha$

Reloj

C. HORIZONTALES
 ACIMUT : A (0, 360°)
 ALTURA : a (-90, +90)
 DIST. CENTRAL : z (0, 180)

C. ECLATORIALES
 α (0, 24h)
 δ (-90, +90)

H : ANGULO HORARIO (0h, 24h)

$$\cos z = \cos(90-\phi) \cdot \cos(90-\delta) + \sin(90-\phi) \sin(90-\delta) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : z = 90 ⇒

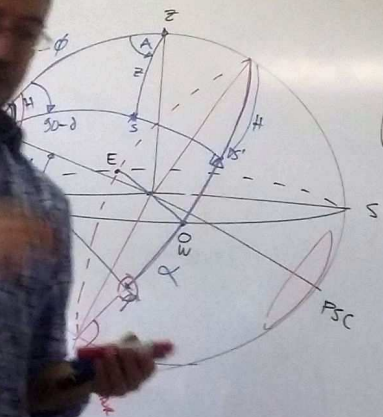
$$\cos H_{salida/puesta} = -\tan \phi \tan \delta \quad 0, 12$$

H > 0 ?

H < 0 ?

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TRIÁNGULO DE POSICIÓN



C. HORIZONTALES

- ACIUT : A ($0, 360^\circ$)
- ALTURA : a ($-90, +90$)
- DIST. CENTAL : z ($0, 180$)

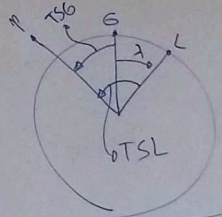
C. ECLATORIALES

- α ($0, 24^h$)
- δ ($-90, +90$)
- H : ANGOLO HORARIO ($0^h, 24^h$)

$$\cos z = \cos(90-\phi) \cdot \cos(90-\delta) + \sin(90-\phi) \sin(90-\delta) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : $z = 90 \Rightarrow \cos H_{sal/puesta} = -\tan \phi \tan \delta$ 0, 12

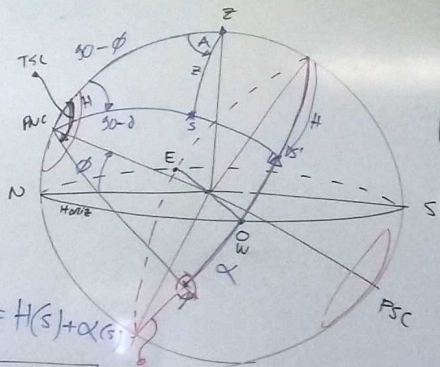


$H > 0 ?$
 $H < 0 ?$

$TSL = TSG + \lambda$

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 - SIDEREAL CLOCK

TRIÁNGULO DE POSICIÓN



C. HORIZONTALES
 ACIMUT : A (0, 360°)
 ALTURA : a (-90, +90)
 DIST. CENTRAL : z (0, 180)

C. ECUATORIALES
 α (0, 24h)
 δ (-90, +90)

H : ANGULO HORARIO (0°, 24h)

$H(\eta) = H(S) + \alpha(S)$

$TSL = H + \alpha$

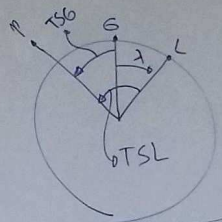
Reloj

$\cos z = \cos(90-\phi) \cdot \cos(90-d) + \sin(90-\phi) \sin(90-d) \cdot \cos H$

$\cos z = \sin \phi \cdot \sin d + \cos \phi \cdot \cos d \cdot \cos H$

SALIDA, PUESTA : z = 90 ⇒

$\cos H_{sal/puesta} = -\tan \phi \tan \delta$ 0, 12



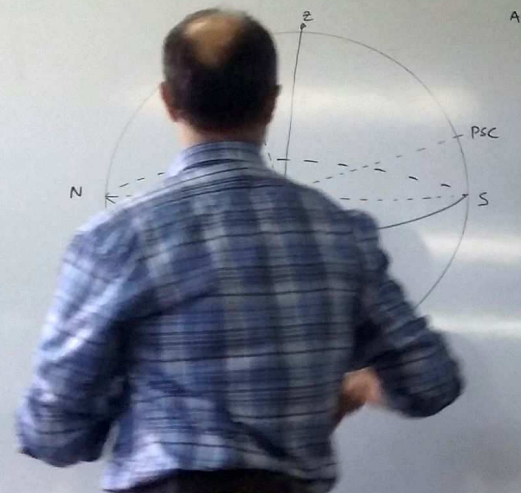
H > 0 ?

H < 0 ?

$TSL = TSG + \lambda$

Fórmula → DADA Hora y día → TSG

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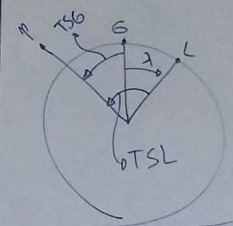
ALTURA RC = -35°

$$\cos z = \cos(90-\phi) \cdot \cos(90-\delta) + \sin(90-\phi) \sin(90-\delta) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : z = 90

$$\cos H_{\text{salida/puesta}} = -\tan \phi \tan \delta \quad 0,12$$



H > 0 ?

H < 0 ?

$$TSL = TSG + \lambda$$

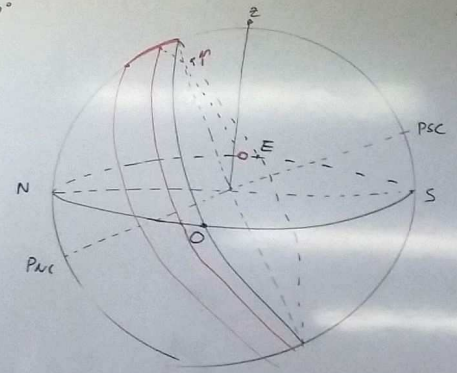
DADA Hora y día
 FÓRMULA
 TSG

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$$\int_{0}^{\infty} \cos \theta = 0^\circ$$

$$\frac{\Delta \delta_a}{\Delta t} > 0$$

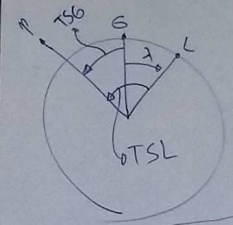
ALTURA $\alpha_x = -35^\circ$



$$\cos z = \cos(90-\phi) \cdot \cos(90-\delta) + \sin(90-\phi) \sin(90-\delta) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : $z = 90 \Rightarrow \cos H_{\text{salida/puesta}} = -\tan \phi \tan \delta$ 0,12



$H > 0 ?$
 $H < 0 ?$

$$TSL = TSG + \lambda$$

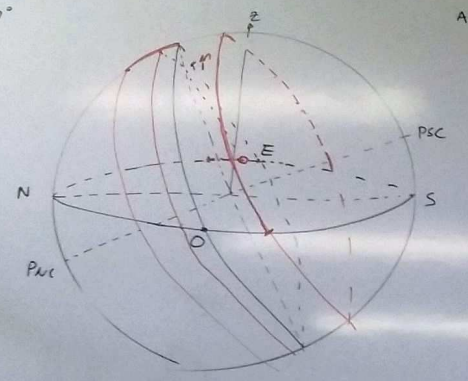
Fórmula $\begin{cases} \text{DADA Hora y día} \\ \text{TSG} \end{cases}$

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$$\int_{0}^{H_0} H_0 \gamma = 0^\circ$$

$$\frac{\Delta \delta_0 > 0}{\Delta t}$$

$$H_0^{SALIDA} = -6^h$$

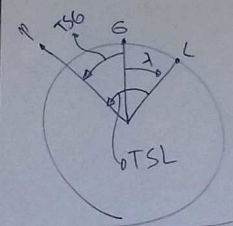


ALTURA RC = -35°

$$\cos z = \cos(90 - \phi) \cdot \cos(90 - \delta) + \sin(90 - \phi) \sin(90 - \delta) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA : $z = 90 \Rightarrow \cos H_{SAL PUESTA} = -\tan \phi \tan \delta$ 0,12



$H > 0 ?$

$H < 0 ?$

$$TSL = TSG + \lambda$$

Fórmula → DADA Hora y día → TSG

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$$\int_{0}^{24} \dot{H} \, dt = 0^\circ$$

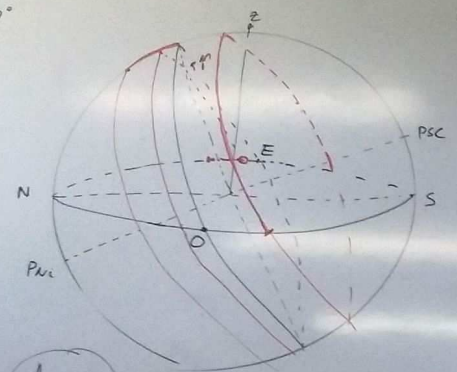
$$\frac{\Delta \delta_a}{\Delta t} > 0$$

$$LDA = -6^h$$

$$6^h$$

$$\Rightarrow H_{SP} = -\frac{1}{2} \cdot \delta = 0$$

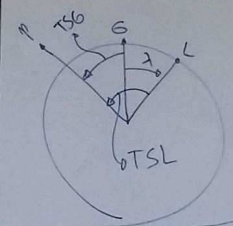
ALTURA RC = -35°



$$\cos z = \cos(90 - \phi) \cdot \cos(90 - \delta) + \sin(90 - \phi) \sin(90 - \delta) \cdot \cos H$$

$$\cos z = \sin \phi \cdot \sin \delta + \cos \phi \cdot \cos \delta \cdot \cos H$$

SALIDA, PUESTA: $z = 90 \Rightarrow \cos H_{SAL/PUESTA} = -\frac{\sin \phi \sin \delta}{\cos \phi}$ 0, 12



$H > 0 ?$

$H < 0 ?$

$$TSL = TSG + \lambda$$

Fórmula → DADA Hora y día → TSG

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$$\int_{0}^{\infty} \cos \theta = 0^\circ$$

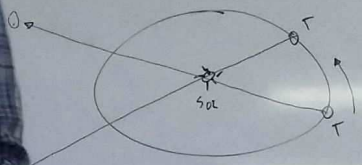
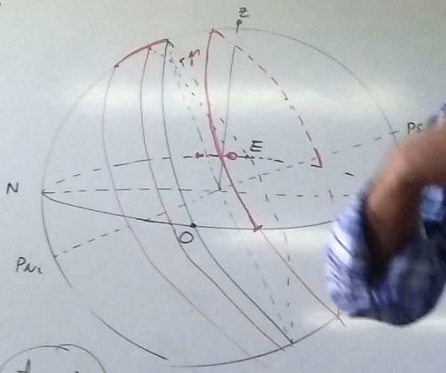
$$\frac{\Delta \alpha}{\Delta t} > 0$$

$$H_0^{(SALIDA)} = -6^h$$

$$H_0^{(PUERTA)} = +6^h$$

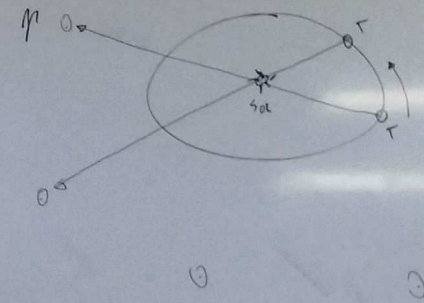
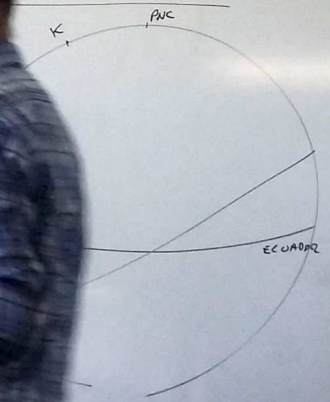
$$\cos H_{SP} = -\frac{1}{2} \Rightarrow H_{SP} = 90^\circ$$

ALTIMA RC



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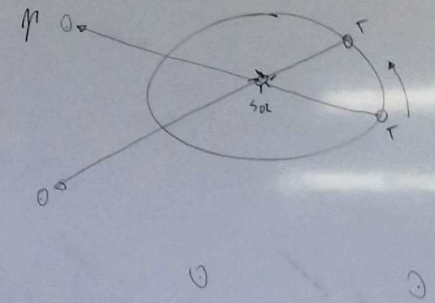
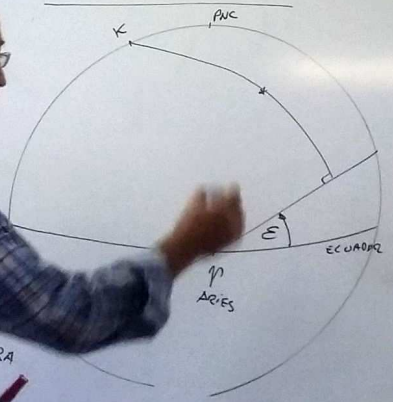
ZENADAS ECLIPTICAS



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COORDENADAS ECLIPTICAS

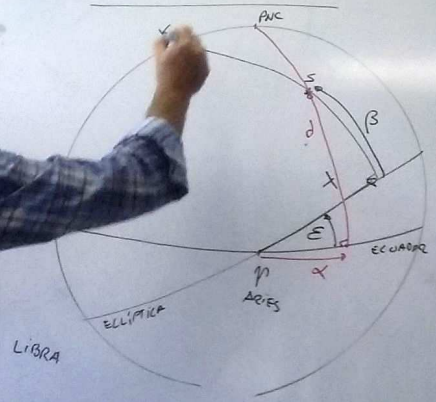
ϵ : OBLICUIDAD $23^{\circ} 27' \dots$



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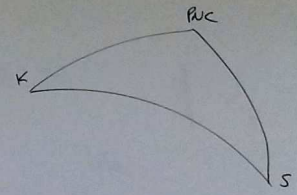
COORDENADAS ECLIPTICAS

ϵ : OBLICUIDAD $23^{\circ} 27'$...



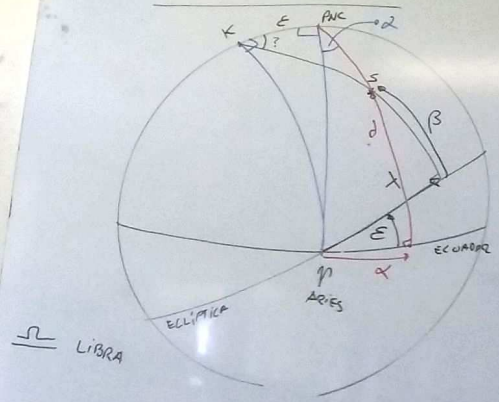
λ = LONGITUD ECLIPTICA
($0^{\circ}, 360^{\circ}$)

β = LATITUD ECLIPTICA
($-90, +90$)



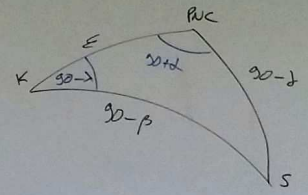
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COORDENADAS ECLIPTICAS



ϵ : ORLICUIDAD $23^{\circ} 27'$...

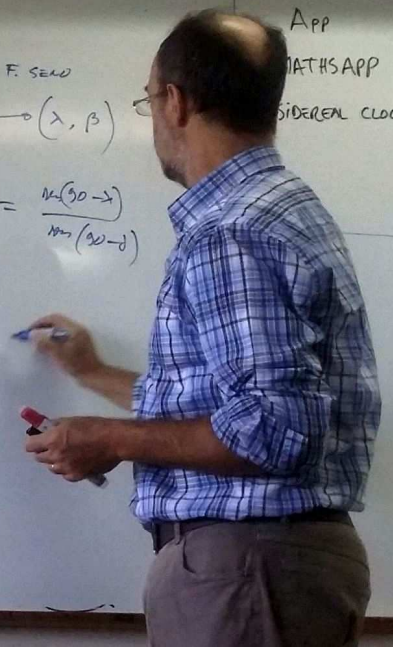
λ = LONGITUD ECLIPTICA
 $(0^{\circ}, 360^{\circ})$
 β = LATITUD ECLIPTICA
 $(-90, +90)$



$F. \cos + F. \text{sen}$
 $\Rightarrow (\alpha, \delta) \leftrightarrow (\lambda, \beta)$
 $\frac{\text{sen}(90 + \alpha)}{\text{sen}(90 - \beta)} = \frac{\text{sen}(90 - \lambda)}{\text{sen}(90 - \delta)}$

$\Rightarrow \frac{\cos \alpha}{\cos \beta} =$

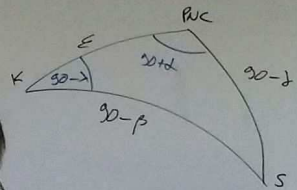
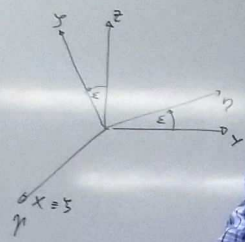
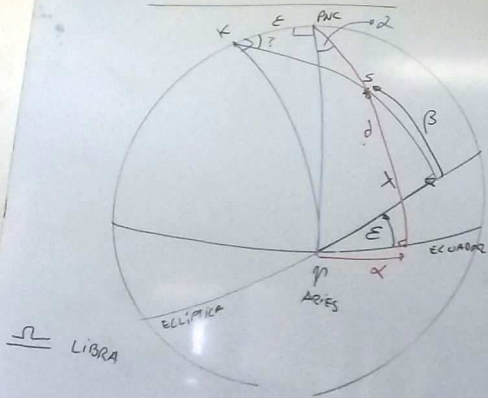
APP
 MATHSAPP
 SIDEREAL CLOCK



COORDENADAS ECLIPTICAS

ϵ : ORLICUIDAD $23^{\circ} 27'$...

(x, y, z) : RECT. ECUALT



$$= R_x(\epsilon) \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

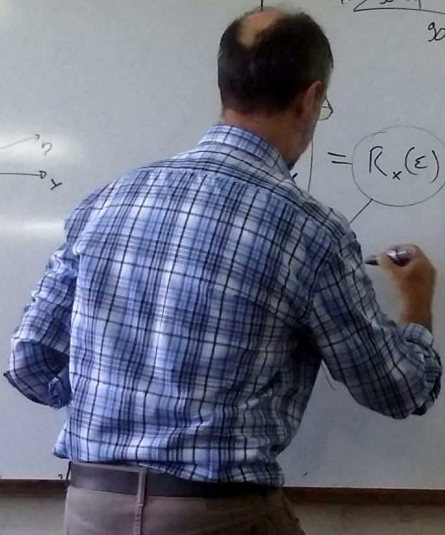
F. COS + F. SEN

$$\Rightarrow (\alpha, \delta) \leftrightarrow (\lambda, \beta)$$

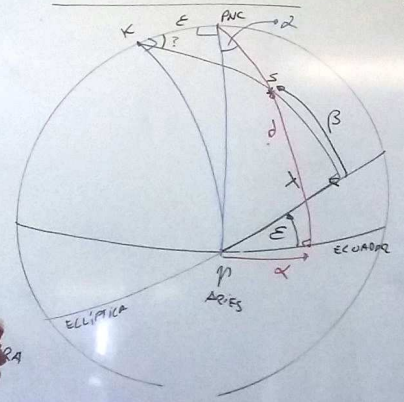
$$\frac{\sin(90 + \alpha)}{\sin(90 - \beta)} = \frac{\sin(90 - \lambda)}{\sin(90 - \delta)}$$

$$\Rightarrow \frac{\cos \alpha}{\cos \beta} = \frac{\cos \lambda}{\cos \delta} \Rightarrow \cos \alpha \cdot \cos \delta = \cos \lambda \cdot \cos \beta$$

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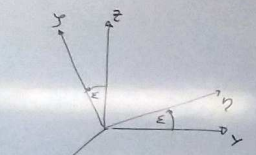


COORDENADAS ECLIPTICAS

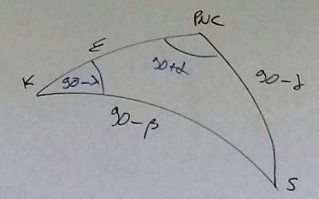


ϵ : OBLICUIDAD $23^\circ 27'$...

(x, y, z) : RECT. ECUAT



$$\beta_0 \equiv 0^\circ$$



F. COS + F. SEN

$$\Rightarrow (\alpha, \delta) \leftrightarrow (\lambda, \beta)$$

$$\frac{\sin(90+\alpha)}{\sin(90-\beta)} = \frac{\sin(90-\delta)}{\sin(90-\lambda)}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_x(\epsilon) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$$

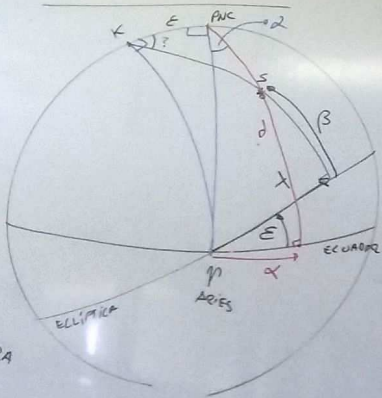
"xi"

$$\xi = X = \cos \delta \cdot \sin \beta$$

$$\Rightarrow \frac{\cos \alpha}{\cos \beta} = \frac{\cos \lambda}{\cos \delta} \Rightarrow \cos \alpha \cdot \cos \delta = \cos \lambda \cdot \cos \beta$$

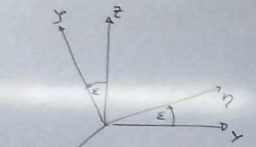
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COORDENADAS ECLIPTICAS



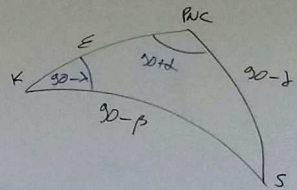
ϵ : ORLICUIDAD $23^\circ 27'$...

(x, y, z) : RECT. ECUAT



$$\beta_0 \equiv 0^\circ$$

$\lambda_0 \neq a.t$



$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R_x(\epsilon) \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

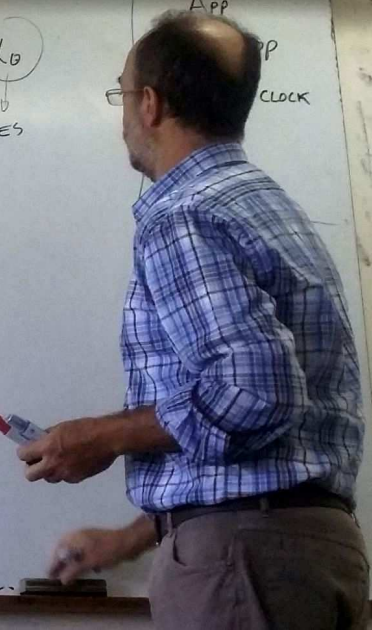
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = X =$$

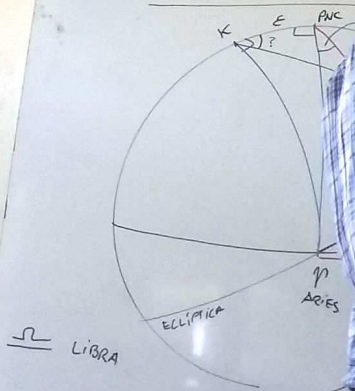
$$TSL = H_0 + \alpha_0$$

NO ES

APP
APP
CLOCK



COORDENADAS ECLIPTICAS

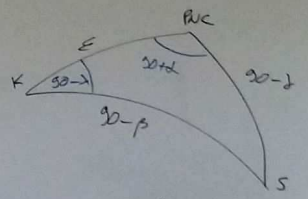


ALICUIDAD $23^\circ 27' \dots$

(λ, β) : RECT. ECLIPT

$$\beta_0 \equiv 0^\circ$$

$$\lambda_0 \neq a.t$$



$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = R_x(\epsilon) \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$$

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = X = \text{"xi"}$$

$$TSL = H_0 + \alpha_0$$

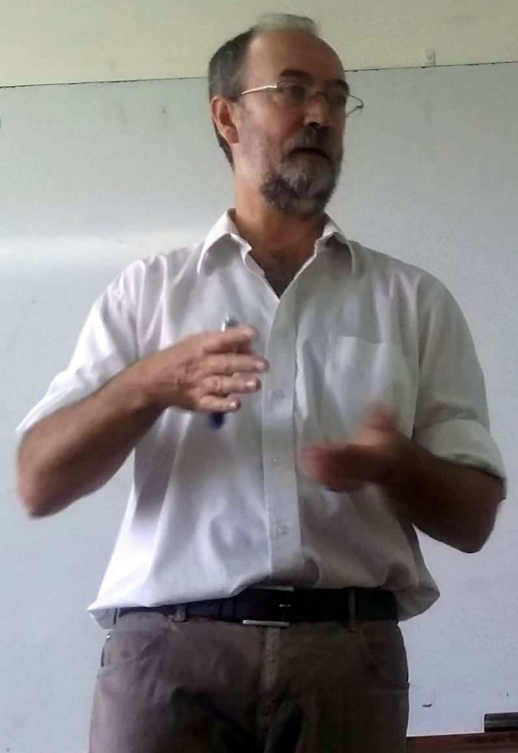
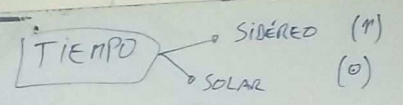
NO ES LINEAL con t

$$\cos \alpha_0 \cdot \cos \delta_0 = \cos \lambda_0 \cdot \sin \beta_0$$

$$\cos \delta_0 = \frac{\cos \lambda_0}{\cos \beta_0}$$

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- MATHSAPP
 - SIDEREAL CLOCK

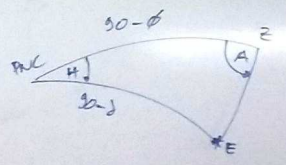
T. SOLAR MEDIO



TIEMPO

- SIDÉREO (π)
- SOLAR (θ)

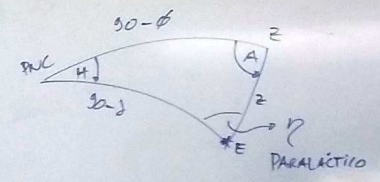
$A = 60^\circ$
 $z = 50^\circ$
 $\alpha = 3^h$
 $\delta = -30^\circ$



$\phi, \lambda?$

TIEMPO

- SIDÉREO (π)
- SOLAR (θ)



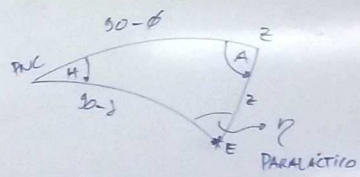
$$\frac{\sin H}{\sin z} = \frac{\sin A}{\cos \delta} = \frac{\sin \gamma}{\cos \phi}$$

$$H = \frac{\sin A}{\cos \delta} \cdot \sin z$$

TIEMPO

- SIDÉREO (π)
- SOLAR (θ)

$A = 60^\circ$
 $z = 50^\circ$
 $\alpha^h = 3^h$
 $\delta = -30^\circ$



$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cdot \cos H$$

$$\cos(90 - \delta) = \cos(90 - \delta) \cdot \cos z + \sin(90 - \delta) \cdot \sin H$$

$$\frac{\sin H}{\cos z} = \frac{\sin A}{\cos \delta} = \frac{\sin \eta}{\cos \phi}$$

$\phi, \lambda?$

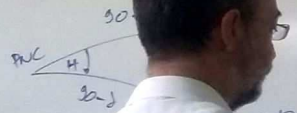
$$\Rightarrow \sin H = \frac{\sin A}{\cos \delta} \cdot \cos z$$

- H_1
- $H_2 = 180 - H_1$

TIEMPO

- SIDÉREO (μ)
- SOLAR (λ)

$A = 60^\circ$
 $z = 50^\circ$
 $\alpha = 3^h$
 $\delta = -30^\circ$



$$\textcircled{1} \cos z = \underbrace{\sin \delta}_{X?} + \underbrace{\cos \delta}_{Y?} \cos H$$

$$\cos(90-\delta) = \cos(90-\delta) \cdot \cos z + \sin(90-\delta) \cdot \sin z \cdot \cos \gamma$$

$$\rightarrow \cos(90-\delta) = \cos z \cdot \cos(90-\delta) + \sin z \cdot \sin(90-\delta) \cdot \cos A$$

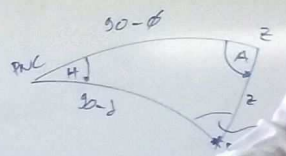
$$\textcircled{2} \sin \delta = \underbrace{\cos z}_{X?} \cdot \underbrace{\sin \delta}_{Y?} + \sin z \cdot \underbrace{\cos \delta}_{Y?} \cos A$$

$$H_2 = 180 - H_1$$

$\delta, \lambda?$

TIEMPO
 SIDÉREO (π)
 SOLAR (θ)

$A = 60^\circ$
 $z = 50^\circ$
 $\alpha = 3^h$
 $\delta = -30^\circ$



$$\frac{\sin H}{\sin z} = \frac{\sin A}{\sin \delta} \cdot \cos \phi$$

$$\Rightarrow \sin H = \frac{\sin A}{\sin \delta} \cdot \sin z$$

$$\cos(\delta - \delta) \cdot \cos z + \sin(\delta - \delta) \cdot \sin z \cdot \cos \gamma$$

$$\cos(\delta - \delta) + \sin z \cdot \sin(\delta - \delta) \cdot \cos A$$

$$+ \sin z \cdot \cos \phi \cdot \cos A$$

$\Rightarrow \phi$

$$TSL = H + \alpha = TSG + \lambda$$

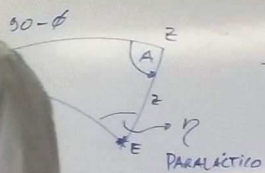
$$\Rightarrow \lambda = TSL - TSG$$

↑
REDUCCION

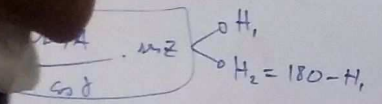
↓
?

$\phi, \lambda?$

SIDÉREO (μ)
 SOLAR (θ)



$$= \frac{\sin \gamma}{\cos \phi} ?$$



$$\textcircled{1} \cos z = \overset{X?}{\sin \phi} \sin \delta + \overset{Y?}{\cos \phi} \cos \delta \cdot \cos H$$

$$\cos(90 - \phi) = \cos(90 - \delta) \cdot \cos z + \sin(90 - \delta) \cdot \sin z \cdot \cos \gamma$$

$$\rightarrow \cos(90 - \delta) = \cos z \cdot \cos(90 - \phi) + \sin z \cdot \sin(90 - \phi) \cdot \cos A$$

$$\textcircled{2} \sin \delta = \cos z \cdot \overset{X?}{\sin \phi} + \sin z \cdot \overset{Y?}{\cos \phi} \cos A$$

$$\textcircled{1} + \textcircled{2} \Rightarrow \boxed{X = \sin \phi} \Rightarrow \phi$$

$Y =$

$$TSL = H + \alpha = TSG + X$$

$$\Rightarrow X = TSL - TSG$$

↑
DIFERENÇA

↓
?

TIEMPO SOLAR

→ Medio (Relojes)

$$TSL = H + \alpha = TSG + \lambda$$

$$\Rightarrow \lambda = TSL - TSG$$

↑
CORRECCION

↓
?

TIEMPO SOLAR

- MEDIO (RELOJES)
- APARENTE (SOL VISIBLE)

$$T_{\text{SOLAR APARENTE LOCAL}} = H_{\odot} + 12^{\text{HS}}$$

$$TSL = H + \alpha = TSG + \lambda$$

$$\Rightarrow \lambda = TSL - TSG$$

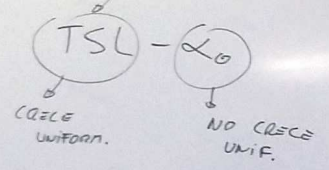
↑ REDUCCION

↓ ?

TIEMPO SOLAR

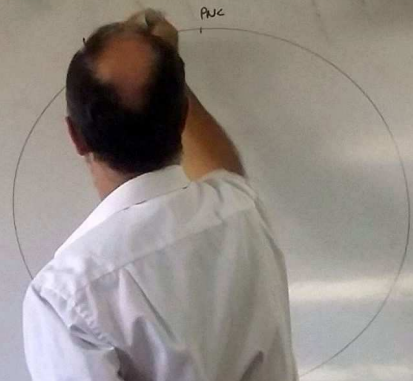
- MEDIO (RELOJES)
- APARENTE (SOL VISIBLE)

$$T_{\text{SOLAR APARENTE LOCAL}} = H_0 + 12^{\text{HS}}$$



→ T. Solar Ap. NO CRECE UNIF.

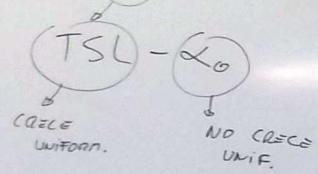
↓ NO RELOJES



TIEMPO SOLAR

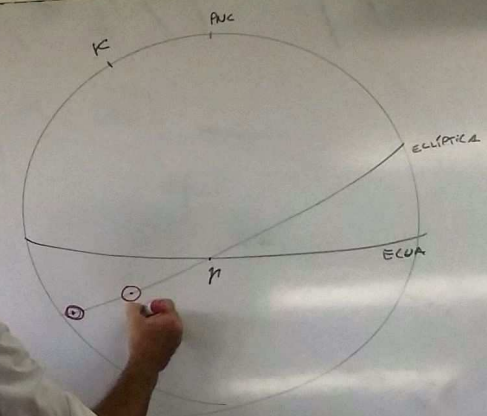
- MEDIO (RELOJES)
- APARENTE (SOL VISIBLE)

$$T_{\text{Solar Aparente Local}} = H_0 + 12^{\text{HS}}$$

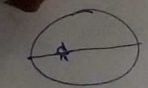


→ T. Solar Ap. NO CRECE UNIF.

↓ NO RELOJES



SOL MEDIO DINAMICO



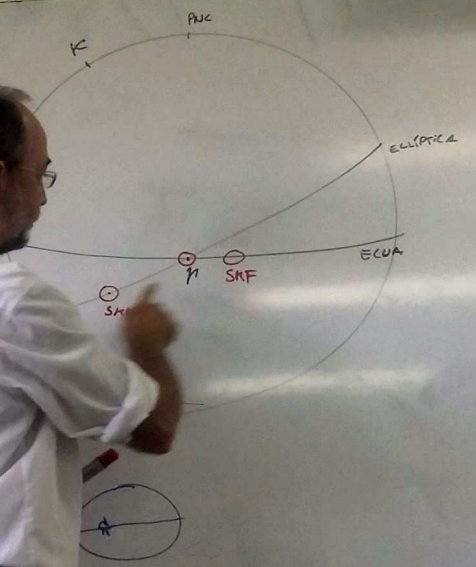
TIEMPO SOLAR

- MEDIO (RELOJES)
- APARENTE (SOL VISIBLE)

$$T_{\text{Solar Aparente Local}} = H_0 + 12^{\text{HS}}$$

H_0 → TSL → CRECE UNIFORM.
 L_0 → NO CRECE UNIF.

→ T. Solar Ap. NO CRECE UNIF.
 ↓
 NO RELOJES



SOL MEDIO DINAMICO
 SOL MEDIO FICTICIO

TIEMPO SOLAR

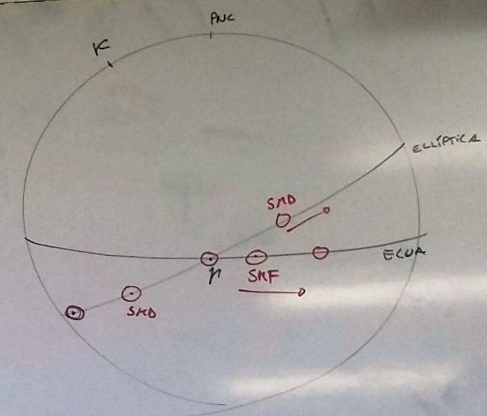
- MEDIO (RELOJES)
- APARENTE (SOL VISIBLE)

$$T_{\text{Solar Aparente Local}} = H_0 + 12^{\text{HS}}$$

H_0 (circled) → TSL (circled) → CRECE UNIFORM.
 12^{HS} (circled) → L_0 (circled) → NO CRECE UNIF.

→ T. Solar Ap. NO CRECE UNIF.

↓ NO RELOJES



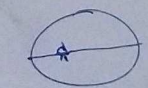
SOL MEDIO DIAFNICO
 SOL MEDIO FICTICIO

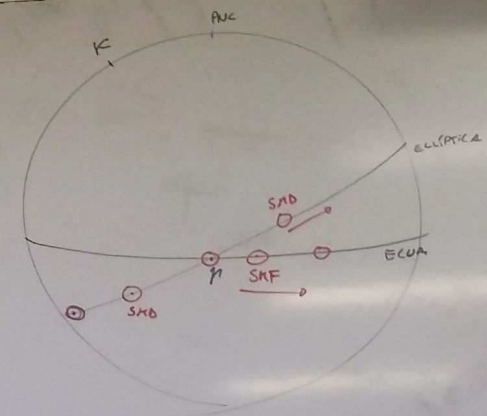
$$\frac{\Delta \alpha_{\text{SMF}}}{\Delta t} = \text{CTE}$$

$$24^{\text{HS}} (360') \rightarrow 1 \text{ AÑO} \sim 365.25$$

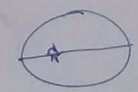
$$\Rightarrow \Delta \alpha_{\text{SMF}} \sim$$

PERIHELIO





PERIHELIO



SOL MEDIO DYNAMICO
SOL MEDIO FICTICIO

$$\frac{\Delta \alpha_{SMF}}{\Delta t} = CTE$$

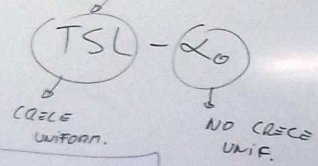
$$24^{hs} (360') \rightarrow 1 \text{ AEO} \sim 365.25$$

$$\Rightarrow \Delta \alpha_{SMF} \sim 1^\circ / \text{dia} = 4'' / \text{dia}$$

TIEMPO SOLAR

- MEDIO (RELOJES)
- APARENTE (SOL VISIBLE)

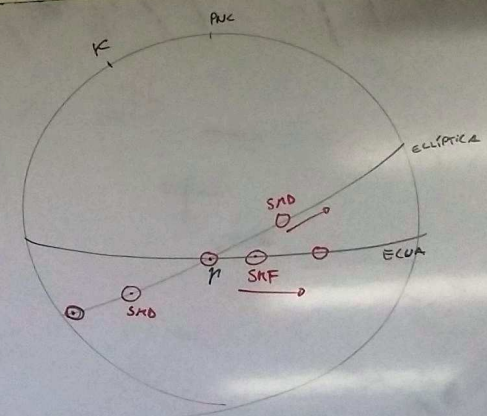
$$T_{\text{Solar Aparente Local}} = H_{\odot} + 12^{\text{hs}}$$



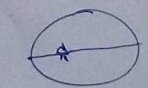
$$T_{\text{Solar Medio}} = H_{\text{SFM}} + 12^{\text{hs}}$$

→ T. Solar Ap. NO CRECE UNIF.

NO RELOJES



PERIHELIO



SOL MEDIO DINAMICO

SOL MEDIO FICTICIO

$$\frac{\Delta \alpha_{\text{SFM}}}{\Delta t} = \text{CTE}$$

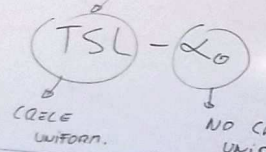
$$24^{\text{hs}} (360') \rightarrow 1 \text{ AED} \sim 365.25$$

$$\Rightarrow \Delta \alpha_{\text{SFM}} \sim 1' / \text{DIA} = 4'' / \text{DIA}$$

TIEMPO SOLAR

- MEDIO (RELOJES)
- APARENTE (SOL VISIBLE)

APARENTE LOCAL = $H_0 + 12^{\text{HS}}$



$H_{SUF} + 12^{\text{HS}}$

→ T. Solar Ap. NO CRECE UNIF.

NO RELOJ

$$T_{\text{Solar Ap}} - T_{\text{Sol Medio}} = \underbrace{H_0}_{TSL - L_0} - \underbrace{H_{SUF}}_{TSL - L_{SUF}} = \alpha_{SUF} - \alpha_0 = \text{Ecuación del tiempo}$$

TIEMPO SOLAR

- MEDIO (RELOJES)
- APARENTE (SOL VISIBLE)

$$T_{\text{Solar Aparente Local}} = H_0 + 12^{\text{HS}}$$

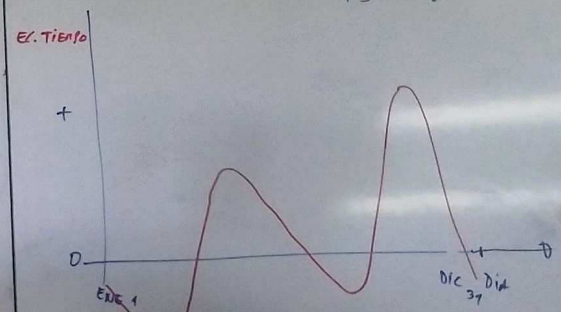
H_0 (CICLO UNIFORM.)
 $TSL - \alpha_0$ (NO CRECE UNIF.)

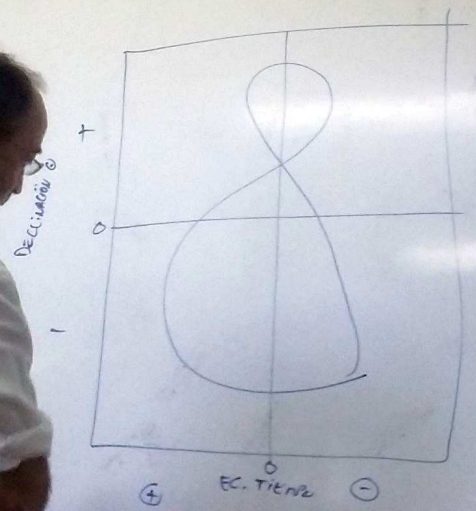
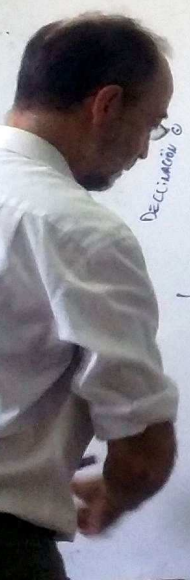
$$T_{\text{Solar Medio}} = H_{\text{SMF}} + 12^{\text{HS}}$$

→ T. Solar Ap. NO CRECE UNIF.

$$T_{\text{Solar Ap}} - T_{\text{Solar Medio}} = H_0 - H_{\text{SMF}} = \alpha_{\text{SMF}} - \alpha_0 = \text{Ecuación del Tiempo}$$

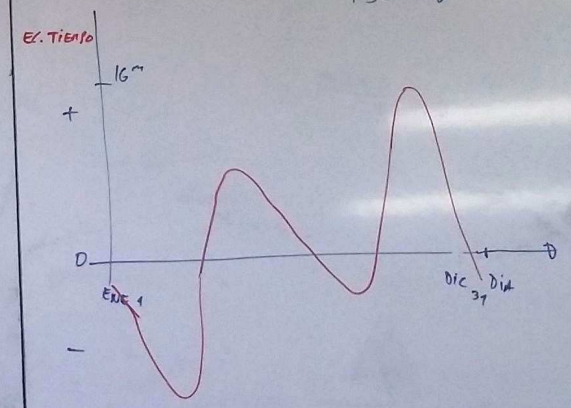
H_0 (CICLO UNIFORM.)
 H_{SMF} (CICLO UNIFORM.)
 $TSL - \alpha_0$ (NO CRECE UNIF.)

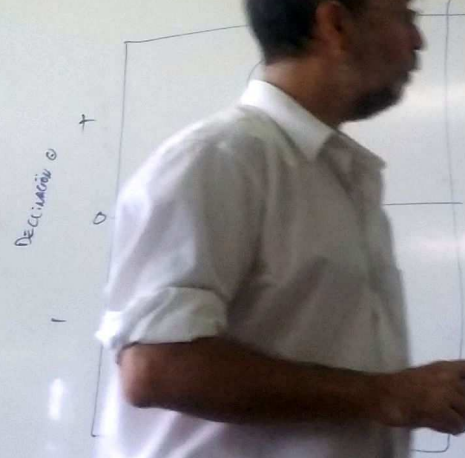




$$TS_{\text{Soln Ap}} - TS_{\text{Sol Medio}} = H_{\odot} - H_{\text{SNF}} = \alpha_{\text{SNF}} - \alpha_{\odot} = \text{Ecuación del tiempo}$$

\downarrow $TSL - \alpha_{\odot}$ \downarrow $TSL - \alpha_{\text{SNF}}$

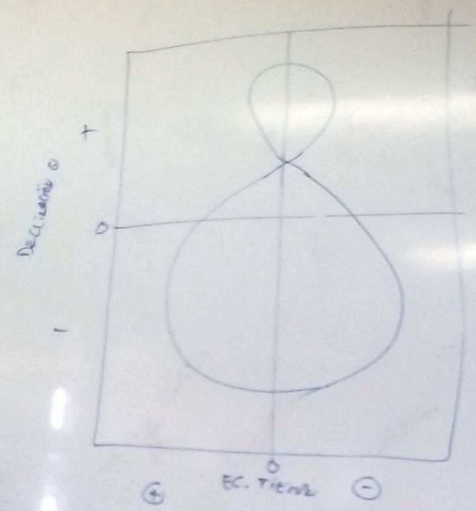




ANALEMA

$$T_{\text{Soln Ap}} - T_{\text{Sol Medio}} = \underbrace{H_{\odot}}_{TSL - \alpha_{\odot}} - \underbrace{H_{\text{SMF}}}_{TSL - \alpha_{\text{SMF}}} = \boxed{\alpha_{\text{SMF}} - \alpha_{\odot}} = \text{Ecuación del Tiempo}$$

$T_{\text{Soln Medio}}$



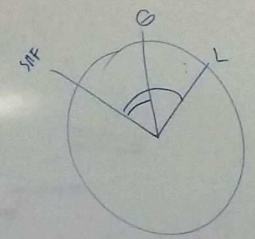
ANALEMA

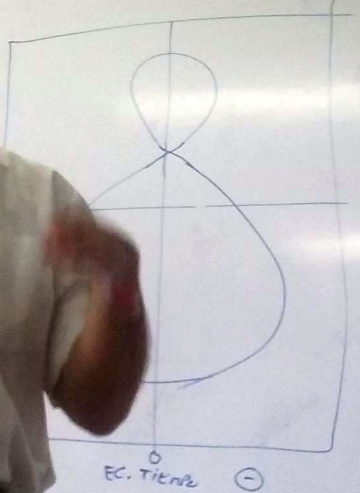
$$T_{Soln Ap} - T_{Sol Medio} = H_{\odot} - H_{SMF} = \alpha_{SMF} - \alpha_{\odot} = \text{Ecuación del Tiempo}$$

\downarrow \downarrow
 $TSL - \alpha_{\odot}$ $TSL - \alpha_{SMF}$

$$T_{Sol Medio Greenwich} = TU$$

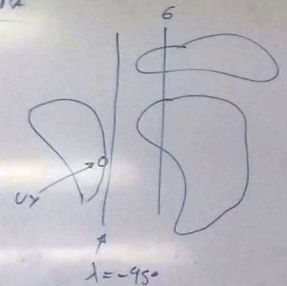
$$T_{Sol Local} = TU + \lambda$$





$$\lambda_m \sim -96^\circ$$

ANALEMA



$$HLU = TU - 3 \text{hs}$$

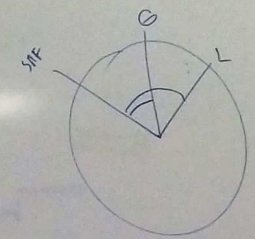
$$T_{\text{Sol. Ap}} - T_{\text{Sol. Medio}} = H_{\odot} - H_{\text{SNF}} = \alpha_{\text{SNF}} - \alpha_{\odot} = \text{Ecuación del Tiempo}$$

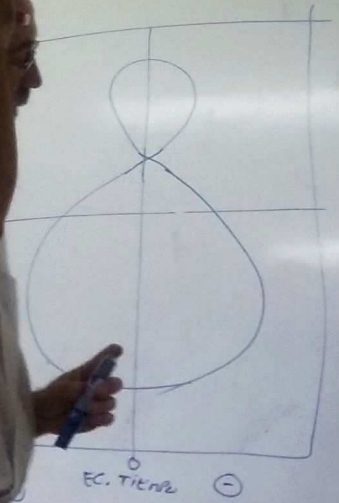
\downarrow $TSL - \alpha_{\odot}$ \downarrow $TSL - \alpha_{\text{SNF}}$

$$T_{\text{Sol. Medio Greenwich}} = TU$$

$$T_{\text{Sol. Medio Local}} = TU + \lambda$$

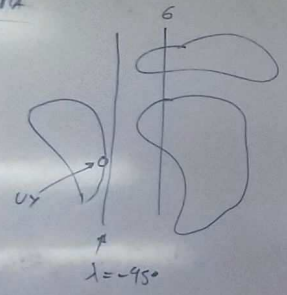
HORA LEGAL : $T_{\text{Sol. Medio}} (\text{MERIDIANO REF.})$





ANALEMA

$$\lambda_n \sim -56^\circ$$



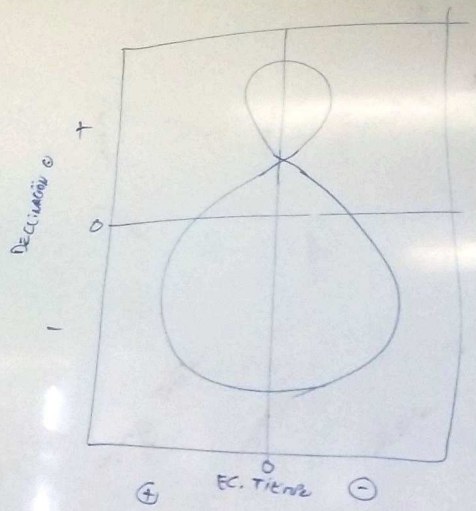
$$HLU = TU - 3 \text{hs}$$

H_0 ?

$$HLU = \textcircled{TU} - 3^{hs}$$

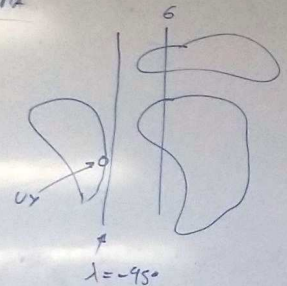
↑
RELOJES

↓
T. SOLAR MEDIO GREEN.



$\lambda_n \sim -56^\circ$

ANALEMA



$$HLU = TU - 3^{\text{hs}}$$

H_0 ?

RELOJES \uparrow

$$HLU = \textcircled{TU} - 3^{\text{hs}}$$

\downarrow T. SOLAR MEDIO GREENW.

$$= \textcircled{\text{T. SOLAR MEDIO MONTEVIDEO}} - \lambda_{\text{MONT.}}$$

\downarrow T. SOLAR @ MONT - E.T.

$$\Rightarrow HLU = T_{\text{Sol. @}} - ET - \lambda_n - 3^{\text{hs}} = T_{\text{Sol. @}} - ET + 44^{\text{m}}$$

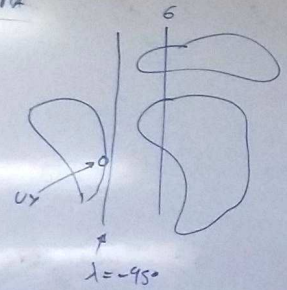
\uparrow visible

$$\lambda_n = -3^{\text{h}} 44^{\text{m}}$$

DIA SOLAR

ANALEMA

$$\lambda_n \sim -56^\circ$$



$$HLU = TU - 3^{\text{hs}}$$

H₀ ?

$$HLU = \text{RELOJES } \left(\text{TU} - 3^{\text{hs}} \right) \text{ T. SOLAR MEDIO GREENW.}$$

$$= \text{T. SOLAR MEDIO MONTEVIDEO} \rightarrow \lambda_{\text{MONT.}}$$

$$\text{T. SOLAR } \odot \text{ MONT.} - \text{E.T.}$$

$$\Rightarrow HLU = T_{\text{Sol } \odot} - \text{ET} - \lambda_n - 3^{\text{hs}} = T_{\text{Sol. } \odot} - \text{ET} + 44^{\text{m}}$$

$$\lambda_n = -3^{\text{h}} 44^{\text{m}}$$

$$\text{RELOJ } \left(HLU = T_{\text{Sol } \odot} + 44^{\text{m}} - \text{ET} \right)$$

visible RELOJ SOLAR

DIA : 2 PASAJES CONSECUTIVAS DE SMF POR MERIDIANO } 0,24 HS SOLARES

: 2 PASAJES DE η POR MERIDIANO } 0,24 HS SIERRAS

DIA SOLAR
T. SOLAR MEDIO

$$\lambda_n \sim -56^\circ$$

$$HLU = \text{RELOJES} \left(\text{TU} - 3^{45} \right)$$

$$= T. \text{Solar Medio GREEN.} = T. \text{Solar Medio MONTEVIDEO} - \lambda_{\text{MONT.}}$$

$$= T_{\text{Sol.} \odot \text{Mont}} - E.T.$$

$$\Rightarrow HLU = T_{\text{Sol.} \odot} - E.T. - \lambda_n - 3^{45} = T_{\text{Sol.} \odot} - E.T. + 44^m$$

$$\lambda_n = -3^h 49^m$$

Diagram illustrating the relationship between HLU, T. Solar Medio Montevideo, and T. Solar Medio Green. The equation $HLU = T_{\text{Sol.} \odot} - E.T. + 44^m$ is shown with annotations: "RELOJ" points to HLU, "VISIBLE" points to $T_{\text{Sol.} \odot}$, and "VISIBLE RELOJ SOLAR" points to the entire equation.

SOL $\rightarrow \beta_0 = 0$

EQUINOCCIO

~ 21 MARZO : SOL EN η

$$\alpha_0 = \lambda_0 = 0^\circ, \delta_0 = 0^\circ$$

~ 21 JUNIO : SOLSTICIO

$$\alpha_0 = \lambda_0 = 90^\circ, \delta_0 = +23^\circ 27'$$

SOL $\rightarrow \beta_0 = 0^\circ$

EQUINOCCIO

\sim 21 MARZO : SOL EN \uparrow

$$\alpha_0 = \lambda_0 = 0^\circ, \delta_0 = 0^\circ$$

\sim 21 JUNIO : SOLSTICIO

$$\alpha_0 = \lambda_0 = 90^\circ, \delta_0 = +23^\circ 27'$$

\sim 22 SET : EQUINOCCIO

SOL EN \downarrow

$$\alpha_0 = \lambda_0 = 180^\circ, \delta_0 = 0^\circ$$

\sim 21 DIC :

SOLSTICIO

$$\alpha_0 = \lambda_0 = 270^\circ, \delta_0 = -23^\circ 27'$$

SOL

EQUINOCCIO

SOL EN π

$$\alpha_{\odot} = \lambda_{\odot} = 0^{\circ}, \quad d_{\odot} = 0^{\circ}$$

SOLSTICIO

$$\alpha_{\odot} = \lambda_{\odot} = 90^{\circ}, \quad d_{\odot} = +23^{\circ} 27'$$

SOLSTICIO

SOL EN 2π

$$\alpha_{\odot} = \lambda_{\odot} = 180^{\circ}, \quad d_{\odot} = 0^{\circ}$$

$$\alpha_{\odot} = \lambda_{\odot} = 270^{\circ}, \quad d_{\odot} = -23^{\circ} 27'$$

COORDENADAS GALÁCTICAS

SOL $\rightarrow \delta_0 = 0^\circ$

\sim 21 MARZO

$\delta_0 = 0^\circ, \alpha_0 = 0^\circ$

\sim 21 JUNIO

$\delta_0 = +23^\circ 27'$

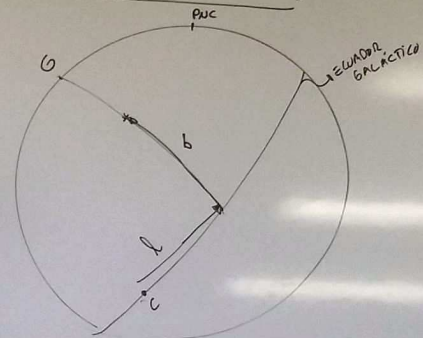
\sim 22

0°

\sim 21 DIC :

$-23^\circ 27'$

COORDENADAS GALÁCTICAS



$l = \text{LONG. GALÁCTICA}$
 $b = \text{LAT. GALÁCTICA}$

SOL $\rightarrow \beta_0 = 0^\circ$

EQUINOCCIO

\sim 21 MARZO : SOL EN \uparrow

$$\alpha_0 = \lambda_0 = 0^\circ, \delta_0 = 0^\circ$$

\sim 21 JUNIO : SOLSTICIO

$$\alpha_0 = \lambda_0 = 90^\circ, \delta_0 = +23^\circ 27'$$

\sim 22 SET : EQUINOCCIO

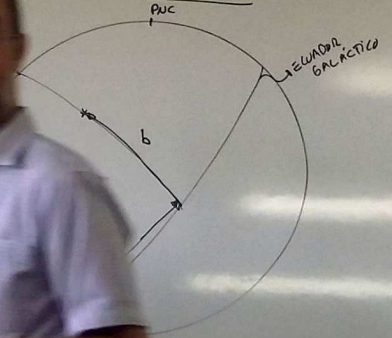
SOL EN \downarrow

$$\alpha_0 = \lambda_0 = 180^\circ, \delta_0 = 0^\circ$$

\sim 21 DIC : SOLSTICIO

$$\alpha_0 = \lambda_0 = 270^\circ, \delta_0 = -23^\circ 27'$$

COORDENADAS GALÁCTICAS



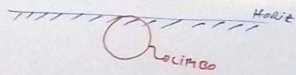
$$\alpha_G = 12^h 51^m$$

$$\delta_G = +27^\circ 8'$$

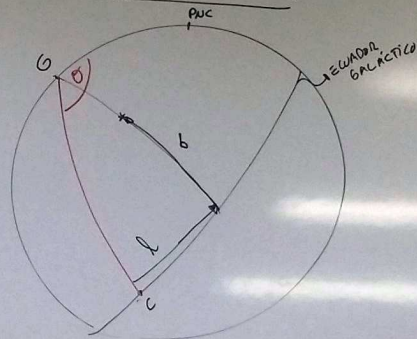
l = LONG. GALÁCTICA

b = LAT. GALÁCTICA

CREPÚSCULOS



COORDENADAS GALÁCTICAS



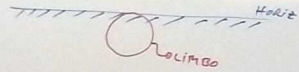
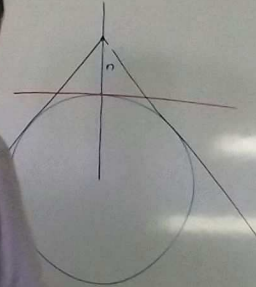
$$\alpha_G = 12^h 51^m$$

$$\delta_G = +27^\circ 8'$$

$$\theta = 123^\circ$$

l = LONG. GALÁCTICA

b = LAT. GALÁCTICA

CREPÚSCULOSDEPRESIÓN DEL HORIZONTE

$$\alpha_G = 12^h 51^m$$

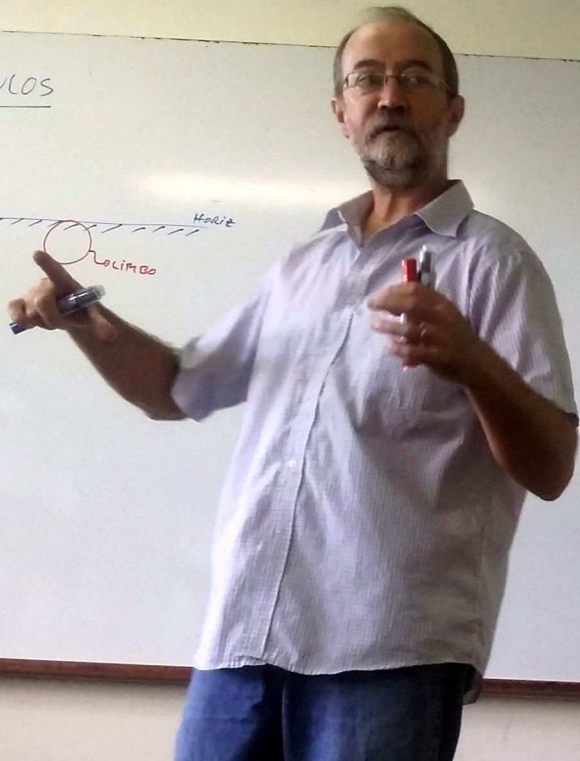
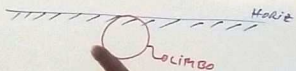
$$d_G = +27^\circ 8'$$

$$\theta = 123^\circ$$

l = LONG. GALÁCTICA

b = LAT. GALÁCTICA

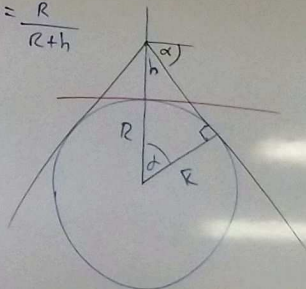
CREPÚSCULOS



DEPRESIÓN DEL HORIZONTE

$\alpha (h)$

$$\cos \alpha = \frac{R}{R+h}$$



$$\alpha_6 = 12^h 51^m$$

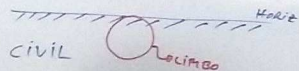
$$d_6 = +27^{\circ} 8'$$

$$\theta = 123^{\circ}$$

$l = \text{LONG. GALÁCTICA}$

$b = \text{LAT. GALÁCTICA}$

CREPÚSCULOS



$$a = -6^\circ$$

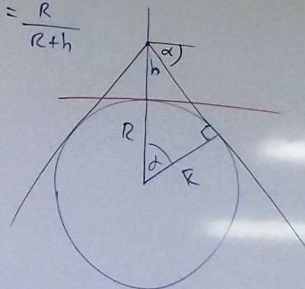
$$a = -12^\circ$$

$$a = -18^\circ$$

DEPRESIÓN DEL HORIZONTE

$$\alpha(h)$$

$$\cos \alpha = \frac{R}{R+h}$$



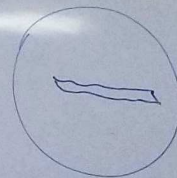
$$\alpha_6 = 12^\circ 51''$$

$$d_6 = +27^\circ 8'$$

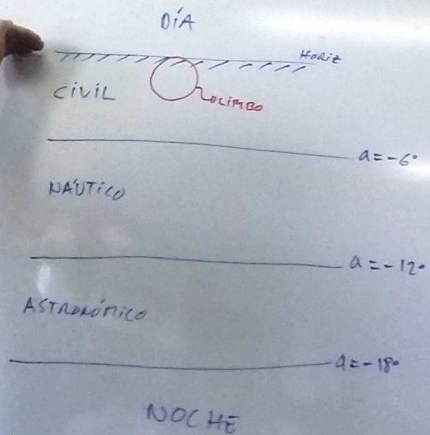
$$\theta = 123^\circ$$

l = LONG. GALÁCTICA

b = LAT. GALÁCTICA



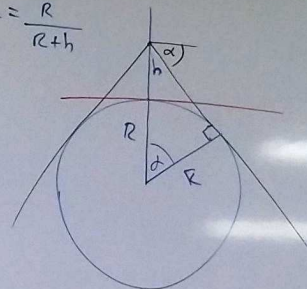
CREPÚSCULOS



DEPRESIÓN DEL HORIZONTE

$\angle (h)$

$$\cos d = \frac{R}{R+h}$$

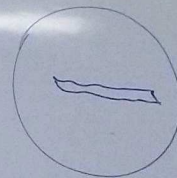


$$\alpha_6 = 12^\circ 51''$$

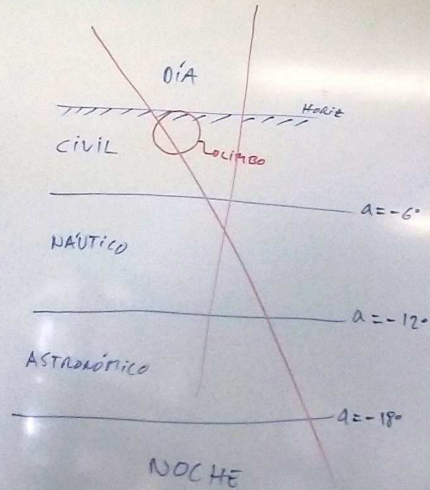
$$d_6 = +27^\circ 8'$$

$$\theta = 123^\circ$$

$l = \text{LONG. GALÁCTICA}$
 $b = \text{LAT. GALÁCTICA}$



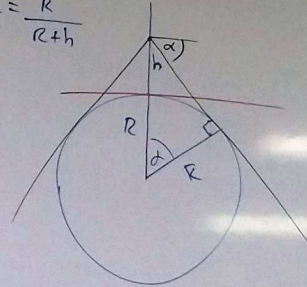
CREPÚSCULOS



DEPRESIÓN DEL HORIZONTE

$\alpha(h)$

$$\cos \alpha = \frac{R}{R+h}$$

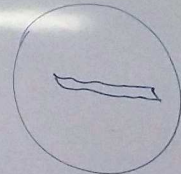


$$\alpha_G = 12^\circ 51''$$

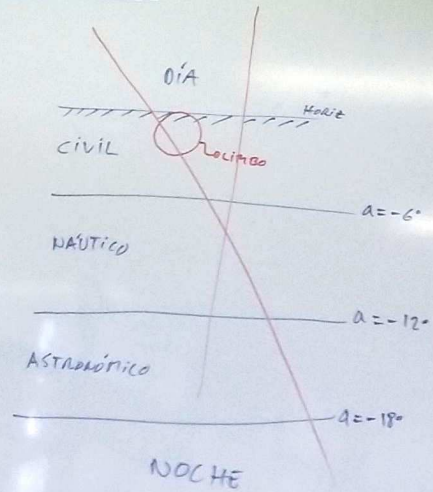
$$\delta_G = +27^\circ 8'$$

$$\theta = 123^\circ$$

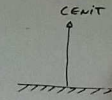
$l = \text{LONG. GALÁCTICA}$
 $b = \text{LAT. GALÁCTICA}$



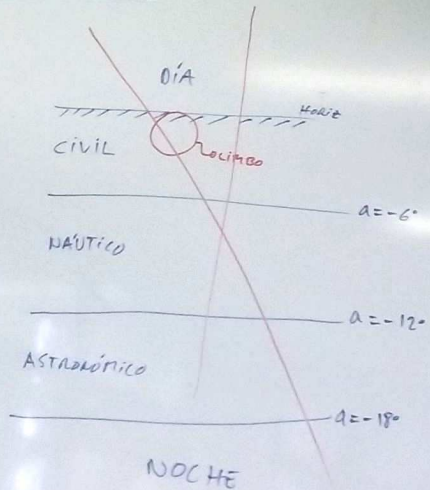
CREPÚSCULOS



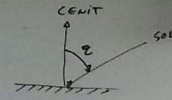
INSOLACIÓN



CREPÚSCULOS

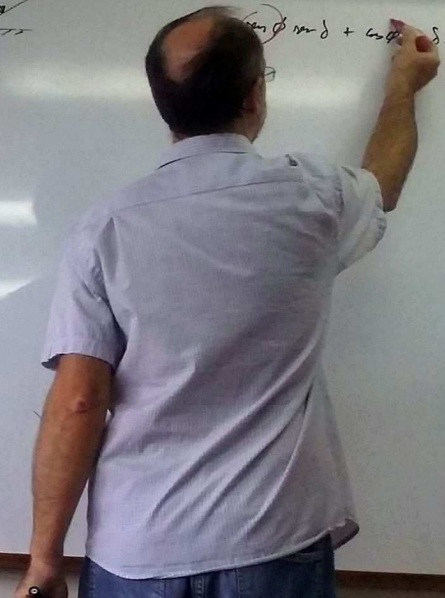


INSOLACIÓN

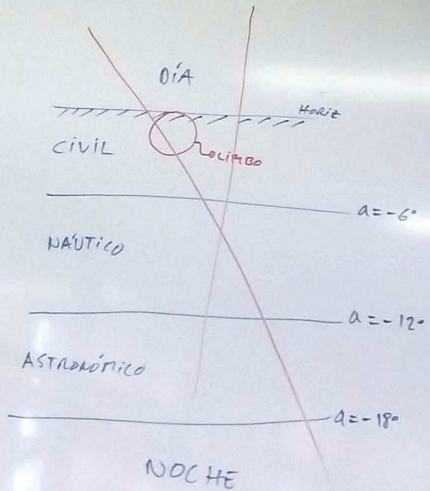


$$\Delta Q = \Delta t \cdot \frac{C_{TE}}{r_0^2} \cdot \cos(z) \cdot z(t)$$

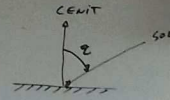
$$\cos(\delta) \cdot \cos(\phi) \cdot \cos(H)$$



CREPÚSCULOS



INSOLACIÓN



$\Delta Q = \Delta \cos(z) \cdot z(t)$

$\cos z = \cos \phi \cos \delta_0 + \sin \phi \sin \delta_0 \cos H_0$

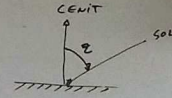
$$\Delta Q = \int_{T_{SOL}}^{T_{AER}} \Delta C_p \cdot \rho \cdot dz + \cos \phi \cdot \sin \delta_0 \cdot \cos H(t) \cdot \rho \cdot dz$$

\downarrow
 RAD.S.

$$dH = \frac{2\pi}{\text{Período Rot.}} \cdot dt$$

$$Q = \frac{CTE}{r_0^2}$$

INSOLACION



$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot \cos(z) \cdot z(t)$$

$$\cos z = \sin \phi \sin \delta_0 + \cos \phi \cos \delta_0 \cos H_0$$

$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot (\sin \phi \sin \delta_0 + \cos \phi \cos \delta_0 \cos H(t))$$

$$dH = \frac{2\pi}{\text{Periodo rot.}} \cdot dt$$

$$\int_{T.SOL}^{T.PUESTA} \Delta Q = \frac{CTE}{r_0^2} \cdot \int_{T.SOL}^{T.PUESTA} (\sin \phi \cdot \sin \delta_0 + \cos \phi \cos \delta_0 \cos H(t)) dt = \frac{CTE}{r_0^2} \cdot \frac{\text{Periodo rot.}}{2\pi} \cdot \int_{H_{SOL}}^{H_{PUESTA}} (\sin \phi \cdot \sin \delta_0 + \cos \phi \cos \delta_0 \cos H) dH$$

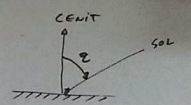
$$Q = \frac{CTE}{r_0^2} \cdot \frac{P_{in}}{2\pi} \cdot Z \left[\sin\phi \cdot \sin\delta_0 (0 - H_{sac}) + \int_0^{H_{sac}} \cos\phi \cdot \cos\delta_0 \cdot \cos H \cdot dH \right]$$

Asociación

$\left. \cos\phi \cos\delta_0 \cos H \right|_0^{H_{sac}} = \cos\phi \cos\delta_0 (-\sin H_{sac})$

$$\frac{Q}{\pi} \cdot \frac{P_{in}}{\pi} \left[\dots \right]$$

INSOLACION



$$\cos z = \sin\phi \sin\delta_0 + \cos\phi \cos\delta_0 \cos H_0$$

$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot \cos(z) \cdot Z(t)$$

$$dH = \frac{2\pi}{\text{Periodo rot.}} \cdot dt$$

$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot \left(\sin\phi \sin\delta_0 + \cos\phi \cos\delta_0 \cos H(t) \right)$$

$$\int_{T_{sac}}^{T_{puesta}} \Delta Q = \frac{CTE}{r_0^2} \cdot \int_{T_{sac}}^{T_{puesta}} \left(\sin\phi \sin\delta_0 + \cos\phi \cos\delta_0 \cos H(t) \right) dt = \frac{CTE}{r_0^2} \cdot \frac{P_{in}}{2\pi} \cdot \int_{H_{sac}}^{H_{puesta}} \left(\sin\phi \sin\delta_0 + \cos\phi \cos\delta_0 \cos H \right) dH$$

$$Q = \frac{CTE}{r_0^2} \cdot \frac{P_{in}}{2\pi} \cdot Z \left[\sin\phi \cdot \sin\delta_0 \cdot (0 - H_{sac}) + \cos\phi \cdot \sin\delta_0 \cdot \cos H \cdot dH \right]$$

Notación

H_{pas} H_{sac}

$$Q = \frac{CTE}{r_0^2} \cdot \frac{P}{\pi} \left[\sin\phi \cdot \sin\delta_0 \cdot H_{pas} + \cos\phi \cdot \sin\delta_0 \cdot \cos H_p \right]$$

(φ, δ₀)

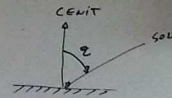
cos φ cos δ₀ sin H |_{H_{sac}}⁰ = cos φ sin δ₀ (-sin H_{sac})

sin H_p

$$Q(r_0^2, \phi, \delta_0)$$

φ, δ₀

INSOLACION



$$\cos z = \sin\phi \sin\delta_0 + \cos\phi \cos\delta_0 \cos H_0$$

$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot \cos(z) \cdot Z(t)$$

$$dH = \frac{2\pi}{\text{Periodo}} \cdot dt$$

Periodo
rot.

$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot (\sin\phi \sin\delta_0 + \cos\phi \sin\delta_0 \cos H(t))$$

$$\int_{T_{sac}}^{T_{puesta}} \Delta Q = \frac{CTE}{r_0^2} \cdot \int_{T_{sac}}^{T_{puesta}} (\sin\phi \cdot \sin\delta_0 + \cos\phi \sin\delta_0 \cos H(t)) dt = \frac{CTE}{r_0^2} \cdot \frac{P_{in}}{2\pi} \cdot \int_{H_{sac}}^{H_{puesta}} (\sin\phi \cdot \sin\delta_0 + \cos\phi \sin\delta_0 \cos H) dH$$

T. SAC T. PUESTA H_{SAC} H_{PUESTA}

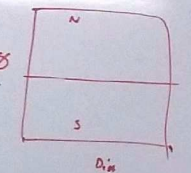
$$Q = \frac{CTE}{r_0^2} \cdot \frac{P_{in}}{2\pi} \cdot Z \left[\sin\phi \cdot \sin\delta_0 \cdot (0 - H_{sac}) + \int_0^0 \cos\phi \cdot \cos\delta_0 \cdot \cos H \cdot dH \right]$$

Rotació

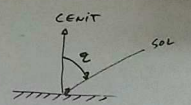
$$Q = \frac{CTE}{r_0^2} \cdot \frac{P}{\pi} \left[\sin\phi \cdot \sin\delta_0 \cdot H_{pmax} + \cos\phi \cos\delta_0 \cos H_p \right]$$

ϕ, δ_0

$$Q(r_0^2, \phi, \delta_0)$$



INSOLACION



$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot \cos(z) \cdot Z(t)$$

$$\cos z = \sin\phi \sin\delta_0 + \cos\phi \cos\delta_0 \cos H_0$$

$$dH = \frac{2\pi}{\text{Període Rot.}} \cdot dt$$

$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot (\sin\phi \sin\delta_0 + \cos\phi \cos\delta_0 \cos H(t))$$

$$\int_{T_{SOL}}^{T_{PUERTA}} \Delta Q = \frac{CTE}{r_0^2} \cdot \int_{T_{SOL}}^{T_{PUERTA}} (\sin\phi \cdot \sin\delta_0 + \cos\phi \cos\delta_0 \cos H(t)) dt = \frac{CTE}{r_0^2} \cdot \frac{P_{in}}{2\pi} \cdot \int_{H_{SOL}}^{H_{PUERTA}} (\sin\phi \cdot \sin\delta_0 + \cos\phi \cos\delta_0 \cos H) dH$$

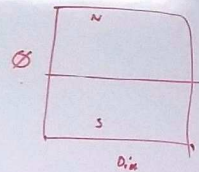
$$Q = \frac{CTE}{r_0^2} \cdot \frac{P_{in}}{2\pi} \cdot Z \left[\sin\phi \cdot \sin\delta_0 \cdot \cos(H_{sac}) + \int_{H_{sac}}^0 \cos\phi \cdot \cos\delta_0 \cdot \cos H \cdot dH \right]$$

Rotación

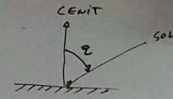
$$Q = \frac{CTE}{r_0^2} \cdot \frac{P}{\pi} \left[\sin\phi \cdot \sin\delta_0 \cdot H_{punta} + \cos\phi \cdot \cos\delta_0 \cdot \cos H_p \right]$$

rotación en H | $H_{sac} = 0 = \cos\phi \cdot \cos\delta_0 \cdot (-\sin H_{sac})$

$$Q(r_0^2, \phi, \delta_0)$$



INSOLACIÓN



$$\cos z = \sin\phi \cdot \sin\delta_0 + \cos\phi \cdot \cos\delta_0 \cdot \cos H_0$$

$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot \cos(z) \cdot Z(t)$$

$$dH = \frac{2\pi}{P_{rot}} \cdot dt$$

$$\Delta Q = \Delta t \cdot \frac{CTE}{r_0^2} \cdot \left(\sin\phi \cdot \sin\delta_0 + \cos\phi \cdot \cos\delta_0 \cdot \cos H(t) \right)$$

$$\int_{T.SAC}^{T.PUNTA} \Delta Q = \frac{CTE}{r_0^2} \cdot \int_{T.SAC}^{T.PUNTA} \left(\sin\phi \cdot \sin\delta_0 + \cos\phi \cdot \cos\delta_0 \cdot \cos H(t) \right) dt = \frac{CTE}{r_0^2} \cdot \frac{P_{rot}}{2\pi} \cdot \int_{H_{sac}}^{H_{punta}} \left(\sin\phi \cdot \sin\delta_0 + \cos\phi \cdot \cos\delta_0 \cdot \cos H \right) dH$$

T. SÍMBOLO MEDIO DE GREENWICH

(TSG6: GMST)

GMST = 18.69

T. SÍMBOLO MEDIO DE GREENWICH

(TSMB: GMST)

$$\text{GMST} = 18^{\text{h}}.69737 + 24^{\text{h}}.0657098 \cdot (\text{JD} - 2451545.0)$$

(Hours)

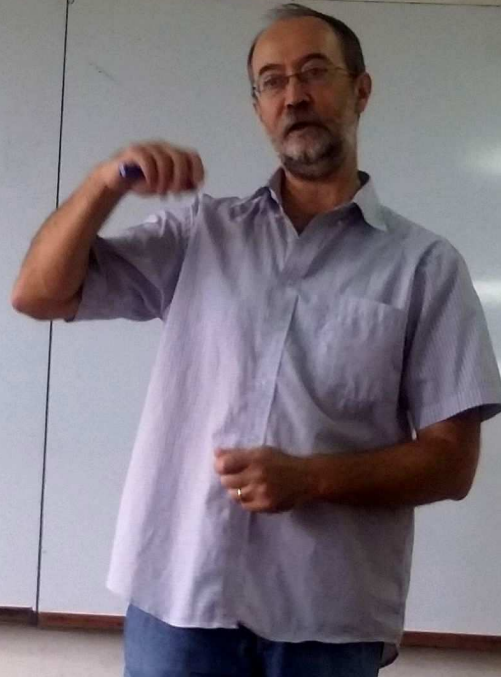
T. SIDEREO MEDIO DE GREENWICH

(TSG ó GMST)

$$\text{GMST} = 18^{\text{h}}.69737 + 24.0657098 \cdot (\text{JD} - 2451545.0)$$

(Horas siderales)

FECHA JULIANA
DEL INSTANTE



T. SIEMERO MEDIO DE GREENWICH

(TSMB ó GMST)

$$\text{GMST} = 18^{\text{h}}.69737 + 24.0657098 \cdot (\text{JD} - 2451545.0)$$

(HORAS SIDERALES)

FECHA JULIANA
DEL INSTANTE

$$\text{JD} = 0$$

-4713 1 ENERO 12h
LUNES

A }
M } → INSTANTE
D } → JD
TU }

T. SÍMBOLO MEDIO DE GREENWICH

(TSMB o GMST)

$$\text{GMST} = 18^{\text{h}}.69737 + 24^{\text{h}}.0657098 \cdot (\text{JD})$$

(Horas siderales)

FECHA JULIANA
DEL INSTANTE

$$\text{JD} = 0$$

$$-4713 \quad 1 \text{ EVE} \\ \text{LUNE}$$

REFRACCIÓN

T. SÍMBOLO MEDIO DE GREENWICH
(TSG6 o GMST)

$$GMST = 18^h.69737 + 24^h.0657098 \cdot (JD - 2451545.0)$$

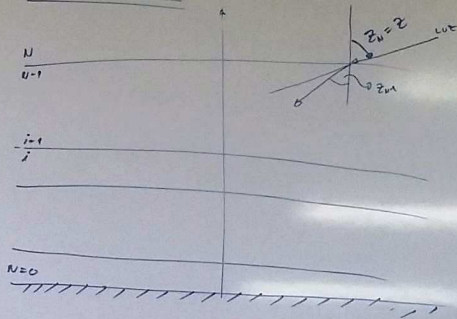
(HORAS SIDERALES)

FECHA JULIANA DEL INSTANTE

$$JD = 0$$

-4713 1 ENERO 12h
LUNES

REFRACCIÓN



SVEL m_w



T. SIMONEO

GREENWICH

(SHA6 o GMT)

GMT

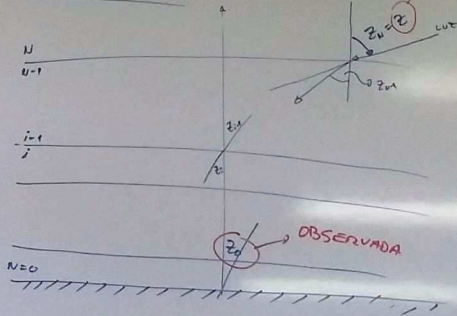
$$+ 24,0657098 \cdot (JD - 2451545.0)$$

FECHA JULIANA DEL INSTANTE

= 0

-4713 1 Enero 12h
LUNES

REFRACCION



SHELL

$$M_n \cdot n \cdot Z_n = M_{n-1} \cdot n \cdot Z_{n-1}$$

$$M_i \cdot n \cdot Z_i = M_{i-1} \cdot n \cdot Z_{i-1}$$

$$M_0 \cdot n \cdot Z_0$$

T. SIEMPRE MEDIO DE GREENWICH
(TSG6 o GMST)

$$GMST = 18^h.69737 + 24.0657098 \cdot (JD - 2451545.0)$$

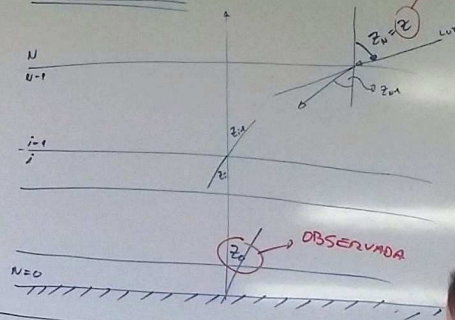
(HORAS SIDERALES)

FECHA JULIANA DEL INSTANTE

JD = 0

-4713 1 Enero 12h LUNES

REFRACCIÓN



SWELL $M_n \cdot n \cdot z_n = M_{n-1} \cdot n \cdot z_{n-1}$

$M_i \cdot n \cdot z_i = M_{i-1} \cdot n \cdot z_{i-1}$

$M_0 \cdot n \cdot z_0$

$z - z_0 = R \rightarrow z = R + z_0$

$n \cdot z = n \cdot (R + z_0) = M_0 \cdot n \cdot z_0$

$\sin R \cdot \cos z_0 + \cos R \cdot \sin z_0 = 1$

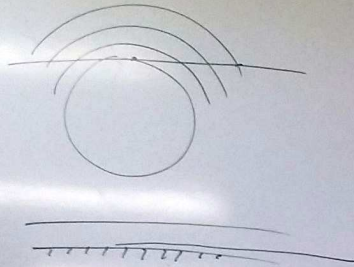
$M_n \cdot n \cdot z_n = M_0 \cdot n \cdot z_0$

$$\Rightarrow R \cdot \cos z_0 + 1 \cdot n z_0 = M_0 \cdot n z_0$$

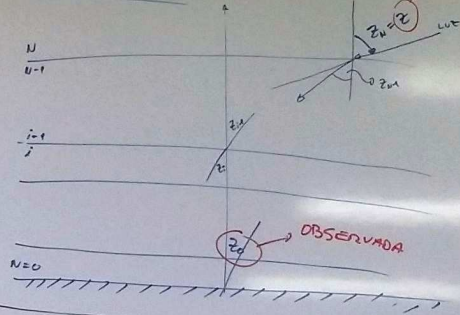
$$R \cos z_0 = (M_0 - 1) \cdot n z_0$$

$$\Rightarrow R = (M_0 - 1) \cdot \frac{1}{\cos z_0} \cdot z_0$$

1.0002527



REFRACCIÓN



SNEL $M_u \cdot n u z_u = M_{u-1} \cdot n z_{u-1}$

$$M_i \cdot n z_i = M_{i-1} \cdot n z_{i-1}$$

$$M_0 \cdot n z_0$$

$$z - z_0 = R \Rightarrow z = R + z_0$$

$$n z = n (R + z_0) = M_0 \cdot n z_0$$

$$R \left(\frac{1}{\cos z_0} \right) \cdot \cos z_0 + \cos z_0 R \cdot n z_0 = M_0 \cdot n z_0$$

$$M_u \cdot n z_u = M_0 \cdot n z_0$$

1 SUP.

$$\Rightarrow R \cdot \cos \alpha + 1 \cdot m \cdot z_0 = M_0 \cdot m \cdot z_0$$

$$R \cos \alpha = (M_0 - 1) \cdot m \cdot z_0$$

$$\Rightarrow R = \frac{(M_0 - 1) \cdot \frac{1}{2} z_0}{\cos \alpha}$$

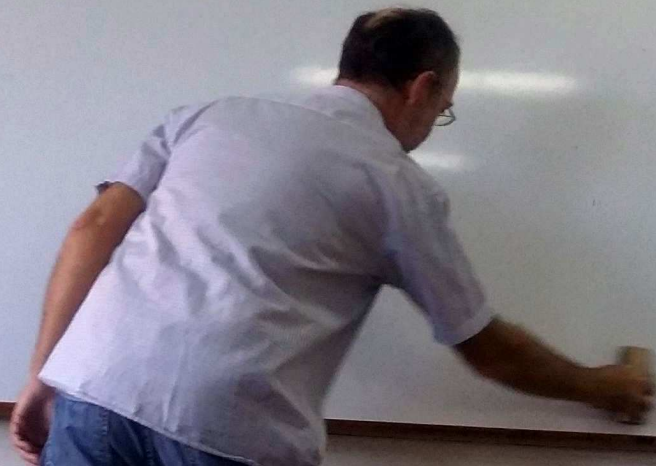
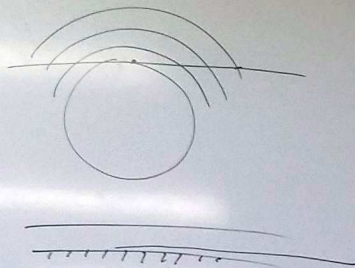
RADIANES

1.0002927

$$R = K \cdot \frac{1}{2} z_0$$

60".4

CONSTANTE DE REFRACCIÓN



REFRACCIÓN

$$R = \underbrace{(K)}_{60''.4} \cdot \frac{1}{y} z \quad (z. < 70^\circ)$$


SEEING



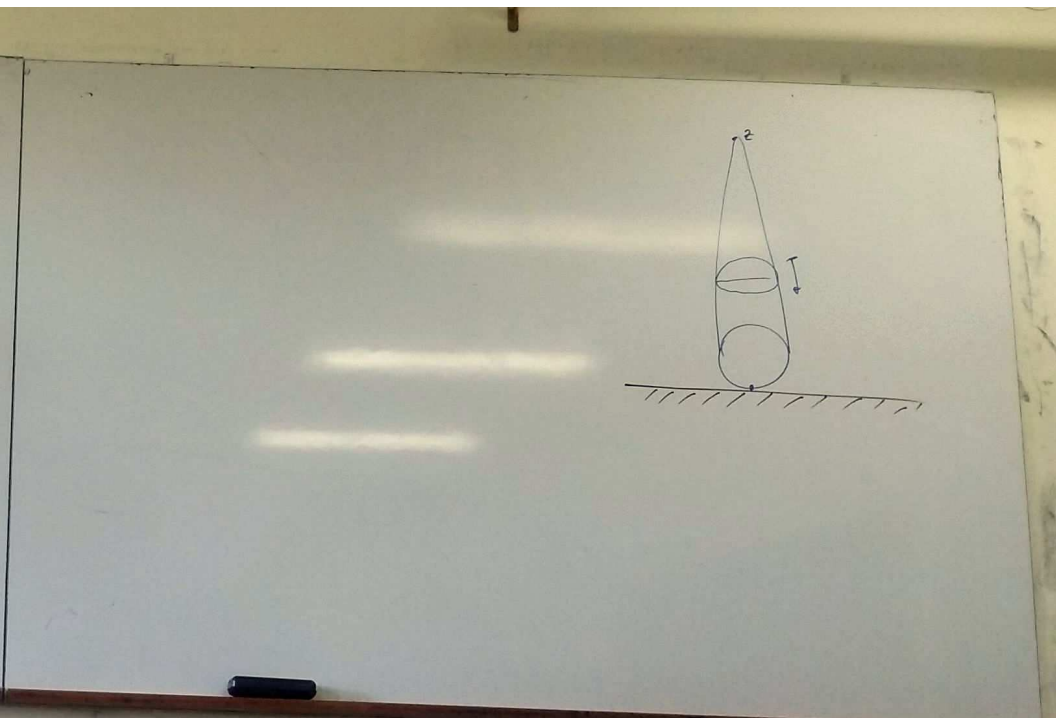
REFRACTION

SEEING

(2.70)



A man in a light blue shirt stands in front of a whiteboard. He is gesturing with his hands. The whiteboard has the word 'REFRACTION' written on the left and 'SEEING' on the right. There is a handwritten number '(2.70)' and a small diagram of a square with a circle and two 'x' marks.

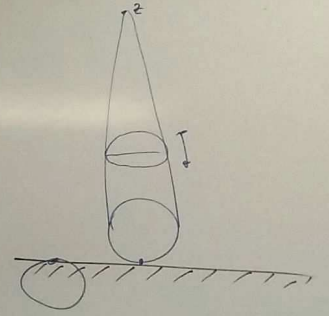
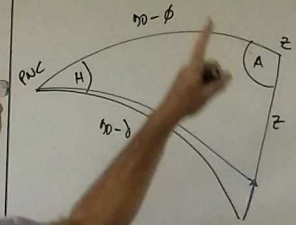


REFRACCIÓN

$$R = \underbrace{(K)}_{60''.4} \cdot \frac{1}{y} z_0 \quad (z_0 < 70^\circ)$$

¿EFECTOS EN α, δ ?

SEEING



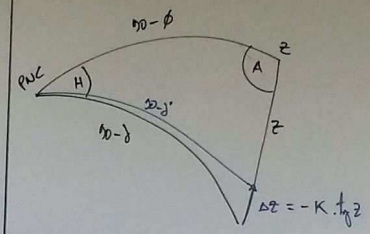
REFRACCIÓN

$\frac{1}{f} z$ ($z_0 < 70^\circ$)

¿EN α, δ ?

$\Delta = \alpha + \dots$

SEEING



METODO (1) : DERIVAR FORMULAS

$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H$

$-\sin z \cdot \Delta z = \sin \phi \cos \delta \cdot \Delta \delta - \cos \phi \cdot \sin \delta \cdot \Delta \delta \cdot \cos H$

$-\cos \phi \cos \delta \cdot \sin H \cdot \Delta H$

REFRACCIÓN

$$R = \textcircled{K} \cdot \frac{1}{\gamma} z \quad (z_0 < 70^\circ)$$

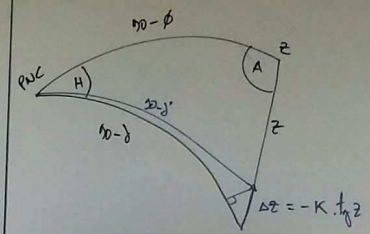
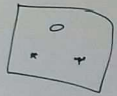
$60'' \cdot 4$

¿EFECTOS EN α, δ ?

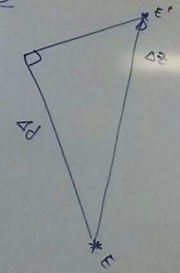
$$TSL = \alpha + \dots$$

$$\Rightarrow \Delta\alpha = -$$

SEEING



MÉTODO (2):



MÉTODO (1): Derivan Fórmulas

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cdot \cos H$$

$$-\sin z \cdot \Delta z = \sin \phi \cos \delta \cdot \Delta \delta - \cos \phi \cdot \sin \delta \cdot \Delta \delta \cdot \cos H$$

$$- \cos \phi \cos \delta \cdot \sin H \cdot \Delta H$$

REFRACCIÓN

$$R = \textcircled{K} \cdot \frac{1}{\gamma} z_0 \quad (z_0 < 70^\circ)$$

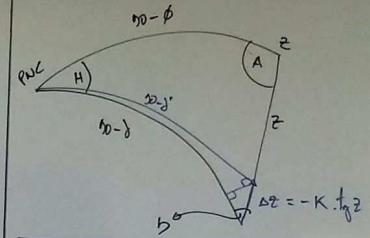
60".4

¿EFECTOS EN α, δ ?

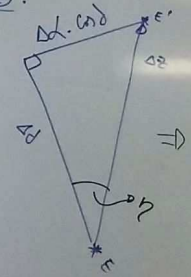
$$TSL = \alpha + H$$

$$\Rightarrow \Delta\alpha = -\Delta H$$

SEEING



MÉTODO ②:



MÉTODO ①: DERIVAR FÓRMULAS

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cdot \cos H$$

$$-\sin z \cdot \Delta z = \sin \phi \cos \delta \cdot \Delta \delta - \cos \phi \sin \delta \cdot \Delta \delta \cdot \cos H - \cos \phi \cos \delta \cdot \sin H \cdot \Delta H$$

REFRACCIÓN

$$R = \textcircled{K} \cdot \frac{1}{\rho} z$$

60".4

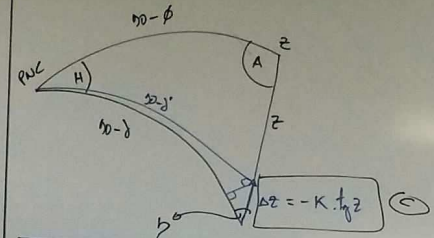
EFECTOS EN

TS

$$\frac{n \cdot \eta}{\cos \phi} = \frac{n \cdot H}{n \cdot z} \quad \textcircled{B}$$

DE $\textcircled{A} > \textcircled{B}$:

$$\Delta \alpha \cdot \cos \delta = \textcircled{\Delta z} \cdot \cos \phi \cdot \frac{n \cdot H}{n \cdot z}$$

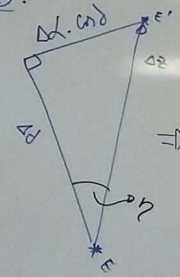


MÉTODO (1) : Derivar Fórmulas

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cdot \cos H$$

$$-n \cdot z \cdot \Delta z = \sin \phi \cos \delta \cdot \textcircled{\Delta \delta} - \cos \phi \cdot n \cdot d \cdot \textcircled{\Delta \delta} \cdot \cos H - \cos \phi \cos \delta \cdot n \cdot H \cdot \textcircled{\Delta H}$$

MÉTODO (2) :



$$\Delta \alpha \cdot \cos \delta = \Delta z \cdot \sin \eta$$

$$\Delta \delta = \Delta z \cdot \cos \eta$$

$$\eta(\phi, z, \delta, \dots)$$

RELACION

$$K \cdot \frac{1}{\sin z} \quad (z < 70^\circ)$$

$$\frac{\sin \gamma}{\cos \phi} = \frac{\sin H}{\sin z} \quad (B)$$

DE (A) > (B):

$$\Delta \alpha \cdot \cos \delta = \Delta z \cdot \cos \phi \cdot \frac{\sin H}{\sin z}$$

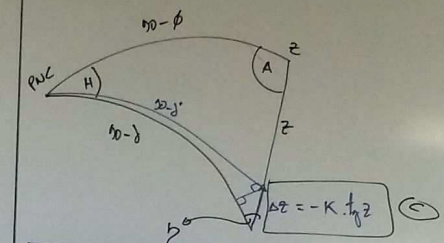
USANDO (C):

$$\Delta \alpha \cdot \cos \delta = -K \frac{\sin \phi \sin H}{\cos z}$$

$$\Rightarrow \Delta \alpha = -K \cdot \frac{\cos \phi \cdot \sin H}{\cos \delta \cos z}$$

$$TSL = \alpha + H$$

$$\Rightarrow \Delta \alpha = -\Delta H$$



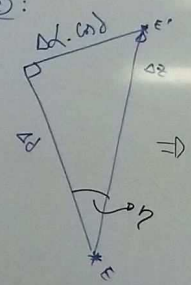
MÉTODO (1): DERIVAR FÓRMULAS

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cdot \cos H$$

$$-\sin z \cdot \Delta z = \sin \phi \cos \delta \cdot \Delta \delta - \cos \phi \cdot \sin \delta \cdot \Delta \delta \cdot \cos H$$

$$-\cos \phi \cos \delta \cdot \sin H \cdot \Delta H$$

MÉTODO (2):



$$\Delta \alpha \cdot \cos \delta = \Delta z \cdot \sin \gamma \quad (A)$$

$$\Delta \delta = \Delta z \cdot \cos \gamma$$

$$\eta(\phi, z, \delta, \dots)$$

REFRACCIÓN

$$R = K \cdot \frac{1}{\rho} z \quad (z_0 < 70^\circ)$$

60".4

¿EFECTOS EN α , δ ?

$$\frac{r - \eta}{\cos \phi} = \frac{m \cdot H}{m \cdot z} \quad \textcircled{B}$$

DE $\textcircled{A} \times \textcircled{B}$:

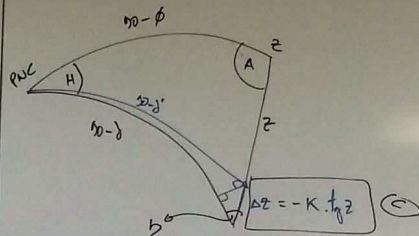
$$\Delta \alpha \cdot \cos \delta = \pm \Delta z \cdot \cos \phi \cdot \frac{m \cdot H}{m \cdot z}$$

USANDO \textcircled{C} :

$$\Delta \alpha \cdot \cos \delta = +K \frac{\cos \phi \cdot m \cdot H}{\cos z}$$

$$\Rightarrow \Delta \alpha = +K \cdot \frac{\cos \phi \cdot m \cdot H}{\cos \delta \cos z}$$

$$\Delta \delta = \dots$$

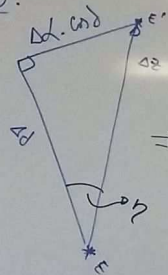


MÉTODO (1): DERIVAR FÓRMULAS

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cdot \cos H$$

$$-m \cdot z \cdot \Delta z = \cos \phi \cos \delta \cdot \Delta \delta - \cos \phi \sin \delta \cdot \Delta \delta \cdot \cos H - \cos \phi \sin \delta \cdot m \cdot H \cdot \Delta H$$

MÉTODO (2):



$$\Delta \alpha \cdot \cos \delta = -\Delta z \cdot m \cdot \eta \quad \textcircled{A}$$

$$\Delta \delta = -\Delta z \cdot \cos \eta$$

$\eta(\phi, z, \delta, \dots)$

↑
OJO

$$\cos(90-\phi) = \cos(90-\delta) \cdot \cos z + \sin(90-\delta) \cdot m \cdot z \cdot \cos \eta$$

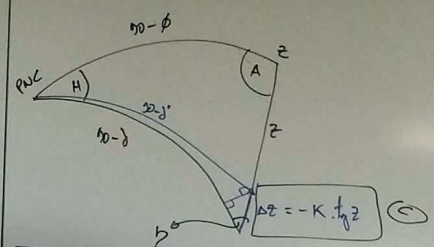
$$\frac{\sin \gamma}{\cos \phi} = \frac{\sin H}{\sin z} \quad \textcircled{B}$$

× ②:

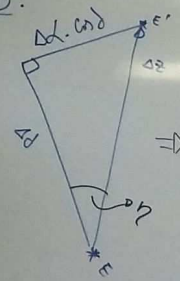
$$\Delta z \cdot \cos \phi \cdot \frac{\sin H}{\sin z}$$

③:

$$+ K \frac{\cos \phi \sin H}{\sin z}$$



MÉTODO ②:



$$\Delta d \cdot \cos \delta = -\Delta z \cdot \sin \gamma \quad \textcircled{A}$$

$$\Delta d = -\Delta z \cdot \cos \gamma$$

↑
ojo

$$\cos(90-\phi) = \cos(90-\delta) \cdot \cos z + \sin(90-\delta) \cdot \sin z \cdot \cos \gamma$$

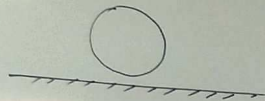
MÉTODO ①: DERIVAR FÓRMULAS

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cdot \cos H$$

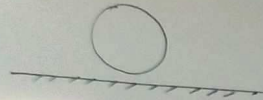
$$-\sin z \cdot \Delta z = \sin \phi \cos \delta \cdot \Delta \delta - \cos \phi \cdot \sin \delta \cdot \Delta \delta \cdot \cos H - \cos \phi \cos \delta \cdot \sin H \cdot \Delta H$$

Acción Horizontal

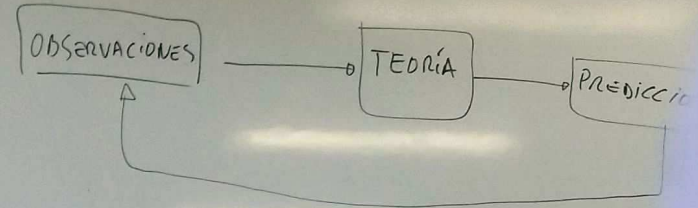
$$R \approx 34'$$



REFRACCIÓN HORIZONTAL
 $R \approx 34'$



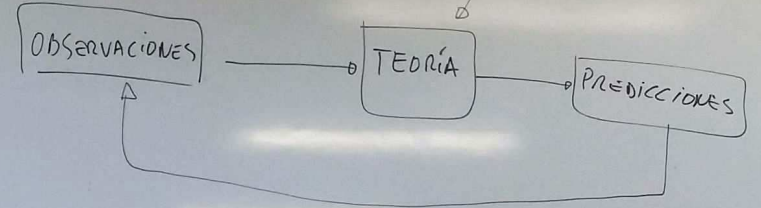
SISTEMAS DE REFERENCIA

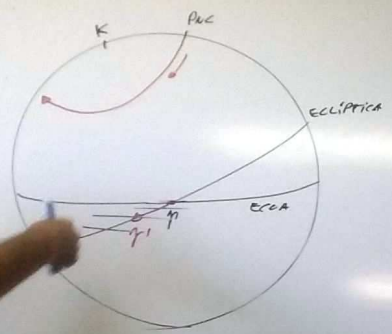


REFRACCIÓN HORIZONTAL
 $R \approx 34'$



SISTEMAS DE REFERENCIA





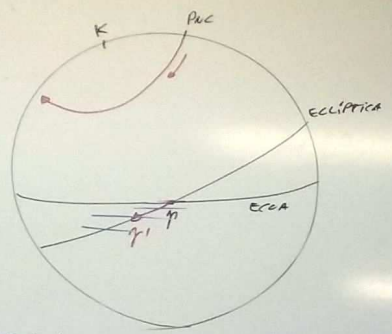
SISTEMAS DE REFERENCIA

OBSERVACIONES

TEORÍA

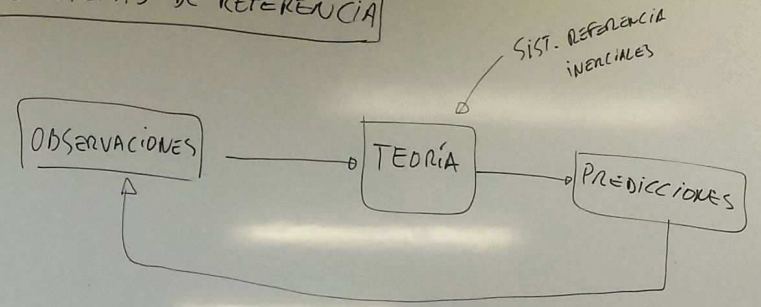
PREDICCIONES

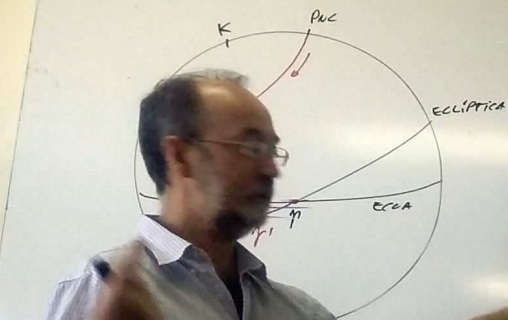
SIST. REFERENCIA INERCIALES



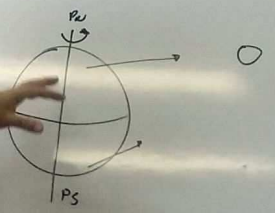
PRECESIÓN
DE LOS
EQUINOCCIOS

SISTEMAS DE REFERENCIA

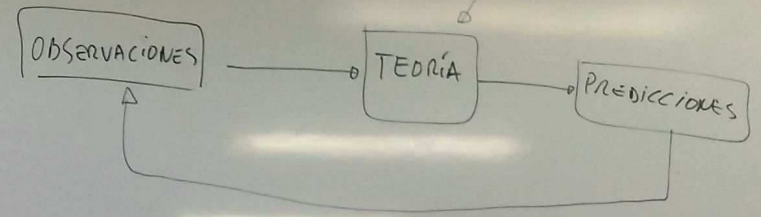


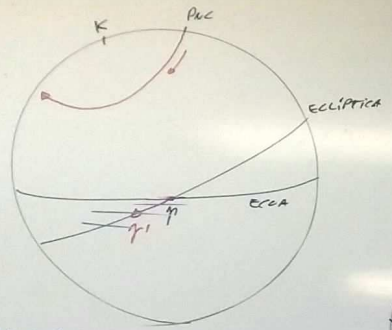


MOV. K → PRECESIÓN PLANETARIA
MOV. P.V.C. → PRECESIÓN LUNAR-SOLAR



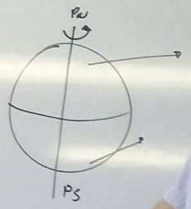
SISTEMAS DE REFERENCIA





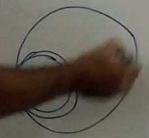
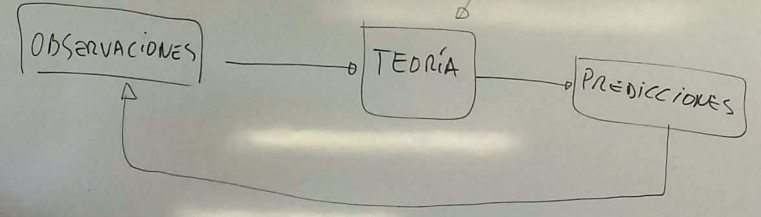
MOV. K → PRECESIÓN PLANETARIA (MOV. ORBITAL T.)

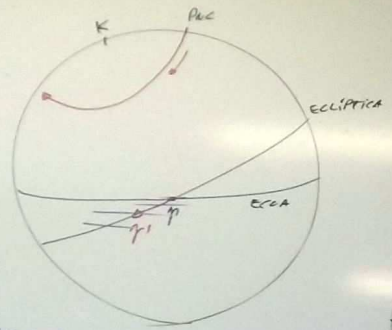
MOV. PNC → PRECESIÓN (ROTACIÓN)



PRECESIÓN DE LOS EQUINOCCIOS

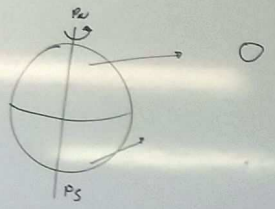
SISTEMAS DE REFERENCIA





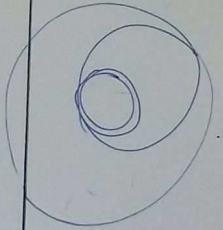
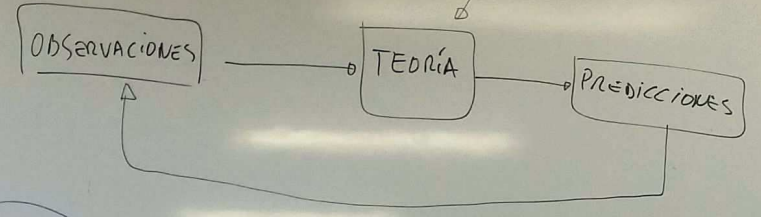
mov. K → PRECESIÓN PLANETARIA (MOV. ORBITAL T.)

MOV. PAC → PRECESIÓN LUNAR-SOLAR (ROTACIÓN T.)

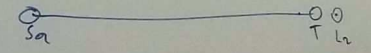


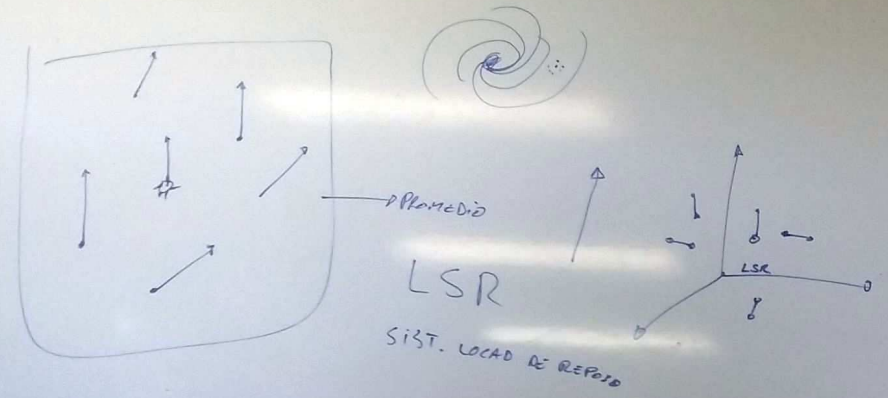
PRECESIÓN DE LOS EQUINOCCIOS

SISTEMAS DE REFERENCIA

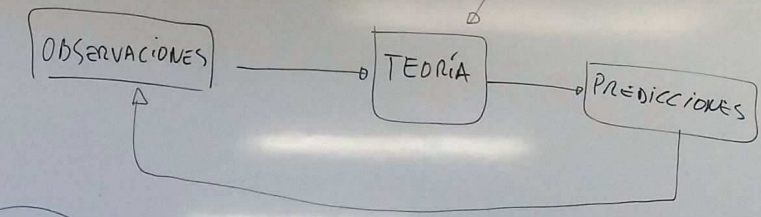


GAIA

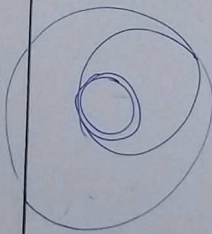
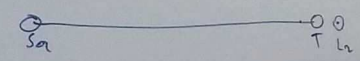


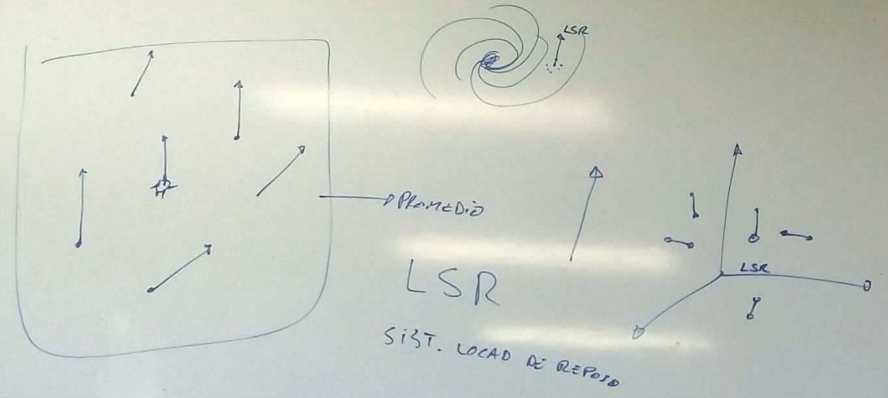


SISTEMAS DE REFERENCIA



GAIA





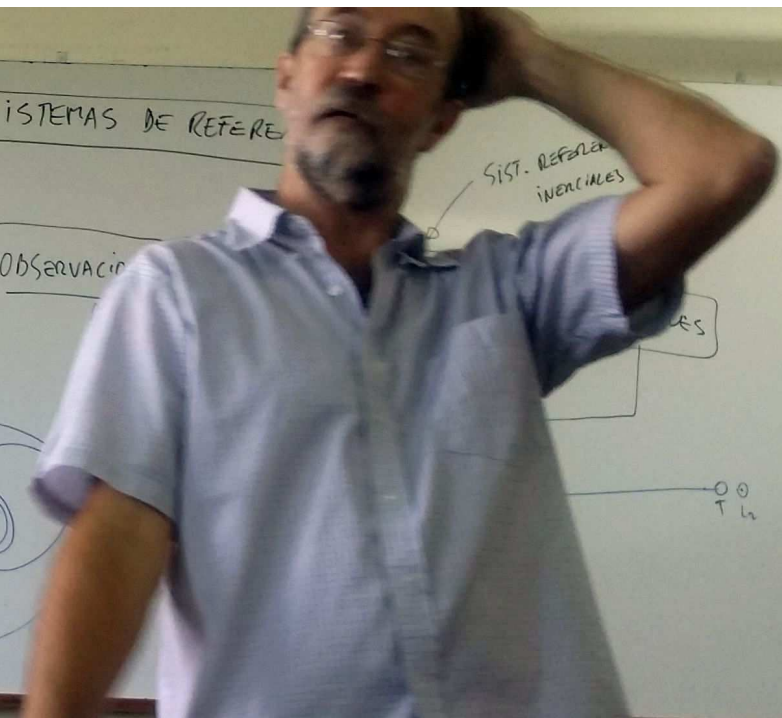
SISTEMAS DE REFERENCIA

OBSERVACIONES

SIST. REFERENCIAL INERCIALES

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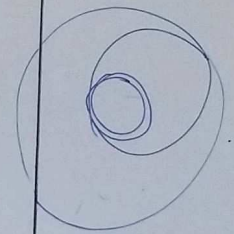
SISTEMAS DE REFERENCIA

OBSERVACIONES

TEORÍA

PREDICCIONES

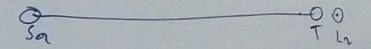
SIST. REFERENCIA INERCIALES

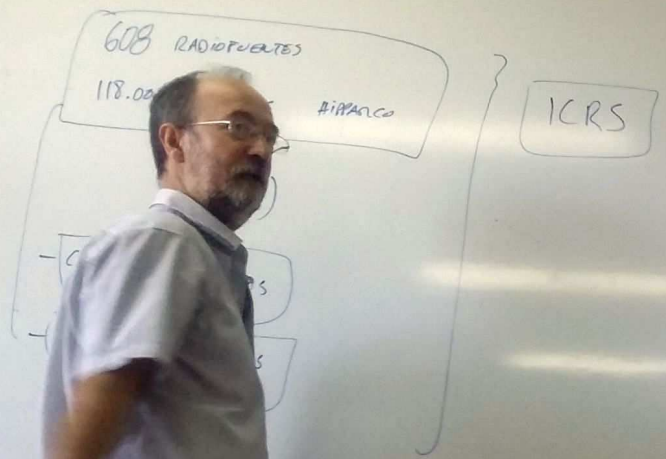


GAIA

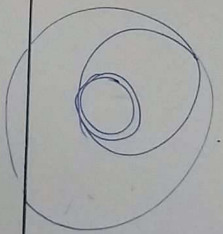
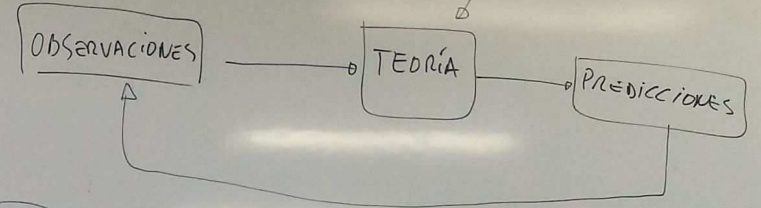
S_{α}

$T L_2$

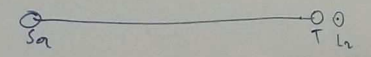




SISTEMAS DE REFERENCIA



GAIA

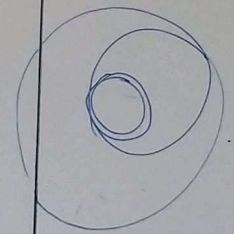
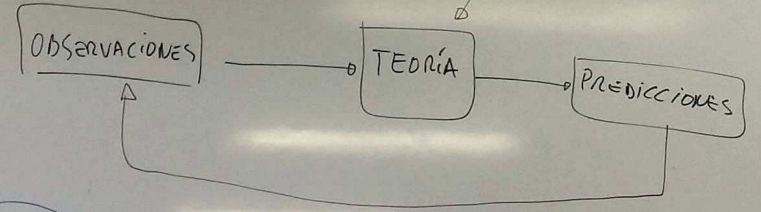


608 RADIOFUENTES
118.000 ESTRELLAS HIPPARCO
MARCO (FRAME)
- CONSTANTES FÍSICAS
- TEORÍAS DINÁMICAS

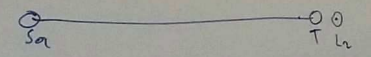
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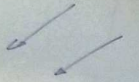
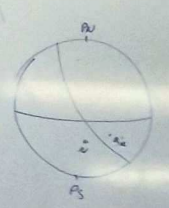
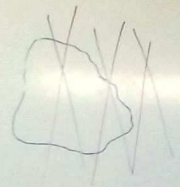
SISTEMAS DE REFERENCIA



GAIA

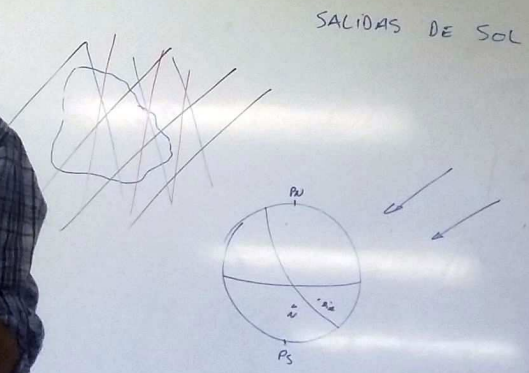


SALIDAS DE SOL



$$\cos A = 0 \Rightarrow A = x$$
$$A = 360 - x$$

Res A =

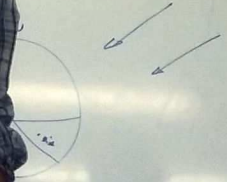


SIST. DE REFERENCIA

- PNC → PRECESIÓN LUNISOLAR



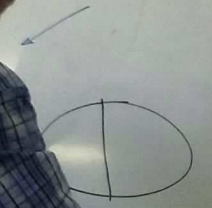
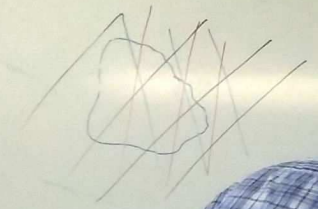
SALIDAS DE SOL



SIST. DE REFERENCIA

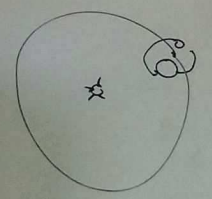
- PNC → PRECESIÓN LUNISOLAR
- K → PRECESIÓN PLANETARIA

SALIDAS DE SOL



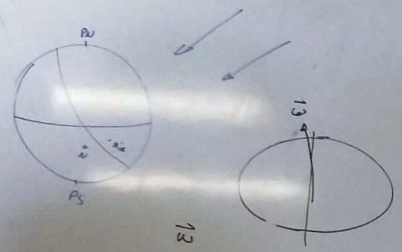
SIST. DE REFERENCIA

- PNC → PRECESIÓN LUNISOLAR
- K → PRECESIÓN PLANAR



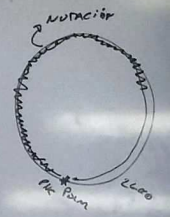
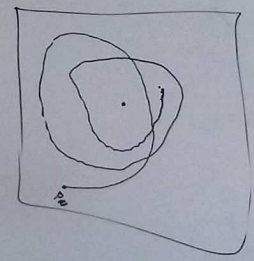
SALIDAS DE SOL

$$\frac{d\vec{L}}{dt} = \vec{M}$$
$$\vec{L} = \pi \vec{\omega}$$



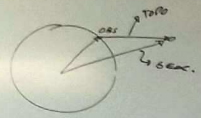
SIST. DE REFERENCIA

- PNC → PRECESIÓN Y NUTACIÓN LUNISOLAR
- K → PRECESIÓN PLANAR
- MOVIMIENTO POLAR



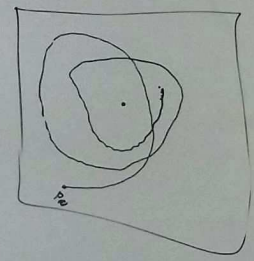
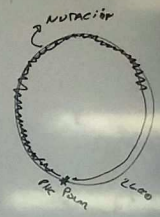
ORIGEN:

- TOPOCÉNTRICAS
(SIN REFRACCIÓ)
- GECÉNTRICAS



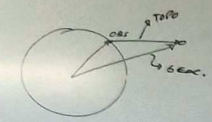
SIST. DE REFERENCIA

- PNC → PRECESSION Y NUTACION LUNISOLAR
- K → PRECESSION PLANETARIA
- MOVIMIENTO POLAR



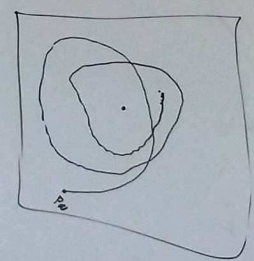
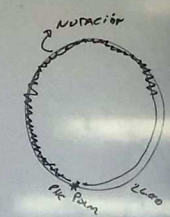
ORIGEN:

- TOPOCÉNTRICAS
(SIN REFRACCIÓ)
- GEOCÉNTRICAS
- HELIOCÉNTRICAS
- BARICÉNTRICAS
(B. DEL S.S.)



SIST. DE REFERENCIA

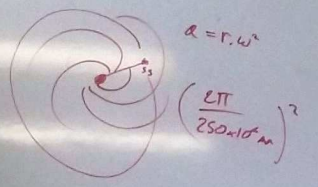
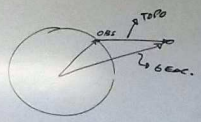
- PNC → PRECESSION Y NUTACIÓ LUNISOLAR
- K → PRECESSION PLANETARIA
- MOVIMIENT POLAR



ORIGEN:

ICS

- TOPOCÈNTRICAS
(SIN REFRACCIÓ)
- GEOCÈNTRICAS
- HELIOCÈNTRICAS
- BARICÈNTRICAS
(B. DEL S.S.)



$$a = r \cdot \omega^2$$

$$\left(\frac{2\pi}{250 \cdot 10^6 \text{ m}} \right)^2$$

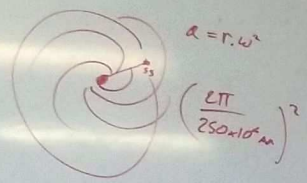
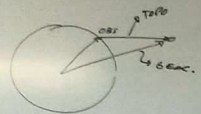
$$a \approx 2 \cdot 10^{-10} \text{ m/s}^2$$

ICRS

ICRF
608 n
118 000 est.

ORIGEN:

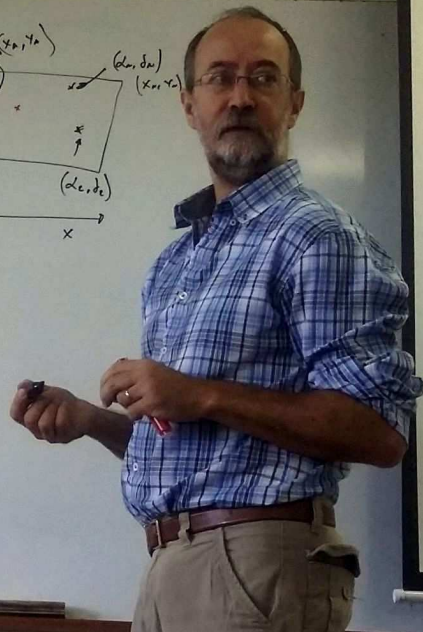
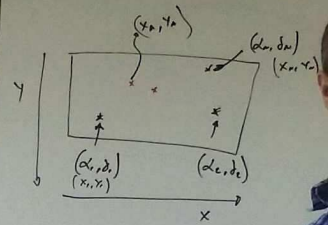
- TOPOCÉNTRICAS
(SIN REFRACCIÓ)
- GEOCÉNTRICAS
- HELIOCÉNTRICAS
- BARICÉNTRICAS
(B. DEL S.S.)



$a \approx 2 \cdot 10^{-10} \text{ m/s}^2$

$a = r \cdot \omega^2$

$\left(\frac{2\pi}{250 \cdot 10^6 \text{ a}} \right)^2$

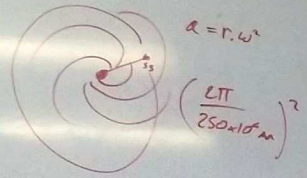
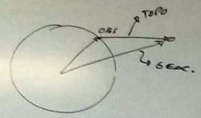


ICRS
 ICRF
 608 A
 118 000 est.

2000.0

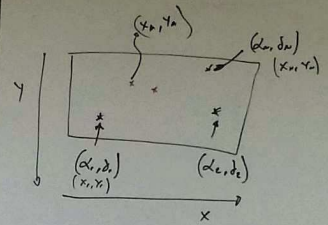
ORIGEN:

- TOPOCÉNTRICAS
(SIN REFRACCIÓ)
- GEOCÉNTRICAS
- HELIOCÉNTRICAS
- BARICÉNTRICAS
(B. DEL S.S.)

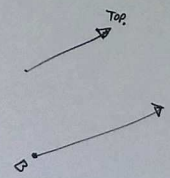


$a \approx 2 \cdot 10^{-10} \text{ m/s}^2$

$a = r \cdot \omega^2$
 $\left(\frac{2\pi}{250 \cdot 10^6 \text{ a}} \right)^2$



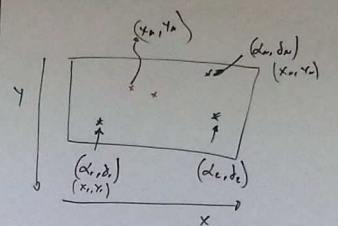
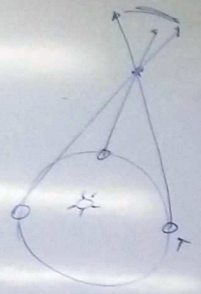
$(\alpha, \delta) \rightarrow (x, y)$



ORIGEN:

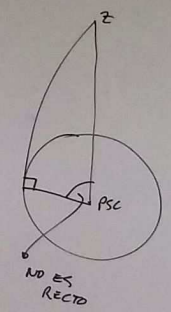
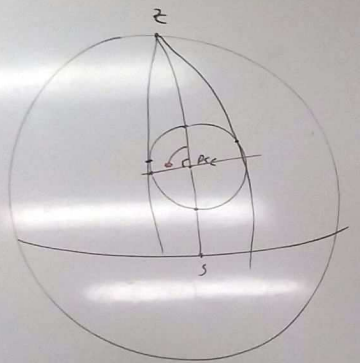
- TOPOCÉNTRICAS (SIN REFRACCIÓ)
- GEOCÉNTRICAS
- HELIOCÉNTRICAS
- BARICÉNTRICAS (B. DEL S.S.)

PARALAJE

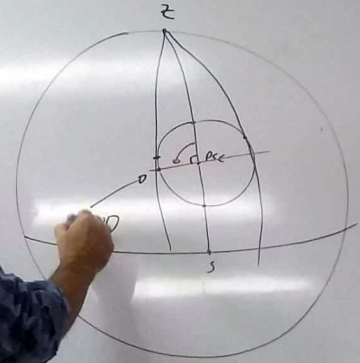


$$(\alpha, \delta) \rightarrow (x, y)$$

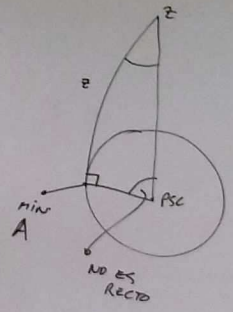




2



1



2

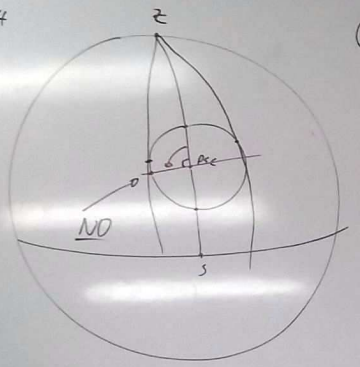
$$\cos z = \cos \delta \cos \alpha + \sin \delta \sin \alpha \cos H$$

SACIDA $z = 90$

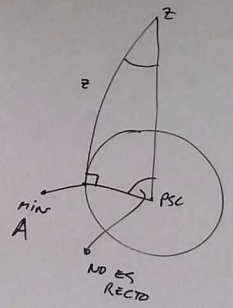
$$\Rightarrow \sin H = -\frac{\sin \delta}{\sin \alpha}$$

$$TSL = H + \alpha$$

$$\Delta TSL = \Delta H + \Delta \alpha$$



1



2

$$\cos z = \cos \phi \cos \delta + \sin \phi \sin \delta \cos H$$

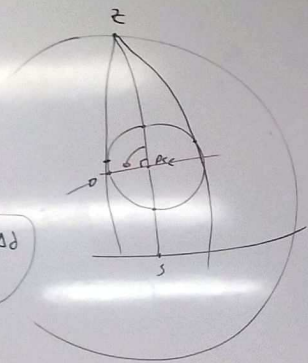
SALIDA $z=90$

$$\Rightarrow \cos H = -\frac{\sin \phi \sin \delta}{\cos \phi}$$

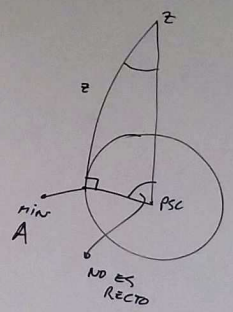
$$TSL = H + \alpha$$

$$\Delta TSL = \Delta H + \Delta \alpha$$

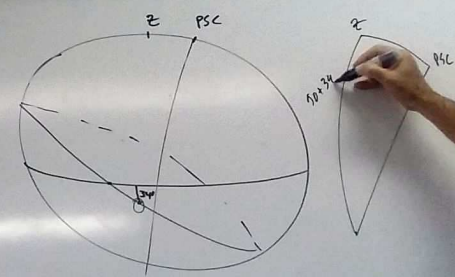
$$-\sin H \cdot \Delta H = -\frac{\sin \phi \cdot 1}{\cos \phi} \cdot \Delta \delta$$



1



3



2

$$\cos z = \cos \phi \cos \delta + \sin \phi \sin \delta \cos H$$

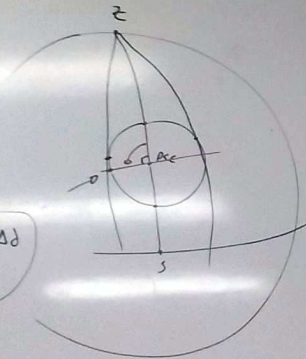
SALIDA $z=90$

$$\Rightarrow \cos H = -\frac{\sin \phi \sin \delta}{\cos \phi}$$

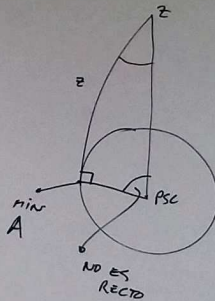
$$TSL = H + \alpha$$

$$\Delta TSL = \Delta H + \Delta \alpha$$

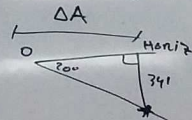
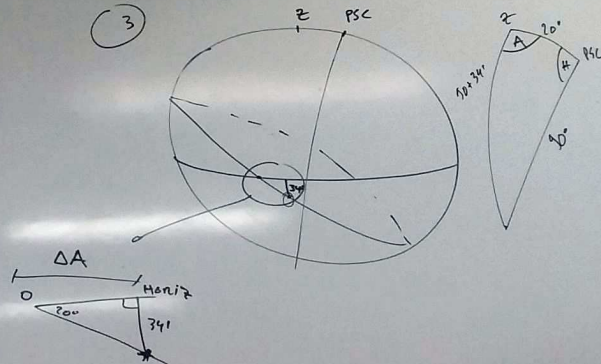
$$-\sin H \cdot \Delta H = -\frac{\sin \phi \cdot \cos \delta}{\cos \phi} \cdot \Delta \delta$$



1



3



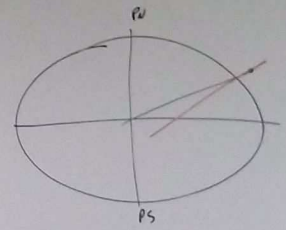
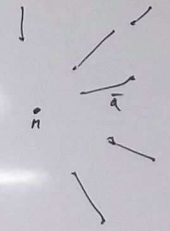
RELACION TOPOCÉNTRICAS — GEOCÉNTRICAS

- POSICIÓN → PARALAJE GEOCÉNTRICA
- VELOCIDAD → ABERRACIÓN DIURNA



RELACION TOPOCÉNTRICAS — GEOCÉNTRICAS

- POSICIÓN → PARALAJE GEOCÉNTRICO
- VELOCIDAD → ABERRACIÓN DIURNA

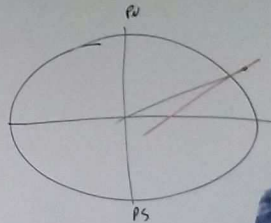
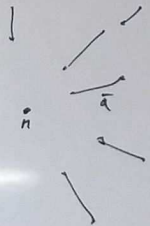


GEOCÉNTRICO



RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS

- POSICIÓN → PARALAJE GEOCÉNTRICO
- VELOCIDAD → ABERRACIÓN DIURNA



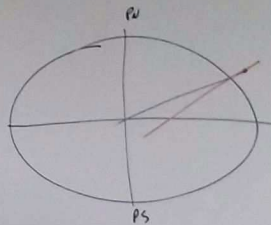
DE: SUP. EQUIPOTENCIAL

≈ ELIPSOIDE DE REVOLUCIÓN
"ESFEROIDE STANDARD"



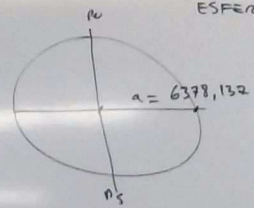
RELACION TROPICAS - GEOCÉNTRICAS

- POSICIÓN → PARA GEOCÉNTRICA
- VELOCIDAD



GEOIDE: SUP. EQUIPOTENCIAL

GEOIDE ≈ ELIPSOIDE DE REVOLUCIÓN
"ESFEROIDE STANDARD"

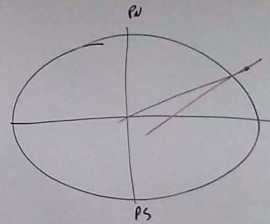
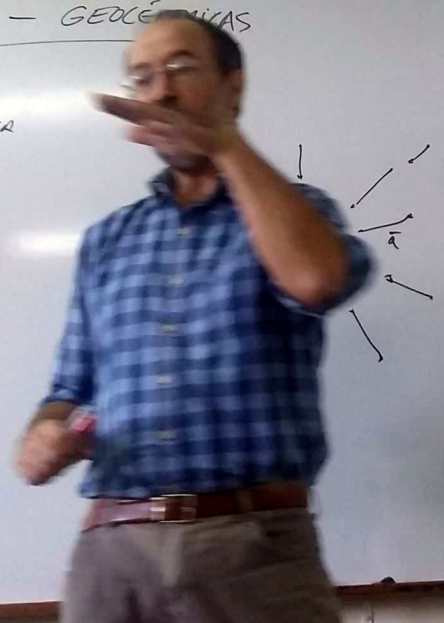


ACHATAMIENTO

$$f = \frac{1}{298,257}$$

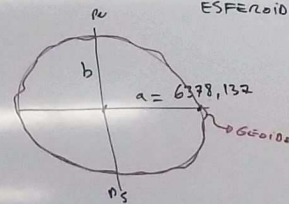
RELACION TOPOCÉNTRICAS — GEOCÉNTRICAS

- POSICIÓN → PARALAJE GEOCÉNTRICO
- VELOCIDAD → ABERRACIÓN DIURNA



GEODE: SUP. EQUIPOTENCIAL

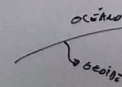
GEODE \approx ELIPSOIDE DE REVOLUCIÓN
"ESFEROIDE STANDARD"



ACHATAMIENTO

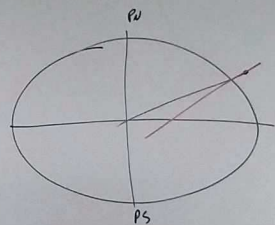
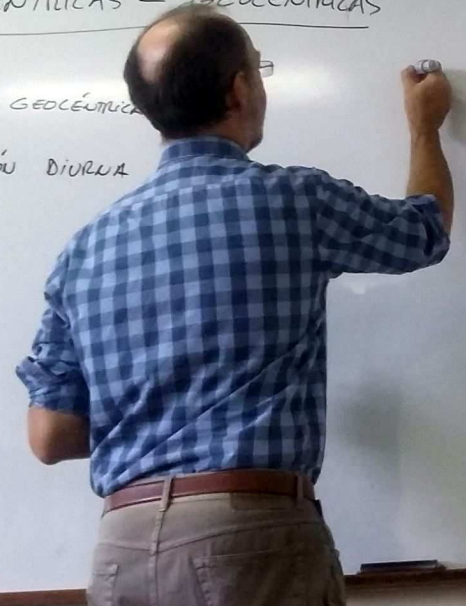
$$f = \frac{1}{298,257}$$

$$b = a(1-f)$$



RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS

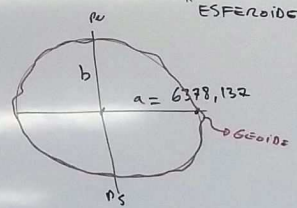
- POSICIÓN → PARALAJE GEOCÉNTRICO
- VELOCIDAD → ABERRACIÓN DIURNA



GEOIDE: SUP. EQUIPOTENCIAL

GRAVEDAD LOCAL |
GEOIDE

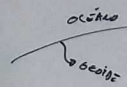
GEOIDE ≈ ELIPSOIDE DE REVOLUCIÓN
"ESFEROIDE STANDARD"



ACHATAMIENTO

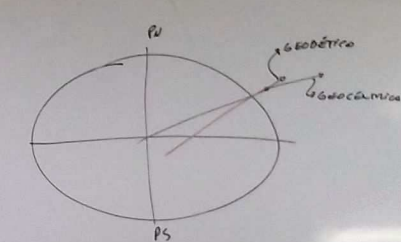
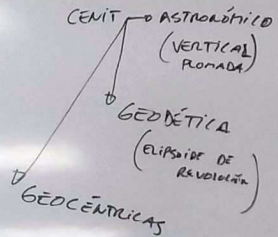
$$f = \frac{1}{298,257}$$

$$b = a(1-f)$$



RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS

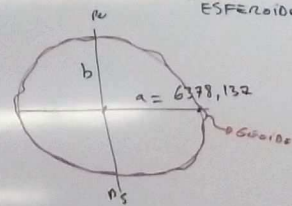
— ión → PARALAJE
 — V



GEOIDE: SUP. EQUIPOTENCIAL

GRAVEDAD LOCAL | GEOIDE

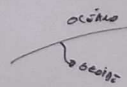
GEOIDE ≈ ELIPSOIDE DE REVOLUCIÓN
 "ESFEROIDE STANDARD"



ACHATAMIENTO

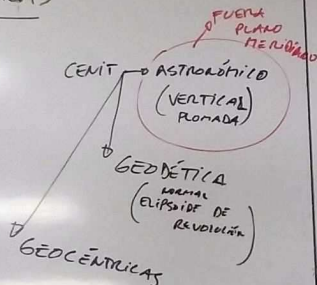
$$f = \frac{1}{298,257}$$

$$b = a(1-f)$$

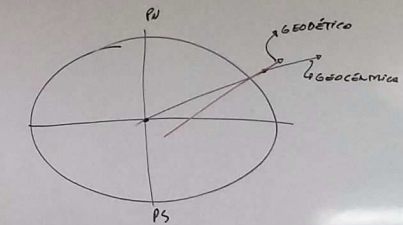


RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS

- POSICIÓN → GEOCÉNTRICA
- VELOCIDAD → DIURNA

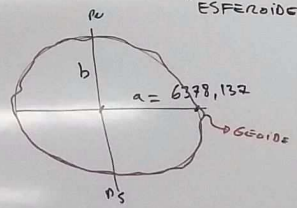


CENI



GEODE: SUP. EQUIPOTENCIAL

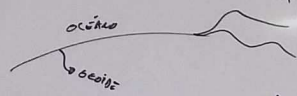
GEODE \approx ELIPSOIDE DE REVOLUCIÓN "ESFEROIDE STANDARD"



ACHATAMIENTO

$$f = \frac{1}{298,257}$$

$$b = a(1-f)$$

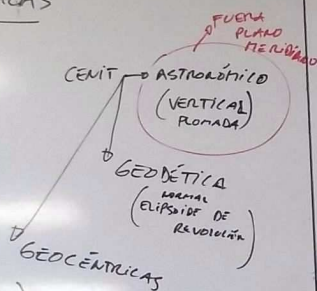


GRAVEDAD LOCAL

GEODE

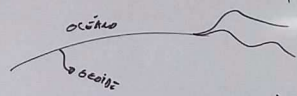
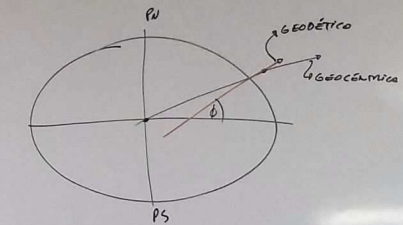
RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS

- POSICIÓN → PARALAJE GEOCÉNTRICO
- VELOCIDAD → ABERRACIÓN DIURNA



ZENIT (AST - GEODÉT) → SEGUNDOS DE ARCO

ZENIT (GEO - GEOCÉNTRICA) → MINUTOS DE ARCO



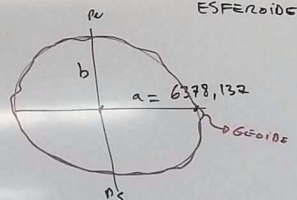
ACHATAMIENTO

$$f = \frac{1}{298,257}$$

$$b = a(1-f)$$

GEODE: SUP. EQUIPOTENCIAL

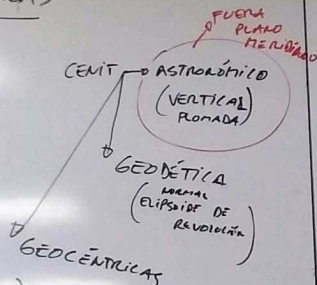
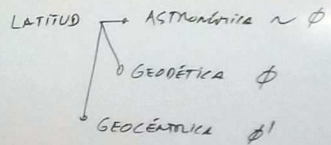
GEODE ≈ ELIPSOIDE DE REVOLUCIÓN
"ESFEROIDE STANDARD"



GRAVEDAD LOCAL

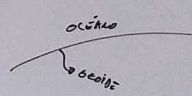
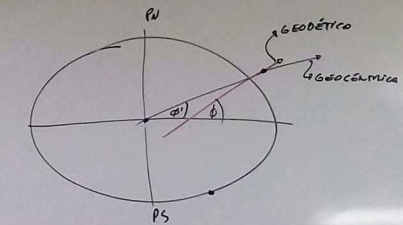
GEODE

RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS



ZENIT (AST - GEODÉT) \rightarrow SEGUNDOS DE ARCO

ZENIT (GEOD - GEOCÉNTRICA) \rightarrow MINUTOS DE ARCO



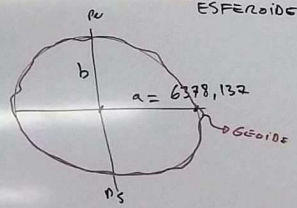
ACHATAMIENTO

$$f = \frac{1}{298,257}$$

$$b = a(1-f)$$

GEODE: SUP. EQUIPOTENCIAL

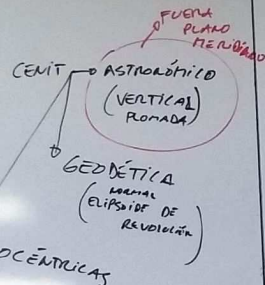
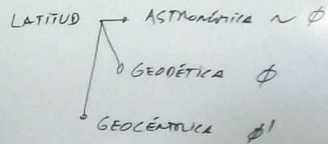
GEODE \approx ELIPSOIDE DE REVOLUCIÓN "ESFEROIDE STANDARD"



GRAVEDAD LOCAL

GEODE

RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS



"DESVIÓ DE LA VERTICAL"

VENTILM AST - VENT. GEODÉTICA

ZENIT (AST - GEODÉT) \rightarrow SEGUNDOS DE ARCO

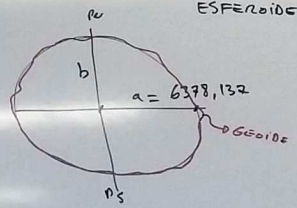
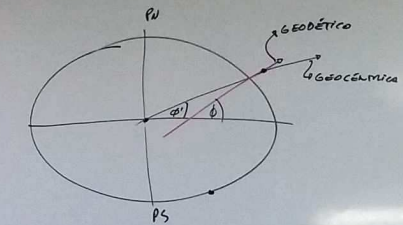
VENT. GEODÉTICA - VENT. GEOCÉNTR.

ZENIT (GEOD - GEOCÉNTRICA) \rightarrow MINUTOS DE ARCO

GRAVEDAD LOCAL
GEÓIDE

GEÓIDE: SUP. EQUIPOTENCIAL

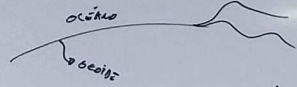
GEÓIDE \approx ELIPSOIDE DE REVOLUCIÓN
"ESFEROIDE STANDARD"



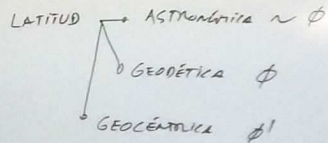
ACHATAMIENTO

$$f = \frac{1}{298,257}$$

$$b = a(1-f)$$



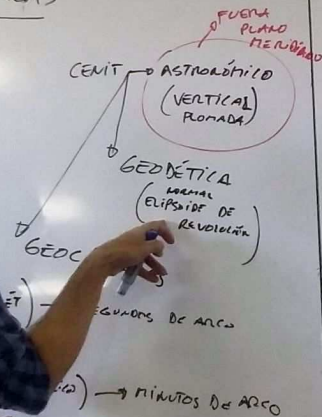
RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS



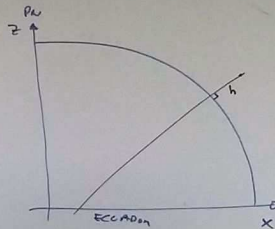
VENTIL. AST - VENT. GEODÉTICA

VENT. GEODÉTICA - VENT. GEOCÉNTRICA

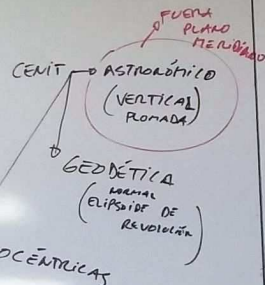
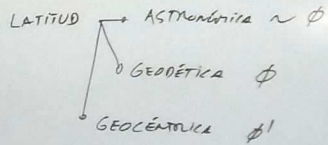
ángulo de la vertical



$h =$ ALTURA SOBRE EL NIVEL DEL MAR



RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS



"DESVIÓ DE LA VERTICAL"

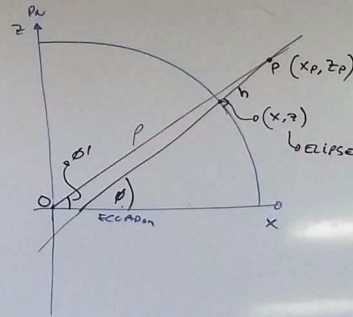
VENTIL AST - VENT. GEODÉTICA

CENIT (AST - GEODÉT) \rightarrow SEGUNDOS DE ARCO

VENT. GEODÉTICA - VENT. GEOCÉNTR.

CENIT (GEOD - GEOCÉNTRICA) \rightarrow MINUTOS DE ARCO

"ÁNGULO DE LA VERTICAL"



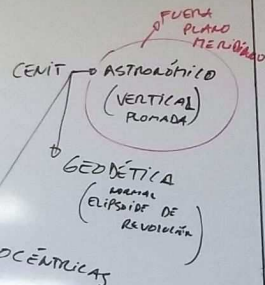
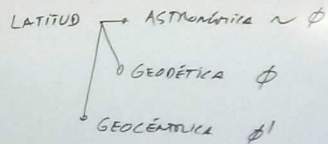
$h =$ ALTURA SOBRE EL NIVEL DEL MAR

$$X_p = OP \cdot \cos \phi' = X + h \cdot \cos \phi$$

$$Z_p = OP \cdot \sin \phi' = Z + h \cdot \sin \phi$$



RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS



"DESVIÓ DE LA VERTICAL"

VENTILM AST - VENT. GEODÉTICA

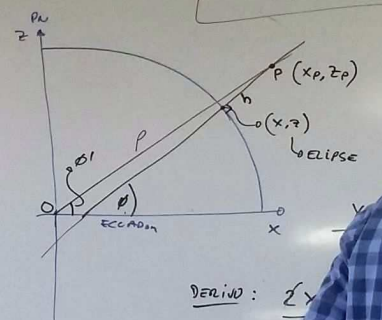
CENIT (AST - GEODÉT) \rightarrow SEGUNDOS DE ARCO

VENT. GEODÉTICA - VENT. GEOCÉNTR.

CENIT (GEO - GEOCÉNTRIC) \rightarrow MINUTOS DE ARCO

"ÁNGULO DE LA VERTICAL"

VÍNCULO (ρ, ϕ') \rightarrow (h, ϕ) $h =$ ALTURA SOBRE EL NIVEL DEL MAR

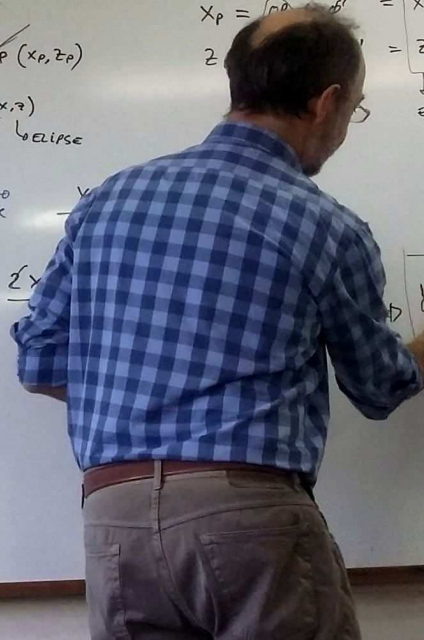


$$X_p = \sqrt{a^2 - b^2} \cos \phi' = \left[\begin{matrix} x \\ z \end{matrix} \right] + h \cdot \begin{matrix} \cos \phi \\ \sin \phi \end{matrix}$$

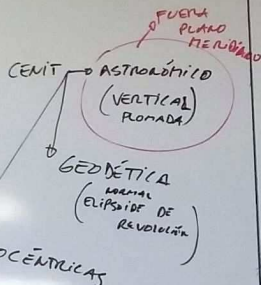
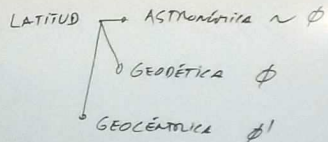
ELIPSE

DERIVADO: $\frac{dz}{dx}$

$$\frac{dx}{dz} = -\frac{z}{x} \cdot \frac{dz}{(a^2 - z^2)}$$



RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS



"DESVIÓ DE LA VERTICAL"

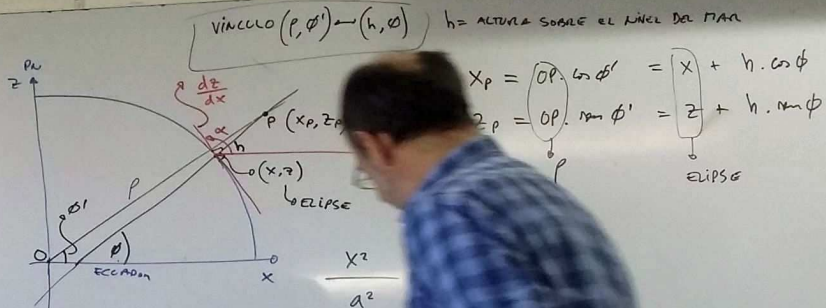
VENTIL AST - VENT. GEODÉTICA

CENIT (AST - GEODÉT) \rightarrow SEGUNDOS DE ARCO

VENT. GEODÉTICA - VENT. GEOCÉNTR.

CENIT (GEOD - GEOCÉNTRICO) \rightarrow MINUTOS DE ARCO

"ÁNGULO DE LA VERTICAL"



DERIVADO: $\frac{z}{x} \cdot dx + \frac{dz}{dx}$

$\tan \alpha = \frac{dz}{dx} = \tan(\theta + \phi)$

$\Rightarrow dx = -\frac{z}{x} \cdot \frac{dz}{(1 - e^2)^2}$

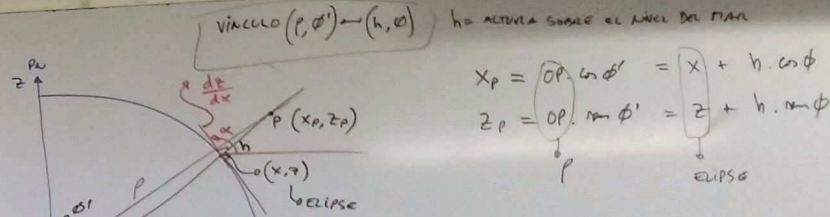
RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS

$$\frac{dx}{dz} = -\frac{z}{x} \frac{1}{(1-f)^2} = -\frac{1}{\tan \phi}$$

$$z = x \frac{1-f^2}{1+f^2}$$

...STITUND EN ... ELIPSE

$$\frac{x^2}{a^2} + \frac{z^2}{(1-f)^2 a^2} = 1$$



$$\frac{x^2}{a^2} + \frac{z^2}{(1-f)^2 a^2} = 1$$

DERIVADO: $\frac{2x}{a^2} \cdot dx + \frac{2z}{(1-f)^2 a^2} \cdot dz = 0 \Rightarrow dx = -\frac{z}{x} \frac{dz}{(1-f)^2}$

$$\tan \alpha = \frac{dz}{dx} = \tan(\phi_0 + \phi) = -\frac{1}{\tan \phi}$$

RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS

$$\frac{dx}{dz} = -\frac{z}{x} \frac{1}{(1-f)^2} = -\frac{z}{x} \phi$$

$$x^2 = \frac{a^2}{[\quad]}$$

$$z = x(1-f)^2 \cdot \frac{z}{x} \phi \rightarrow \text{SUSTITUYO EN EC. ELIPSE}$$

$$\frac{x^2}{a^2} + \frac{x^2(1-f)^4 \frac{z^2}{x^2}}{(1-f)^2 a^2} = 1 \Rightarrow \frac{x^2}{a^2} \left[1 + (1-f)^2 \frac{z^2}{x^2} \right] = 1$$

$$x = a \cdot C \cdot \cos \phi$$

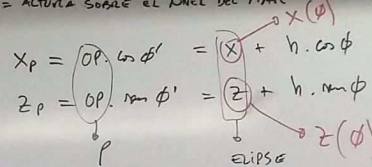
$$z = a \cdot S \cdot \sin \phi$$

$$S = \left[\cos^2 \phi + (1-f)^2 \sin^2 \phi \right]^{-1/2}$$

$$S = (1-f)^2 \cdot C$$

VÍNCULO $(p, \phi) \rightarrow (h, \phi)$

$h =$ ALTURA SOBRE EL NIVEL DEL MAR



$$\Rightarrow dx = -\frac{z}{x} \frac{dz}{(1-f)^2}$$

RELACION TOPOCÉNTRICAS - GEOCÉNTRICAS

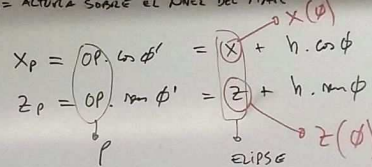
$$\frac{dx}{dz} = -\frac{z}{x} \frac{1}{(1-f)^2} = -\frac{z}{x} \phi$$

$$x^2 = \frac{a^2}{[\quad]}$$

$$z = x(1-f)^2 \cdot \frac{z}{x} \phi \rightarrow \text{SUSTITUYENDO EN EC. ELIPSE}$$

$$+ \frac{x^2(1-f)^4 \frac{z^2}{x^2}}{(1-f)^2 a^2} = 1 \Rightarrow \frac{x^2}{a^2} \left[1 + (1-f)^2 \frac{z^2}{x^2} \phi^2 \right] = 1$$

VÍNCULO (p, ϕ') \rightarrow (h, ϕ) $h =$ ALTURA SOBRE EL NIVEL DEL MAR



$$x = a \cdot C \cdot \cos \phi$$

$$z = a \cdot S \cdot \sin \phi$$

$$C = \left[\cos^2 \phi + (1-f)^2 \sin^2 \phi \right]^{-1/2}$$

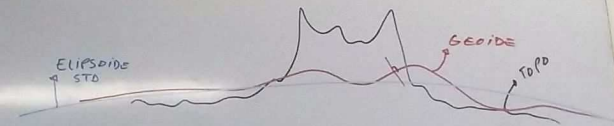
$$S = (1-f) \cdot C$$

$$\Rightarrow X_p = p \cdot \cos \phi' = a \cdot \cos \phi \left(C + \frac{h}{a} \right)$$

$$Z_p = p \cdot \sin \phi' = a \cdot \sin \phi \left(S + \frac{h}{a} \right)$$

$$\Rightarrow dx = -\frac{z}{x} \frac{dz}{(1-f)^2}$$

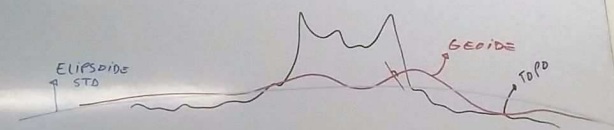
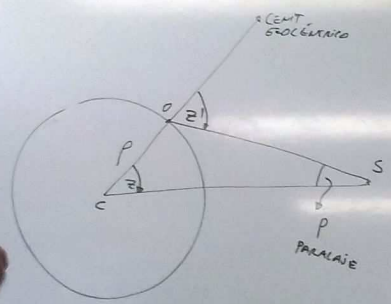
PARALAJE GEOCÉNTRICA



- PARALAJE GEOCÉNTRICA (o DIURNA)
ABERRACIÓN DIURNA

PARALAJE

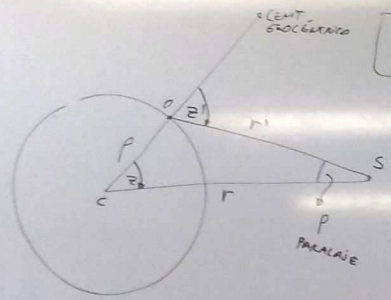
$$z' = z + p$$



- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

PARALAJE

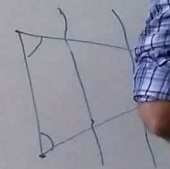
$$z' = z + p$$



$$\frac{\Delta \alpha p}{p} = \frac{\Delta \alpha z}{r'} = \frac{\Delta \alpha z}{r}$$

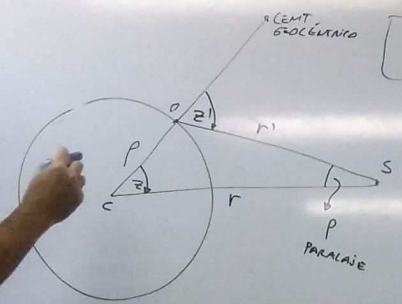
$$\Delta \alpha p = \frac{p}{r} \cdot \Delta \alpha z'$$

PARALAJE



- PARALAJE GEOCÉNTRICA (o' DIURNA)
ABERRACIÓN DIURNA

PARALAJE



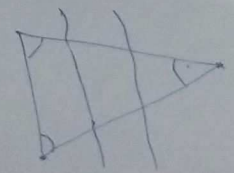
$$z'' = z + p$$

$$\frac{\sin p}{p} = \frac{\sin z}{r'} = \frac{\sin z'}{r} \rightarrow \boxed{\sin p = \frac{p}{r} \sin z'}$$

PARALAJE HORIZONTAL ECUATORIAL
 $\hookrightarrow z' = 90$ $\hookrightarrow p = a$

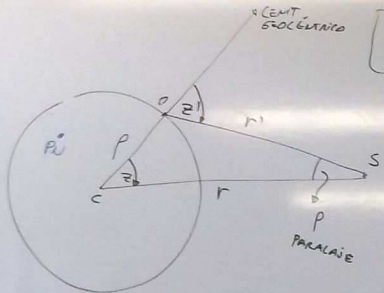
$$p (p=a, z'=90)$$

$$\boxed{\sin p = \frac{a}{r}}$$



- PARALAJE GEOCÉNTRICA (o: DIURNA)
- ABERRACIÓN DIURNA

PARALAJE



$z' = z + p$

$\frac{\sin p}{r} = \frac{\sin z'}{r}$

$\sin p = \frac{r}{r} \sin z'$

PARALAJE HORIZONTAL ECUATORIAL

$P (p=a, z'=90)$

$\sin p = \frac{a}{r}$

$P_{LUNA} = 57'$

$P_{SOL} = 8''.8$

- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

PARALAJE

Form.

$$\frac{\sin p}{r} = \frac{\sin z}{r'} = \frac{\sin z'}{r}$$

$$\sin p = \frac{p}{r} \quad \sin z'$$

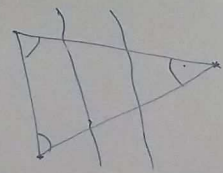
PARALAJE HORIZONTAL ECUATORIAL

$$P (p=a, z'=90)$$

$$\sin p = \frac{a}{r}$$

$$P_{LUNA} = 57'$$

$$P_{SOL} = 8''.8$$



- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

EFFECTO $\Delta\alpha, \Delta\delta$

PARALAJE

F. DIST. APOX (LEJANOS)
 F. FIGURAS VECTORIALES

$\overline{PP'}$

$$\frac{M P}{P} = \frac{M Z}{r'} = \frac{M Z'}{r}$$

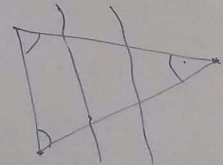
$$M P = \frac{P}{r} M Z'$$

PARALAJE HORIZONTAL ECUATORIAL
 $\hookrightarrow Z' = 90$ $\hookrightarrow P = a$

$$M P = \frac{a}{r}$$

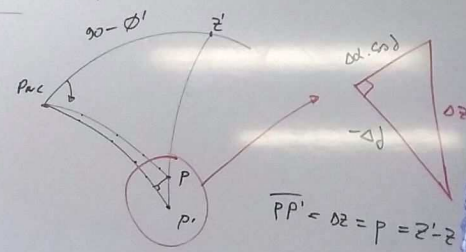
$$P_{LUNA} = 57'$$

$$P_{SOL} = 8'',8$$



- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

EFFECTO $\Delta\alpha, \Delta\delta$



$$\overline{PP'} = \Delta z = r - z = z' - z$$

$$\Delta\delta = \delta' - \delta < 0$$

PARALAJE

FORM. Aprox (LEJANOS)
 F. RIGIDA VECTORIAL

$$z' - z$$

$$\frac{\Delta z}{r} = \frac{z' - z}{r}$$

$$\Delta z = \frac{r}{r} \Delta z'$$

PARALAJE HORIZONTAL ECUATORIAL

$$P(p=a, z'=90)$$

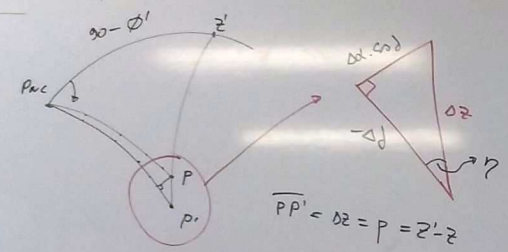
$$\Delta z = \frac{a}{r}$$

$$P_{LUNA} = 57'$$

$$P_{SOL} = 8''{,}8$$

- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

EFFECTO $\Delta\alpha, \Delta\delta$



$$\Delta\delta = \delta' - \delta < 0$$

PARALAJE

F. DIST. APROX (LEJANOS)
 F. FIGURAS VECTORIALES

$$z' - z = p$$

$$\frac{\sin p}{p} = \frac{\sin z}{r} = \frac{\sin z'}{r} \rightarrow \sin p = \frac{p}{r} \sin z'$$

$p \cos \delta$

$$\Delta\alpha \cdot \cos \delta = \Delta z \cdot \sin \delta$$

$$-\Delta\delta = \Delta z \cdot \cos \delta$$

$$\frac{p}{r} \cdot \sin z'$$

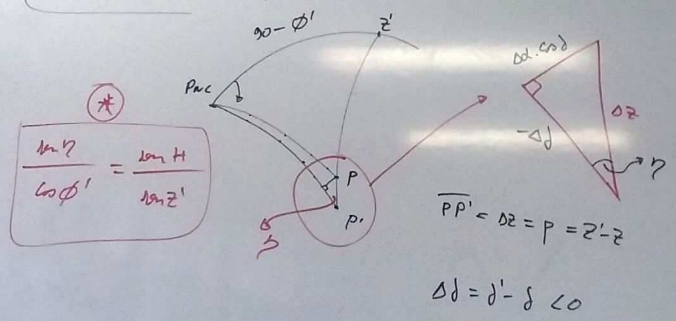
$$\Delta\alpha = \frac{p}{r} \cdot \frac{\sin z' \cdot \sin \delta}{\cos \delta}$$

$$\Delta\delta = -\frac{p}{r} \cdot \cos \delta$$



- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

EFFECTO $\Delta\alpha, \Delta\delta$



$$\frac{\tan \gamma}{\cos \phi'} = \frac{\tan H}{\tan z'}$$

PARALAJE

FORM. APPROX (LEJANOS)
 F. RIGOROSAS VECTORIALES

$$z' - z = p$$

$$\frac{\sin p}{r} = \frac{\sin z}{r'} = \frac{\sin z'}{r}$$

$$\sin p = \frac{r}{r'} \sin z'$$

P (arcos)

$$\Delta\alpha \cdot \cos \delta = \Delta z \cdot \sin \gamma$$

$$-\Delta\delta = \Delta z \cdot \cos \gamma$$

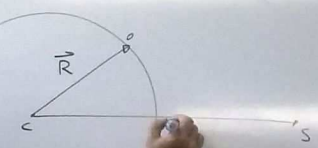
$$\frac{p}{r} \cdot \sin z'$$

$$\Delta\alpha = \frac{p}{r} \cdot \frac{\sin z'}{\cos \delta} \cdot \sin \gamma$$

$$\Delta\delta = -\frac{p}{r} \cdot \cos \gamma \cdot \sin z'$$

$\Delta\alpha = \frac{p}{r} \cdot \frac{\cos \phi \sin H}{\cos \delta}$
 PUNTO EN FUNCIÓN DE ϕ, H, δ
 NIF Tolo-Geoc.

- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA



PARALAJE

F. RIGUROSAS
VECTORIALS

$$z' - z = p$$

$$\frac{m p}{r} = \frac{m z}{r'} = \frac{m z'}{r}$$

$$m p = \frac{p}{r} m z'$$

p (anos)

$$-\Delta \alpha \cdot \cos \delta = \Delta z \cdot \sin \gamma$$

$$-\Delta \delta = \Delta z \cdot \cos \gamma$$

$$\frac{p}{r} \cdot m z'$$

$$-\Delta \alpha = \frac{p}{r} \cdot \frac{m z' \sin \gamma}{\cos \delta}$$

$$\Delta \delta = -\frac{p}{r} \cdot \cos \gamma \cdot m z'$$

$$-\Delta \alpha = \frac{p}{r} \cdot \frac{\cos \delta \sin H}{\cos \delta}$$

NIF Total - Geoc.

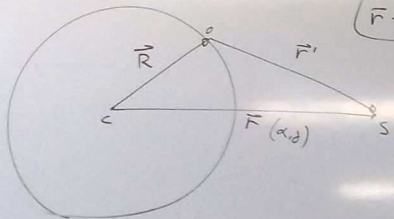
Power en Funció de δ, H, d

- PARALAJE GEOCÉNTRICA (o diurna)
- ABERRACIÓN DIURNA

PARALAJE

F. RIGUROSAS
VECTORIALES

$$\vec{r} = \vec{R} + \vec{F}'$$



COORD. RECT. ECATORIALES:

$$\vec{r} = r (\cos \delta \cos \alpha, \cos \delta \sin \alpha, \sin \delta)$$



DIF. TOPO-GEOC.

$$\Delta \alpha = \frac{p}{r} \cdot \frac{\sin \eta'}{\cos \delta}$$

$$-\Delta \alpha = \frac{p}{r} \cdot \frac{\cos \phi \sin H}{\cos \delta}$$

$$\Delta \delta = -\frac{p}{r} \cdot \cos \eta' \sin \eta'$$

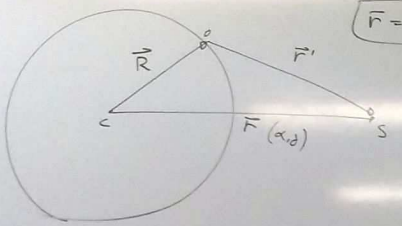
POWER EN FUNCIÓN DE ϕ, H, δ

- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

PARALAJE

F. RIGURASAS VECTORIALS

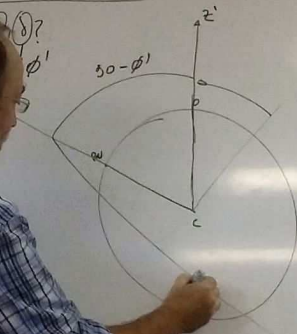
$$\vec{r} = \vec{R} + \vec{r}'$$



COORD. RECT. ECATORIALES

$$\vec{r} = r(\cos\delta\cos\alpha, \cos\delta\sin\alpha, \sin\delta)$$

POS. OBSERVADOR \vec{R} :



DIF TOPO-GEOC.

$$\Delta\alpha = \frac{\rho}{r} \cdot \frac{\sin\delta'}{\cos\delta} \cdot \sin\eta$$

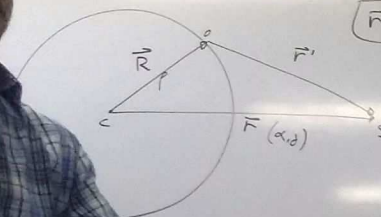
$$-\Delta\alpha = \frac{\rho}{r} \cdot \frac{\cos\delta' \sin H}{\cos\delta}$$

$$\Delta\delta = -\frac{\rho}{r} \cdot \cos\eta \cdot \sin\delta'$$

POWER EN FUNCION DE ϕ, H, δ

PARALAJE GEOCÉNTRICA (o DIURNA)
 PARALAJE DIURNA

PARALAJE



$$\vec{F} = \vec{R} + \vec{F}'$$

COORD. RECT. ECATORIALES:

$$\vec{F} = r (\cos \delta \cos \alpha, \cos \delta \sin \alpha, \sin \delta)$$

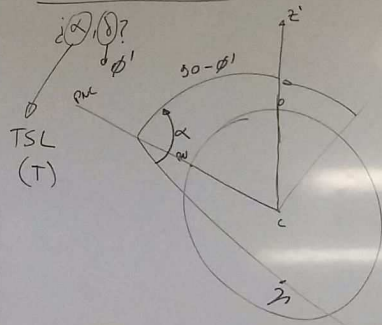
$$\vec{R} = \rho (\cos \phi \cos T, \cos \phi \sin T, \sin \phi)$$

$$\vec{r}' = \vec{F} - \vec{R}$$

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F. RIGURAS VECTORIALES

POS. OBSERVADOR \vec{R} :



DIF TOLO-GEOC.

$$\Delta \alpha = \frac{\rho}{r} \cdot \frac{\sin \delta'}{\cos \delta}$$

$$-\Delta \alpha = \frac{\rho}{r} \cdot \frac{\cos \phi \sin H}{\cos \delta}$$

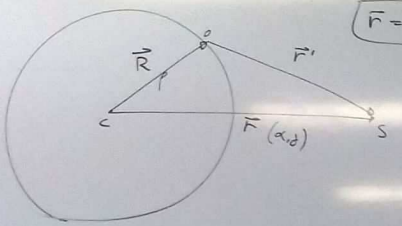
$$\Delta \delta = -\frac{\rho}{r} \cdot \cos \eta \sin \delta'$$

POWER EN FUNCIÓN DE ϕ, H, δ

- PARALAJE GEOCÉNTRICA (o diurna)
- ABERRACIÓN DIURNA

PARALAJE

F. RIGURAS VECTORIALES



$$\vec{r} = \vec{R} + \vec{r}'$$

COORD. RECT. ECATORIALES:

$$\vec{r} = r (\cos \delta \cos \alpha, \cos \delta \sin \alpha, \sin \delta)$$

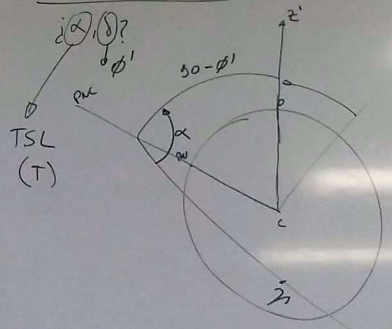
$$\vec{R} = R (\cos \phi \cos T, \cos \phi \sin T, \sin \phi)$$

$$\vec{r}' = \vec{r} - \vec{R}$$

→ Hallar α', δ'

EJ. pág 108

POS. OBSERVADOR \vec{R} :



DIF. TOPO-GEOC.

$$\Delta \alpha = \frac{p}{r} \cdot \frac{\sin \delta'}{\cos \delta}$$

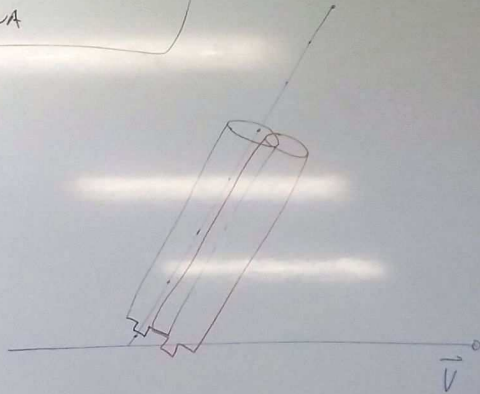
$$-\Delta \delta = \frac{p}{r} \cdot \frac{\cos \phi \sin H}{\cos \delta}$$

POWER EN FUNCIÓN DE ϕ, H, δ

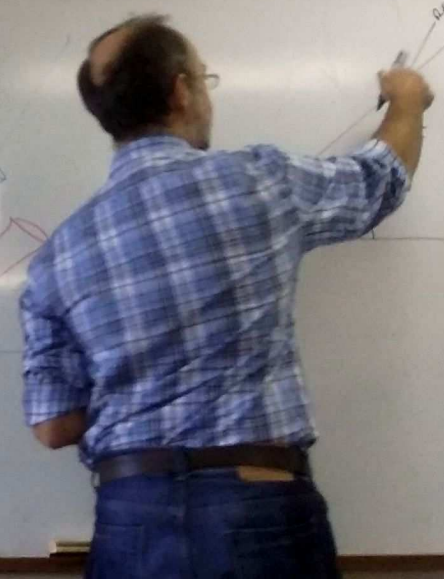
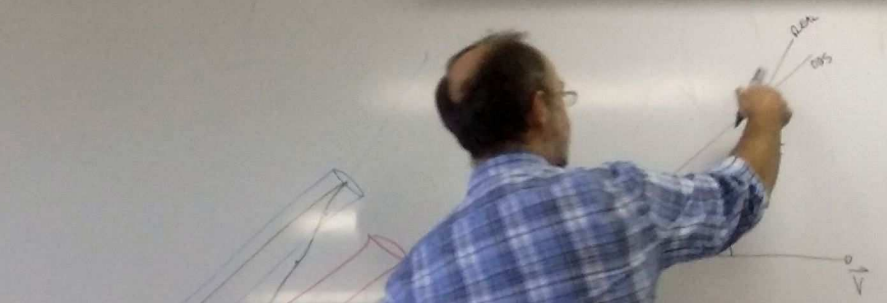
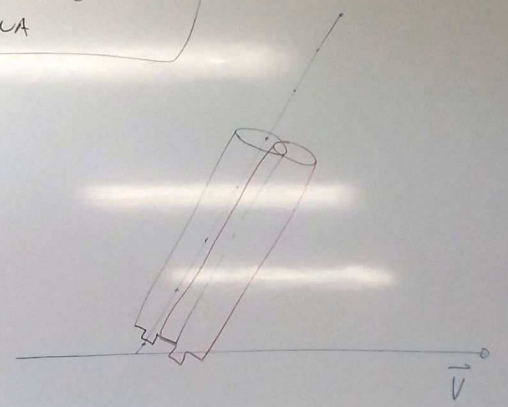
- PARALAJE HÉLÍO-CÉNTRICA (o DIURNA)
- ABERRACIÓN ANUAL



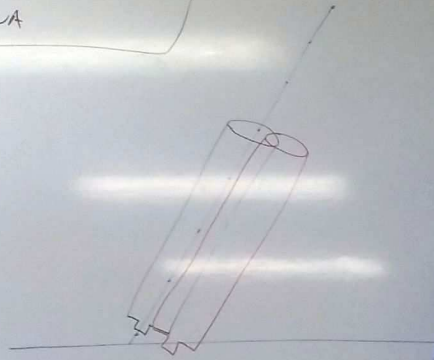
- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA



- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA



- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA



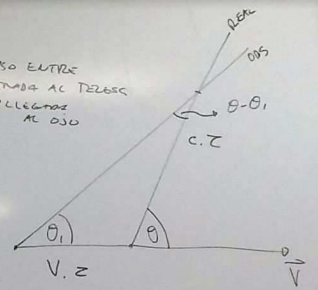
$$\frac{v \sin(\theta - \theta_1)}{Vz} = \frac{v \sin \theta_1}{cZ}$$

$\Delta \theta$ (RADS)

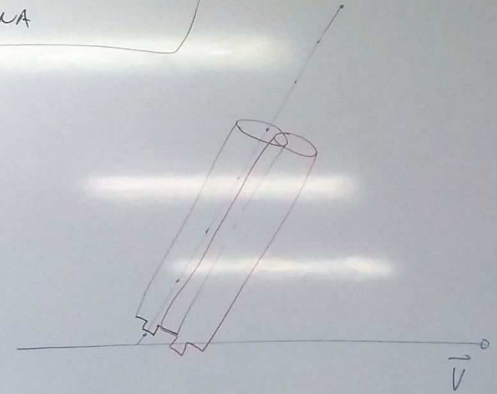
$$\Delta \theta = v \sin \theta_1$$

ABERRACIÓN
ANUAL

Z = LARGO ENTRE
ENTRADA AL TELESCO
Y LUGAR AL SOL



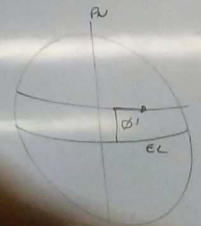
- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA



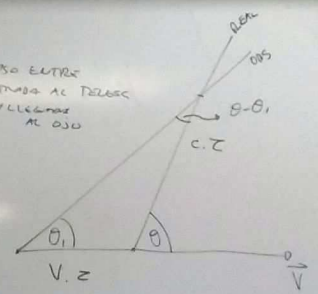
$$\frac{\sin(\theta - \theta_1)}{Vz} = \Delta\theta$$

$$\Delta\theta =$$

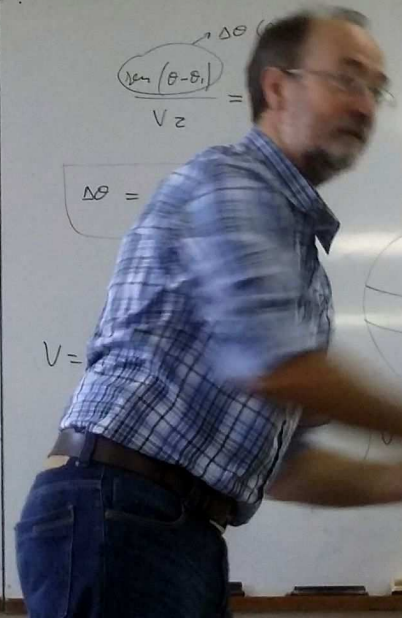
$$V =$$



z = LARGO ENTRE
EXTREMOS AL TELURICO
7 LLEGA AL OJO

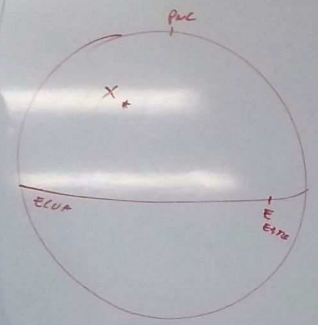


$$\cos\phi'$$



- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN ANUAL

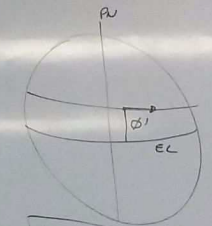
EF $\Delta\alpha, \Delta\delta$



$$\frac{\sin(\theta - \theta_1)}{Vz} = \frac{\sin \theta_1}{cZ}$$

$$\Delta\theta = \frac{V}{c} \cdot \sin \theta_1$$

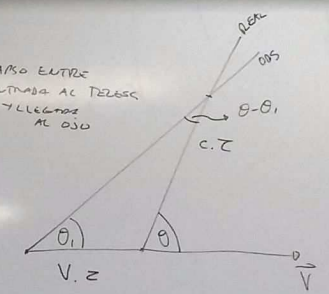
ABERRACIÓN
 ↓
 DIURNA ANUAL



$$V = \frac{\omega}{2\pi} \cdot P \cdot \cos \phi'$$

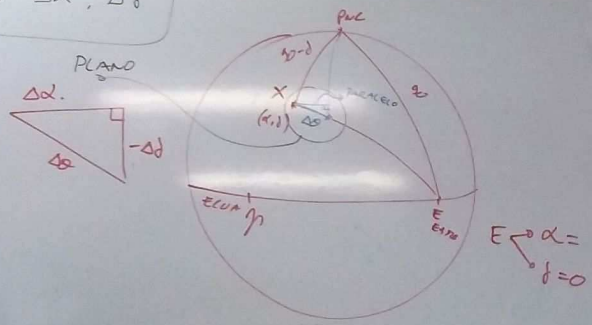
1 día

Z = LARGO ENTRE
 ENTRADA AL TELESCOPIO
 Y LLEGADA AL OJO



- PARALAJE GEOCÉNTRICA (o DIURNA)
 ABERRACIÓN DIURNA

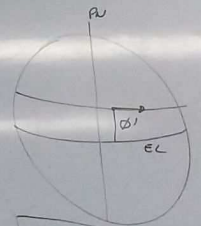
EFFECTOS $\Delta\alpha$, $\Delta\delta$



$$\frac{\sin(\theta - \theta_1)}{Vz} = \frac{\sin \theta_1}{cZ}$$

$$\Delta\theta = \frac{V}{c} \cdot \sin \theta_1$$

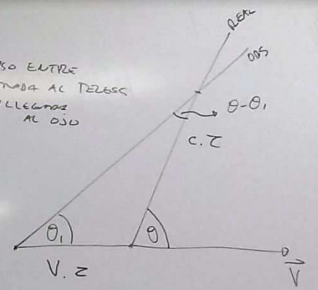
ABERRACIÓN
 ↓
 DIURNA ANUAL



$$V = \frac{\omega}{2\pi} \cdot P \cdot \cos \phi'$$

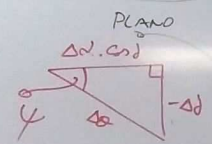
1 día

Z = LARGO ENTRE
 ENTRADA AL TELESCO
 Y LLEGADA AL OJO

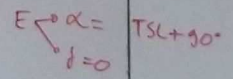
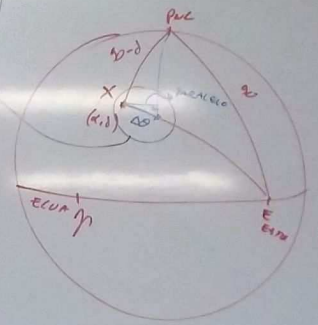


- PARALAJE GEOCÉNTRICA (δ DIURNA)
- ABERRACIÓN DIURNA

EFFECTOS $\Delta\alpha$, $\Delta\delta$

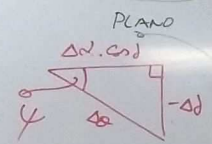


$$\Delta\alpha \cdot \cos\delta = \Delta\theta \cdot \sin\psi$$
$$-\Delta\delta = \Delta\theta \cdot \cos\psi$$



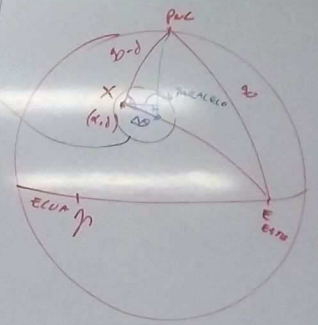
- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

EFFECTOS $\Delta\alpha$, $\Delta\delta$

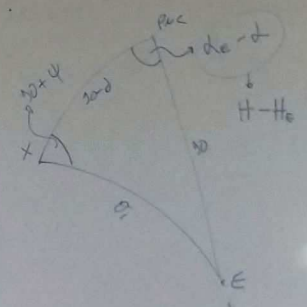


$$\Delta\alpha \cdot \sin\delta = \delta \cdot \sin\psi$$

$$-\Delta\delta = \delta \cdot \cos\psi$$



$E \rightarrow \alpha = TSL + 90^\circ$
 $\delta = 0$

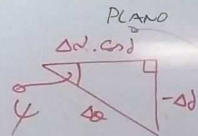


$$\frac{R \sin(\psi + \psi)}{d} = \frac{R \sin(TSL + 90 - \alpha)}{d \cos \psi}$$



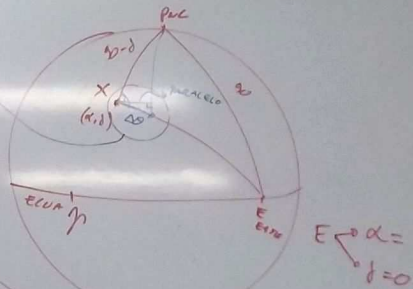
- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

EFFECTOS $\Delta\alpha$, $\Delta\delta$



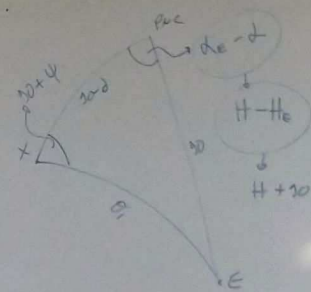
$$\Delta\alpha \cdot \cos\delta = \Delta\theta \cdot \cos\psi$$

$$-\Delta\delta = \Delta\theta \cdot \sin\psi$$



$$\alpha = \lambda + 90^\circ$$

$$\delta = 0$$



$$\frac{v \sin\psi}{1} = \frac{v \sin(TSL + 90^\circ - \alpha)}{1} = \frac{v H}{r_B}$$

$$\Rightarrow \cos\psi = \frac{v H}{r_B}$$

$$\Delta\alpha = \Delta\theta \cdot \frac{v H}{r_B} \cdot \frac{1}{\cos\psi}$$

$$= \frac{v}{c} \cdot \frac{v H}{r_B}$$

$$\Rightarrow \Delta\alpha = \frac{v}{c} \frac{v H}{r_B}$$

$$TSL = \alpha + H$$

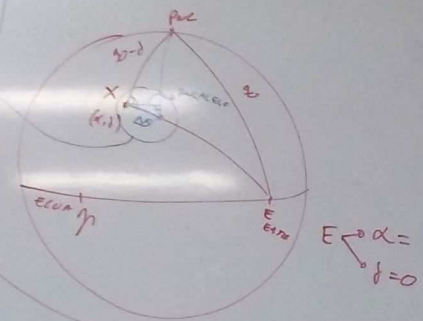
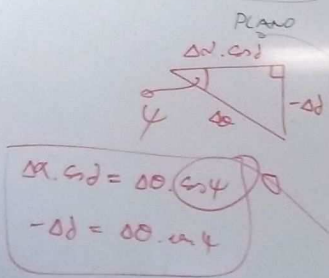
$$H = TSL - \alpha$$

$$H_e = TSL - \alpha_e$$

$$H_e = -50^\circ$$

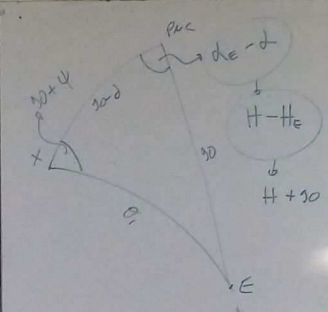
- PARALAJE GEOCÉNTRICA (o DIURNA)
- ABERRACIÓN DIURNA

EFFECTOS $\Delta\alpha$, $\Delta\delta$



$\Delta\alpha \cos \delta = \Delta\theta \cos \psi$
 $-\Delta\delta = \Delta\theta \sin \psi$

$E \rightarrow \alpha = TSL + 90^\circ$
 $\delta = 0$



$$\frac{\sin(90 + \psi)}{1} = \frac{\sin(TSL + 90 - \delta)}{\sin \theta_1} = \frac{\cos H}{\sin \theta_1}$$

$$\Rightarrow \cos \psi = \frac{\cos H}{\sin \theta_1}$$

$$\Delta\alpha = \Delta\theta \cdot \frac{\cos H}{\sin \theta_1} \cdot \frac{1}{\cos \delta}$$

$$\Rightarrow \Delta\alpha = \frac{v}{c} \cdot \frac{\cos H}{\sin \theta_1}$$

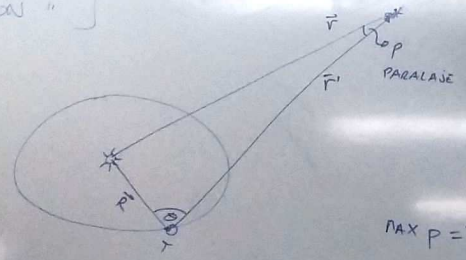
$$\Rightarrow \Delta\alpha = 0.0213 \frac{v \cos H}{c \sin \theta_1}$$

$$\Delta\delta = 0.32 \frac{v \sin \psi}{c \sin \theta_1}$$

$TSL = \alpha + H$
 $H = TSL - \alpha$
 $H_E = TSL - \alpha_E$
 $H_E = -90^\circ$

PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE ANUAL
ABERRACIÓN "



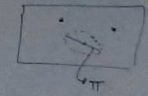
$$\frac{v_p}{R} = \frac{v_\theta}{r}$$

$$\Rightarrow \sin p = \frac{R}{r} \cdot \sin \theta$$

$$\approx p(\text{en } \text{rad}) = \frac{R}{r} \cdot \sin \theta$$

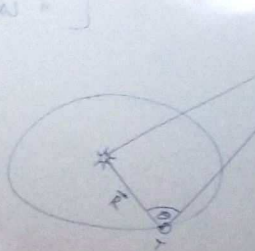
$$\text{MAX } p = \pi = \frac{R}{r} \cdot (\sin 90^\circ)$$

DADO $\pi \rightarrow r$



PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE ANUAL
ABERRACIÓN



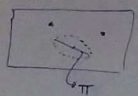
$$\frac{h.p}{R} = \frac{\sin \theta}{r}$$

$$\Rightarrow \sin p = \frac{R}{r} \cdot \sin \theta$$

$$\approx p(\text{en rad}) = \frac{R}{r} \cdot \sin \theta$$

$$\text{MAX } p = \pi = \frac{R}{r} \cdot (\sin 90^\circ)$$

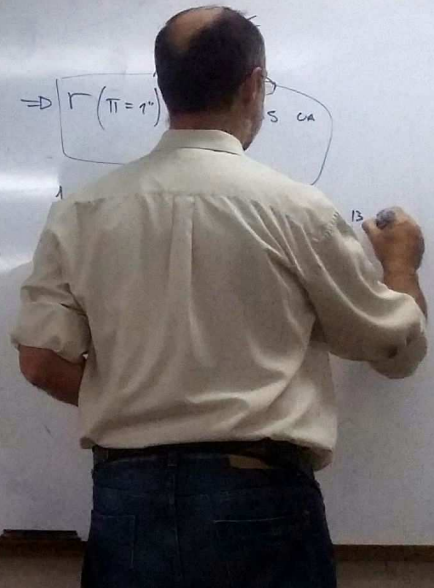
DADO $\pi \rightarrow r$



$$1 \text{ AU} = 300.000 \times 60 \times 60 \times 24 \times 365,25 \text{ Km}$$

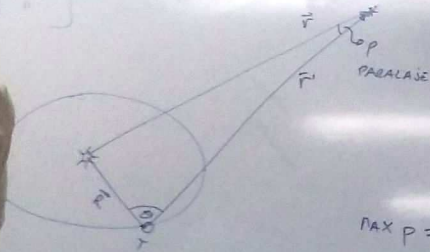
$$\text{si } \pi = 1'' = \frac{1}{206265} = \frac{1 \text{ AU}}{r(\text{en AU})}$$

$$\Rightarrow r(\pi = 1'') = 206265 \text{ AU}$$



ABAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

ALUAC
KÉN



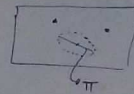
$$\frac{r \cdot p}{R} = \frac{r \cdot \theta}{r}$$

$$\Rightarrow \sin p = \frac{R}{r} \cdot \sin \theta$$

$$\approx p(\cos) = \frac{R}{r} \cdot \sin \theta$$

$$\text{MAX } p = \pi = \frac{R}{r} \cdot (\sin 90^\circ)$$

DADO $\pi \rightarrow r$



$$1 \text{ AL} = 300.000 \times 60 \times 60 \times 24 \times 365,25 \text{ Km} = 3,5 \times 10^{12} \text{ Km}$$

$$\text{si } \pi = 1'' = \frac{1}{206265} = \frac{1 \text{ UA}}{r(\text{UA})}$$

PARSEC

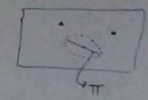
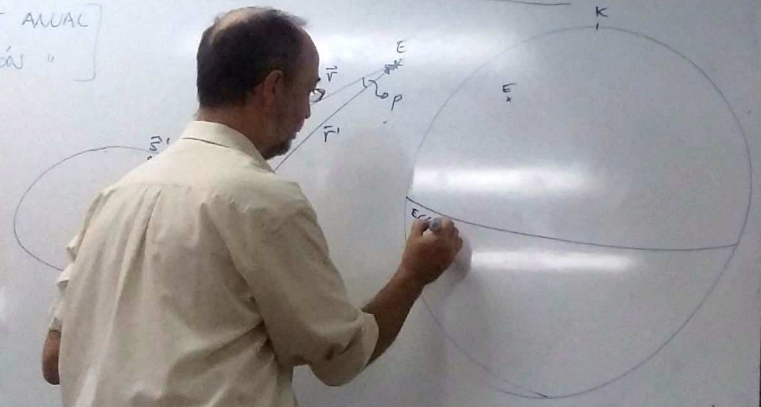
$$\Rightarrow r(\pi = 1'') = 206265 \text{ UA}$$

$$\alpha \text{ CENTAURO} \rightarrow \pi = 0,74$$

$$1 \text{ PC} = 3,26 \text{ AL} = 3,08 \times 10^{13} \text{ Km}$$

PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE ANUAL
ABERRACIÓN "



$$1 \text{ AL} = 300000 \times 60 \times 60 \times 24 \times 365 \text{ Km} = 9.5 \times 10^{12} \text{ Km}$$

$$\text{si } \pi = 1'' = \frac{1}{206265} = \frac{1 \text{ ua}}{r(\text{ua})}$$

$$\Rightarrow r(\pi = 1'') = 206265 \text{ ua}$$

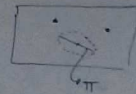
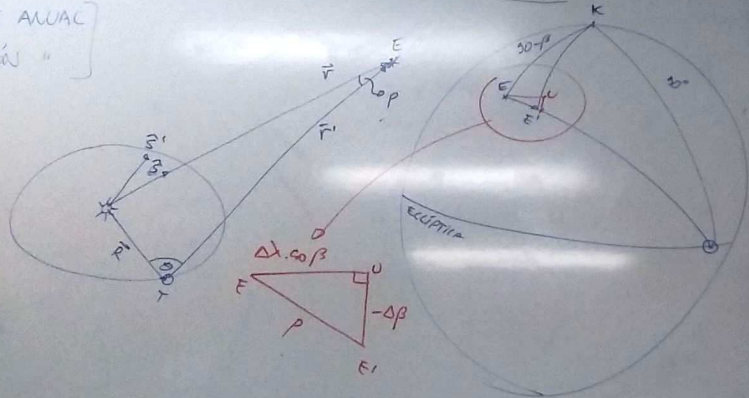
PARSEC

$$\alpha_{\text{CELESTRE}} \rightarrow \pi = 0,74$$

$$1 \text{ Pr} = 3,26 \text{ AL} = 3,08 \times 10^{13} \text{ Km}$$

PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE ANUAL
ABERRACIÓN "



$$1 \text{ AL} = 300000 \times 60 \times 60 \times 24 \times 365.25 \text{ Km} = 3,5 \times 10^{13} \text{ Km}$$

$$\text{si } \pi = 1'' = \frac{1}{206265} = \frac{1 \text{ UA}}{d(\text{UA})} \Rightarrow \Gamma(\pi = 1'') = 206265 \text{ UA}$$

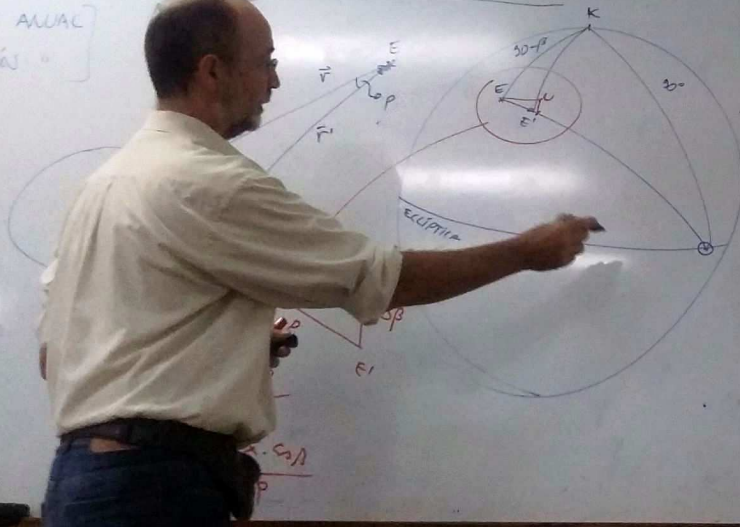
PARSEC

$$\alpha_{\text{CEMERO}} \rightarrow \pi = 0,74$$

$$1 \text{ Pc} = 3,26 \text{ AL} = 3,08 \times 10^{13} \text{ Km}$$

PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE ANUAL
ABERRACIONES



$$1 \text{ AU} = 300.000 \times 60 \times 60 \times 24 \times 365,25 \text{ Km} = 9,5 \times 10^{12} \text{ Km}$$

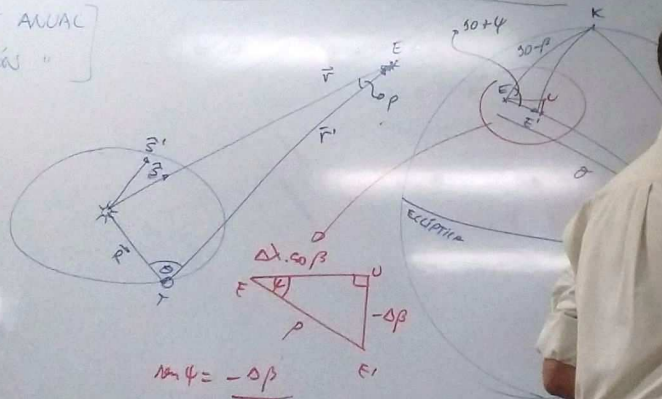
$$\Rightarrow \begin{cases} \Delta \beta = -p \sin \gamma \\ \Delta \lambda \cos \beta = p \cos \gamma \end{cases}$$

$$\frac{\sin p}{R} = \frac{\sin \theta}{r} \Rightarrow \sin p = \frac{R}{r} \cdot \sin \theta = \pi \cdot \sin \theta$$

$$\Rightarrow \begin{cases} \Delta \beta = -\pi \sin \theta \sin \gamma \\ \Delta \lambda \cos \beta = \pi \cdot \sin \theta \cos \gamma \end{cases}$$

PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE ANUAL
ABERRACIÓN "



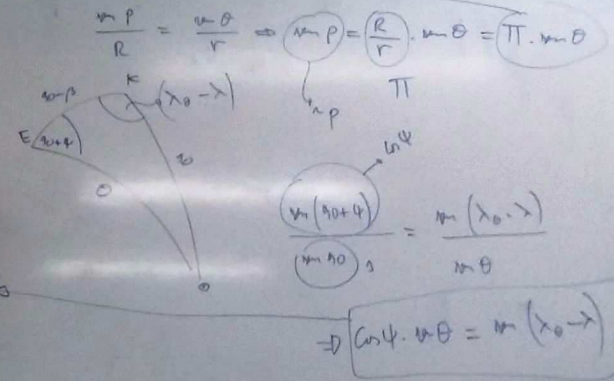
$$\sin \psi = \frac{-\Delta \beta}{p}$$

$$\cos \psi = \frac{\Delta \lambda \cdot \sin \theta}{p}$$

$$\Rightarrow \Delta \beta = -p \sin \psi$$

$$\Delta \lambda \cos \theta = p \cos \psi$$

$$1 \text{ AU} = 300000 \times 60 \times 60 \times 24 \times 365.25 \text{ Km} = 9.5 \times 10^7 \text{ Km}$$



$$\frac{v \cdot p}{R} = \frac{v \cdot \theta}{r} \Rightarrow \sin \psi = \frac{R}{r} \cdot \sin \theta = \pi \cdot \sin \theta$$

$$\frac{\sin(\psi + \theta)}{\sin \theta} = \frac{\sin(\lambda_0 - \lambda)}{\sin \theta}$$

$$\Rightarrow \cos \psi \cdot \sin \theta = \sin(\lambda_0 - \lambda)$$

PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAXIS
ABERRACIÓN

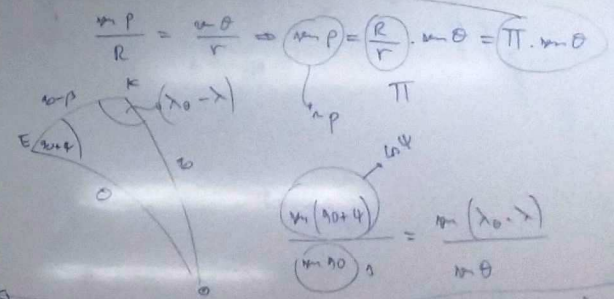


$$\Rightarrow \begin{cases} \Delta\beta = -p \sin \psi \\ \Delta\lambda \cos \beta = p \cos \psi \end{cases}$$

$$\Rightarrow \begin{cases} \Delta\beta = -\pi \mu \theta \sin \psi \\ \Delta\lambda \cos \beta = \pi \mu \theta \cos \psi \end{cases}$$

$$\Rightarrow \begin{cases} \Delta\beta = -\pi \mu \beta \cos(\lambda_0 - \lambda) \\ \Delta\lambda \cos \beta = \pi \mu (\lambda_0 - \lambda) \end{cases}$$

$$1 \text{ AU} = 300000 \times 60 \times 60 \times 24 \times 365.25 \text{ Km} = 1.5 \times 10^8 \text{ Km}$$



$$\frac{\mu P}{R} = \frac{\mu \theta}{r} \Rightarrow \mu P = \frac{R}{r} \cdot \mu \theta = \pi \mu \theta$$

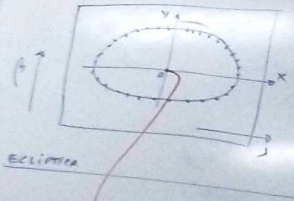
$$\frac{\mu (90 + \psi)}{\mu \theta} = \frac{\mu (\lambda_0 - \lambda)}{\mu \theta}$$

$$\Rightarrow \cos \psi \cdot \mu \theta = \mu (\lambda_0 - \lambda)$$

ELIPSE

PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE ANUAL
ABERRACIONES



$$\frac{x^2}{\pi^2} + \frac{y^2}{(\pi \cdot p)^2} = 1$$

ELIPSE PARALÁCTICA

SEMI EJE MAYOR = π

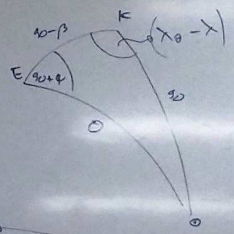
" " " MENOR = $\pi \cdot \cos \beta$

HELIOCÉNTRICA

$$1 \text{ AU} = 300.000 \times 60 \times 60 \times 24 \times 365,25 \text{ Km} = 1,5 \times 10^8 \text{ Km}$$

$$\Rightarrow \begin{cases} \Delta \beta = -p \sin \psi \\ \Delta \lambda \cos \beta = p \cos \psi \end{cases}$$

$$\frac{m \cdot p}{R} = \frac{v \cdot \theta}{r} \Rightarrow m \cdot p = \frac{R}{r} \cdot m \cdot \theta = \pi \cdot m \cdot \theta$$



$$\Rightarrow \begin{cases} \Delta \beta = -\pi \cdot m \cdot \theta \cdot \sin \psi \\ \Delta \lambda \cdot \cos \beta = \pi \cdot m \cdot \theta \cdot \cos \psi \end{cases}$$

$$\frac{m \cdot (90 + \psi)}{\pi \cdot 90} = \frac{m \cdot (\lambda_0 - \lambda)}{m \cdot \theta}$$

$$\Rightarrow \begin{cases} \Delta \beta = -\pi \cdot m \cdot \theta \cdot \sin(\lambda_0 - \lambda) \\ \Delta \lambda \cdot \cos \beta = \pi \cdot m \cdot \theta \cdot (\lambda_0 - \lambda) \end{cases}$$

ELIPSE

$$\Rightarrow \cos \psi \cdot m \cdot \theta = m \cdot (\lambda_0 - \lambda)$$

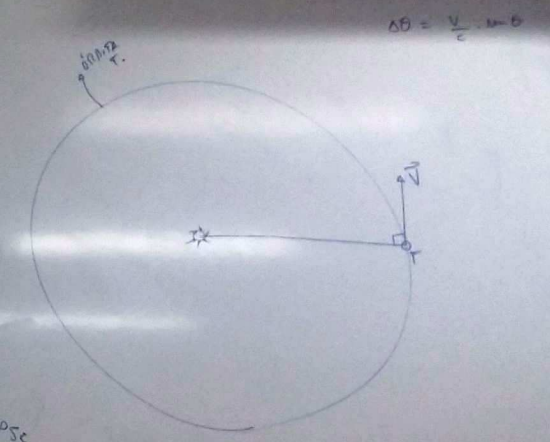
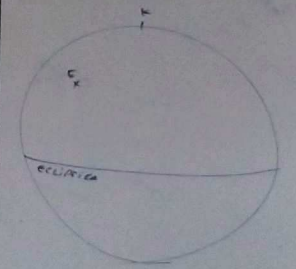
PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE ANUAL
ABERRACIÓN "



$$\frac{x^2}{a^2} + \frac{y^2}{(a \sin \beta)^2} = 1$$

$\omega - \beta$



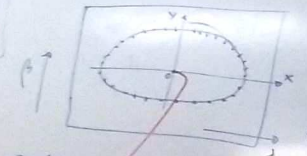
$$\Delta \beta = -\pi \cdot m \cdot \beta \cdot \cos(\lambda_0 - \lambda)$$

$$\Delta \lambda \cos \beta = \pi \cdot m \cdot (\lambda_0 - \lambda)$$

ECLIPSE

PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE ANUAL
ABERRACIONES

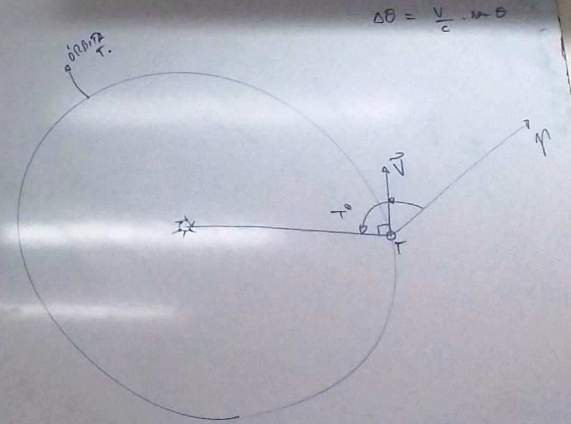
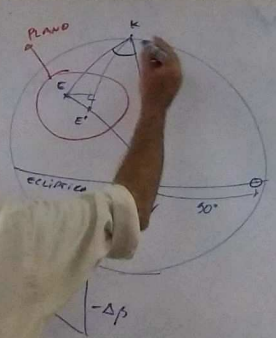


$$\frac{x^2}{\pi^2} + \frac{y^2}{(\pi \mu \rho)^2} = 1$$

ELIPSE PARALÁCTICA

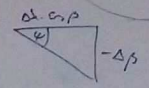
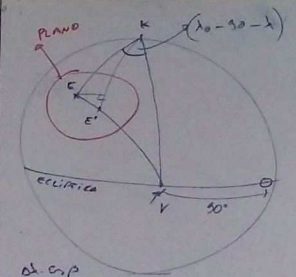
Semi Eje mayor = π

" " menor = $\pi \mu \rho$



PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

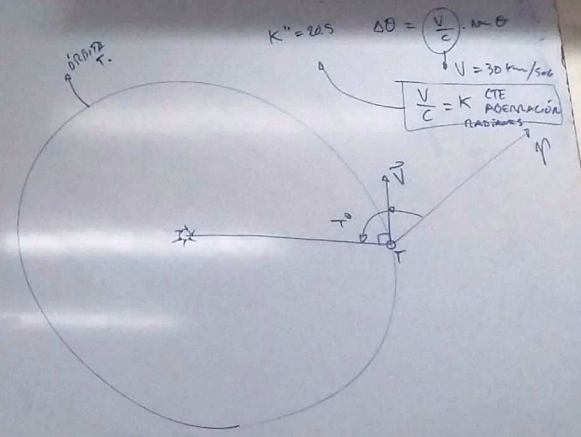
PARALAJ
ABERRAC



$$\Delta\beta = -K \sin\theta \cdot \sin\psi$$

$$\Delta\lambda \cdot \cos\beta = K \sin\theta \cdot \cos\psi$$

ECLIPICUM



$$K'' = 205 \quad \Delta\theta = \left(\frac{V}{C}\right) \cdot \sin\theta$$

$V = 30 \text{ km/s}$

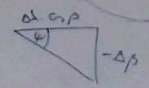
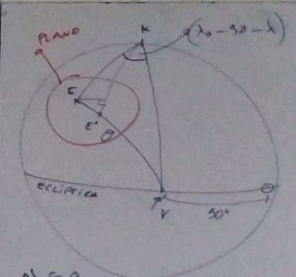
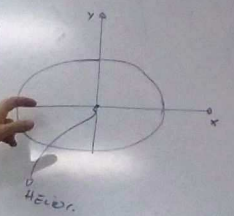
$\frac{V}{C} = K \text{ CTE ABERRACION RADIANES}$

PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARALAJE AU
ABERRACIÓN

$$\Delta\beta = -k \sin\beta \cdot \sin(\lambda_0 - \lambda)$$

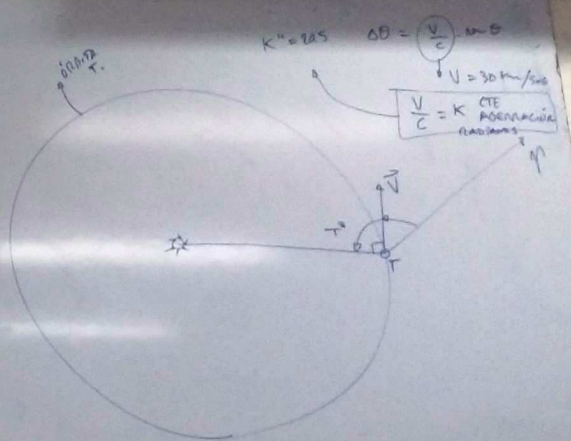
$$\Delta\lambda \cdot \cos\beta = -k \cdot \cos(\lambda_0 - \lambda)$$



$$\Delta\beta = -k \sin\theta \cdot \sin\psi$$

$$\Delta\lambda \cdot \cos\beta = k \sin\theta \cdot \cos\psi$$

- ecliptic



$$k'' = 205 \quad \Delta\theta = \frac{V}{C} \sin\theta$$

$$V = 30 \text{ km/s}$$

$\frac{V}{C} = k \text{ CTE ABERRACION PARALAJE}$

PASAJE

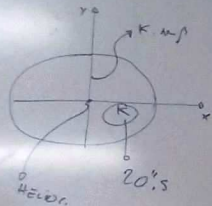
ÓRBITAS - HELIOCÉNTRICAS

PARALAJE ANUAL

ABERRACIÓN

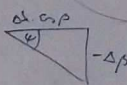
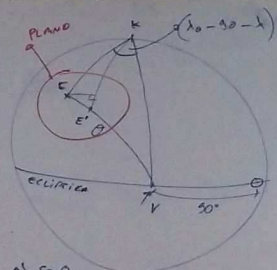
$$\alpha = -K \sin \beta \sin (\lambda_0 - \lambda)$$

$$\beta = -K \cos (\lambda_0 - \lambda)$$



= 1

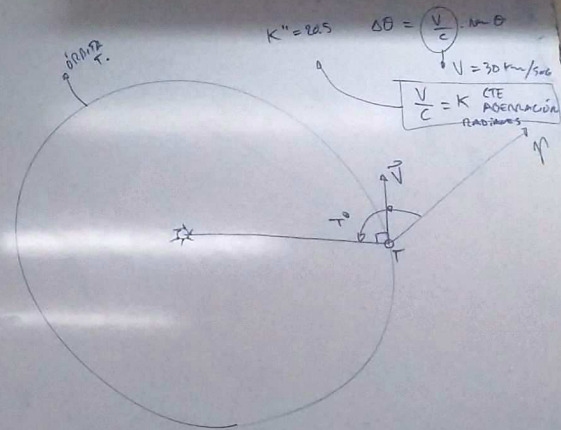
ELIPSE DE ABERRACIÓN



$$\Delta\beta = -K \sin \theta \cdot \sin \psi$$

ECLIPSE

$$\Delta\lambda \cdot \cos \beta = K \sin \theta \cdot \cos \psi$$



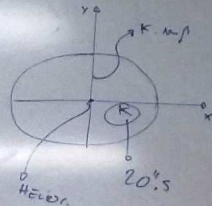
PASAJE GEOCÉNTRICAS - HELIOCÉNTRICAS

PARAJE ANUAL

ABERRACIÓN "

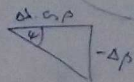
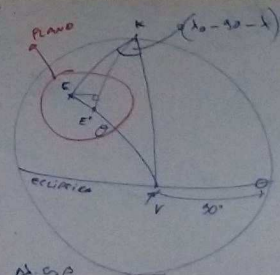
$$\Delta\beta = -K \sin\beta \cdot \sin(x_0 - \lambda)$$

$$\Delta\lambda \cdot \cos\beta = -K \cdot \cos(x_0 - \lambda)$$



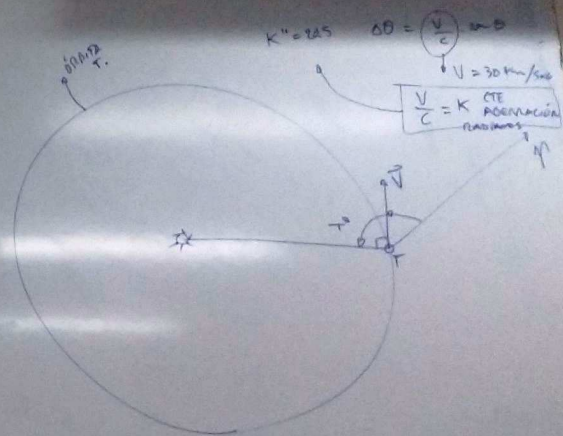
$$\frac{x^2}{K^2} + \frac{y^2}{(K \cos\beta)^2} = 1$$

ELIPSE DE ABERRACIÓN



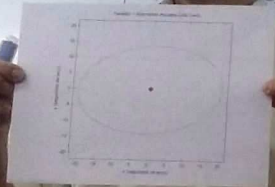
$$\Delta\beta = -K \sin\theta \cdot \sin\psi$$

$$\Delta\lambda \cdot \cos\beta = K \sin\theta \cdot \cos\psi$$



Posición → "TOPOCÉNTRICA" : OBSERVADOR
"APARENTE" : GEOCÉNTRICA
"VERDADERA" : HELIOCÉNTRICA

Posición
PARALAXIS



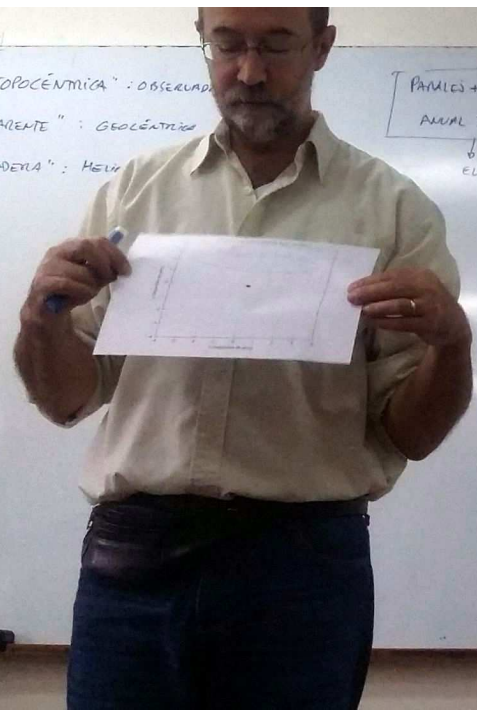
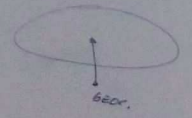
Posición → "TOPOCÉNTRICA" : OBSERVADA
"APARENTE" : GEOCÉNTRICA
"VERDADERA" : HELIO

PARABOLAS + APROXIMACIÓN
ANUAL Y DIARIA
↓
ELIPSAS

P. DIURNA

$$x = -\frac{p}{r} \cos \delta' \sin H$$
$$y = \frac{p}{r} (\sin \delta' \cos \delta' \cos H - \cos \delta' \sin \delta')$$

C.T.C.



POSICIÓN → "TOPOCÉNTRICA" : OBSERVADOR
↳ "APARENTE" : GEOCÉNTRICA
↳ "VERDADERA" : HELIOCÉNTRICA

VECTORIAL

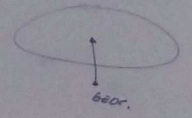
PARÁMETROS + ASINUTACIÓN
ANUAL Y DIARIA
↓
ELIPSES

P. DIURNA

$$x = -\frac{p}{r} \cos \phi \cos H$$

$$y = \frac{p}{r} (\sin \delta \cos \phi' \cos H - \cos \delta \sin \phi')$$

C.T.M.



Posición → "TOPOCÉNTRICA" : OBSERVADOR
 ↳ "GEOCÉNTRICA"
 ↳ "HELIOCÉNTRICA"

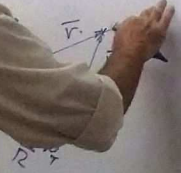
PARÁMETROS + ASOCIACIÓN
 ANUAL Y DIURNA
 ↓
 ELIPSES

$$\vec{r} = \vec{R} + \vec{r}' \rightarrow r \cdot \hat{s} = R \cdot \hat{n} + r' \cdot \hat{s}'$$

$$r' \cdot \hat{s}' = r \cdot \hat{s} - R \cdot \hat{n}$$

$$r' \cdot \hat{s} \wedge \hat{s}' = 0 - R \cdot \hat{s} \wedge \hat{n}$$

PARALAJE

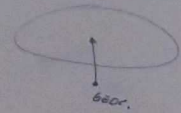


P. DIURNA

$$x = -\frac{p}{r} \cos \delta \cdot \sin H$$

$$y = \frac{p}{r} \left(\sin \delta \cos \delta' \cos H - \cos \delta \sin \delta' \right)$$

CTE

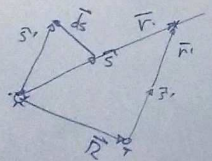


Posición → "TOPOCÉNTRICA": OBSERVADOR
 "APARENTE": GEOCÉNTRICA
 "VERDADERA": HELIOCÉNTRICA

PARABOLA + AGRUPACIÓN
 ANUAL Y DIARIA
 ↓
 ELIPSES

PARALAJE VECTORIAL

$d\vec{s} \cong \perp \vec{s}, \hat{s}$



$\vec{r} = \vec{R} + \vec{r}' \rightarrow r \cdot \vec{s} = R \cdot \hat{n} + r' \cdot \vec{s}'$

$r' \cdot \vec{s}' = r \cdot \vec{s} - R \cdot \hat{n}$

$r' \cdot \vec{s} \wedge \vec{s}' = 0 - R \cdot \vec{s} \wedge \hat{n}$

$r' \cdot \vec{s} \wedge (\vec{s} \wedge \vec{s}') = -R \cdot \vec{s} \wedge (\vec{s} \wedge \hat{n})$

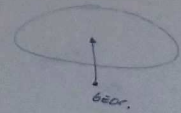
$(\vec{s} \cdot \vec{s}') \cdot \vec{s} - (\vec{s} \cdot \vec{s}') \cdot \vec{s}' = -\frac{R}{r'} \vec{s} \wedge (\vec{s} \wedge \hat{n})$

$\vec{s}' - \vec{s} = \frac{R}{r'} \vec{s} \wedge (\vec{s} \wedge \hat{n})$

P. DIURNA

$x = -\frac{p}{r} \cos \delta \cdot \sin H$

$y = \frac{p}{r} (\sin \delta \cos \delta' \cos H - \cos \delta \sin \delta')$

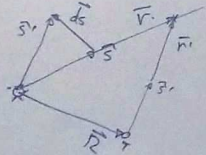


POSICIÓN → "TOPOCÉNTRICA" : OBSERVADOR
 "APARENTE" : GEOCÉNTRICA
 "VERDADERA" : HELIOCÉNTRICA

PARÁMETROS + ASINUTACIÓN
 ANUAL Y DIURNAL
 ↓
 ELIPSES

PARALAJE VECTORIAL

$$\vec{ds} \cong \perp \hat{s}, \hat{s}'$$



$$\vec{r} = \vec{R} + \vec{r}' \rightarrow r \cdot \hat{s} = R \cdot \hat{n} + r' \cdot \hat{s}'$$

$$r' \cdot \hat{s}' = r \cdot \hat{s} - R \cdot \hat{n}$$

$$r' \cdot \hat{s} \wedge \hat{s}' = 0 - R \cdot \hat{s} \wedge \hat{n}$$

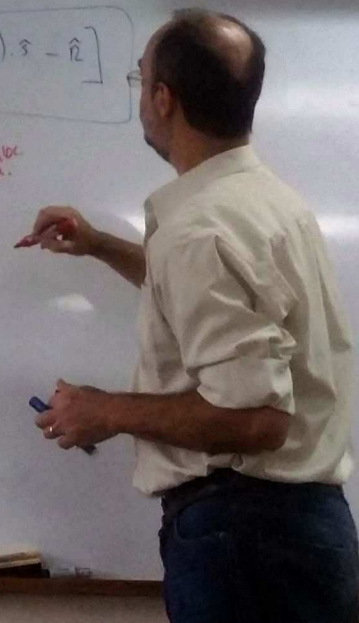
$$r' \cdot \hat{s} \wedge (\hat{s} \wedge \hat{s}') = -R \cdot \hat{s} \wedge (\hat{s} \wedge \hat{n})$$

$$\underbrace{(\hat{s} \cdot \hat{s}')}_{\pm 1} \cdot \hat{s} - \underbrace{(\hat{s} \cdot \hat{s})}_{1} \cdot \hat{s}' = -\frac{R}{r'} \hat{s} \wedge (\hat{s} \wedge \hat{n})$$

$$\vec{ds} = \frac{R}{r'} \hat{s} \wedge (\hat{s} \wedge \hat{n}) \rightarrow (\hat{s} \cdot \hat{n}) \cdot \hat{s} - (\hat{s} \cdot \hat{s}) \cdot \hat{n}$$

$$\vec{ds} = \pi \left[(\hat{s} \cdot \hat{n}) \cdot \hat{s} - \hat{n} \right]$$

DR. HELDO
 ESTA.

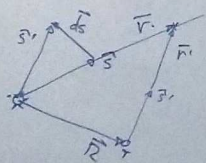


Posición "TOPOCÉNTRICA": OBSERVADOR
 "APARENTE": GEOCÉNTRICA
 "VERDADERA": HELIOCÉNTRICA

PARÁMETROS + ABERRACIÓN ANUAL Y DIURNAL
 ↓
 ELIPSES

PARALAJE VECTORIAL

$$\vec{ds} \cong \perp \vec{s}, \vec{s}'$$



$$\vec{r} = \vec{R} + \vec{r}' \rightarrow r \cdot \vec{s} = R \cdot \hat{n} + r' \cdot \vec{s}'$$

$$r' \cdot \vec{s}' = r \cdot \vec{s} - R \cdot \hat{n}$$

$$r' \cdot \vec{s} \wedge \vec{s}' = 0 - R \cdot \vec{s} \wedge \hat{n}$$

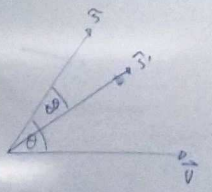
$$r' \cdot \vec{s} \wedge (\vec{s} \wedge \vec{s}') = -R \cdot \vec{s} \wedge (\vec{s} \wedge \hat{n})$$

$$\underbrace{(\vec{s} \cdot \vec{s}')}_{\cong 1} \cdot \vec{s} - \underbrace{(\vec{s} \cdot \vec{s})}_{1} \cdot \vec{s}' = -\frac{R}{r'} \vec{s} \wedge (\vec{s} \wedge \hat{n})$$

$$\vec{ds} = \frac{R}{r'} \vec{s} \wedge (\vec{s} \wedge \hat{n}) \rightarrow (\vec{s} \cdot \hat{n}) \cdot \vec{s} - (\vec{s} \cdot \vec{s}) \cdot \hat{n}$$

$$\vec{ds} = \pi \left[\underbrace{(\vec{s} \cdot \hat{n})}_{\text{DIR. HELIO. EST.}} \cdot \vec{s} - \underbrace{\hat{n}}_{\text{DIRECCIÓN TIERRA}} \right]$$

ABERRACIÓN GRAL



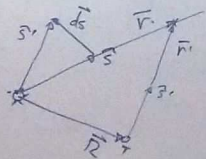
$$\Delta \theta = \frac{v}{c} \approx 0$$

Posición → "TOPOCÉNTRICA": OBSERVADOR
 "APARENTE": GEOCÉNTRICA
 "VERDADERA": HELIOCÉNTRICA

PARALAJE + ABERRACIÓN ANUAL Y DIARIA
 ↓
 ELIPSES

PARALAJE VECTORIAL

$$d\vec{s} \cong \perp \vec{s}, \hat{s}$$



$$\vec{r} = \vec{R} + \vec{F}' \rightarrow r \cdot \hat{s} = R \cdot \hat{n} + r' \cdot \hat{s}'$$

$$r' \cdot \hat{s}' = r \cdot \hat{s} - R \cdot \hat{n}$$

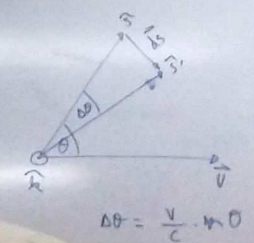
$$r' \cdot \hat{s} \wedge \hat{s}' = 0 - R \cdot \hat{s} \wedge \hat{n}$$

$$r' \cdot \hat{s} \wedge (\hat{s} \wedge \hat{s}') = -R \cdot \hat{s} \wedge (\hat{s} \wedge \hat{n})$$

$$\underbrace{(\hat{s} \cdot \hat{s}')}_{\pm 1} \cdot \hat{s} - \underbrace{(\hat{s} \cdot \hat{s})}_{1} \cdot \hat{s}' = -\frac{R}{r'} \hat{s} \wedge (\hat{s} \wedge \hat{n})$$

$$\hat{s}' - \hat{s} = \frac{R}{r'} \hat{s} \wedge (\hat{s} \wedge \hat{n}) \rightarrow \underbrace{(\hat{s} \cdot \hat{n})}_{\pi} \cdot \hat{s}$$

ABERRACIÓN GRAL



$$d\vec{s} = \hat{s}' - \hat{s} = \frac{v}{c} \cdot \hat{s}$$

$$\Delta\theta = \frac{v}{c} \cdot \sin\theta$$

$$\hat{n} = \hat{s} \wedge \hat{k} \rightarrow \hat{h} = \frac{\vec{v} \wedge \hat{s}}{v \cdot \sin\theta}$$

$$\Rightarrow d\vec{s} = \frac{v}{c} \sin\theta \cdot \hat{s} \wedge (\vec{v} \wedge \hat{s})$$

POSICIÓN "TOPOCÉNTRICA": OBSERVADOR
 "APARENTE": GEOCÉNTRICA
 "VERADERA": HELIOCÉNTRICA

PARALEL + ABERRACIÓN
 PARAL Y ÓRBITA
 ELIPSES

$$\vec{r} = \vec{R} + \vec{F} \rightarrow r \cdot \hat{s} = R \cdot \hat{n} + r' \cdot \hat{s}'$$

$$r' \cdot \hat{s}' = r \cdot \hat{s} - R \cdot \hat{n}$$

$$r' \cdot \hat{s} \wedge \hat{s}' = 0 - R \cdot \hat{s} \wedge \hat{n}$$

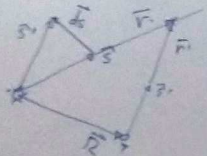
$$r' \cdot \hat{s} \wedge (\hat{s} \wedge \hat{s}') = -R \cdot \hat{s} \wedge (\hat{s} \wedge \hat{n})$$

$$\underbrace{(\hat{s} \cdot \hat{s}')}_{=1} \cdot \hat{s} - \underbrace{(\hat{s} \cdot \hat{s})}_{=1} \cdot \hat{s}' = -\frac{R}{r'} \hat{s} \wedge (\hat{s} \wedge \hat{n})$$

$$\hat{s}' - \hat{s} = \frac{R}{r'} \hat{s} \wedge (\hat{s} \wedge \hat{n}) \rightarrow (\hat{s} \cdot \hat{n}) \cdot \hat{s}$$

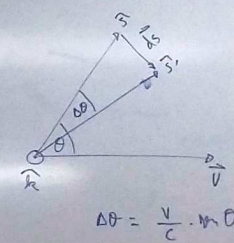
PARALELASE VECTORIAL

$$\vec{ds} \cong \perp \hat{s}, \hat{s}'$$



ABERRACIÓN GRAL

$$\vec{ds} = \frac{1}{c} [\vec{v} - (\hat{s} \cdot \vec{v}) \cdot \hat{s}]$$



$$\vec{ds} = \hat{s}' - \hat{s} = \frac{v}{c} \cdot \sin \theta \cdot \hat{n}$$

$$\hat{n} = \hat{s} \wedge \hat{h} \rightarrow \hat{h} = \vec{v} \wedge \hat{s}$$

$$\Rightarrow \vec{ds} = \frac{v}{c} \cdot \sin \theta \cdot \hat{s} \wedge (\vec{v} \wedge \hat{s}) = \frac{1}{c} \cdot [(\hat{s} \cdot \hat{s}) \vec{v} - (\hat{s} \cdot \vec{v}) \cdot \hat{s}]$$

Posición "TOPOCÉNTRICA" - OBSERVADA

"APARENTE" - GEOMÉTRICA

"VERADERA"

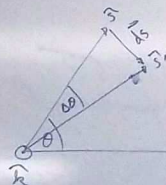
ABERRACIÓN RAYETARIA

+ Corrección Por T. LUZ

Pos. observada

$$\vec{ds} = \frac{1}{c} [\vec{v} - (\vec{s} \cdot \vec{v}) \cdot \vec{s}]$$

ABERRACIÓN GRAL



$$\Delta\theta = \frac{v}{c} \cdot \sin\theta$$

$$\vec{ds} = \vec{s}' - \vec{s} = \frac{v}{c} \cdot \sin\theta \cdot \hat{k}$$

$$\hat{k} = \vec{s} \wedge \hat{h} \rightarrow \hat{h} = \frac{\vec{v} \wedge \vec{s}}{v \cdot \sin\theta}$$

$$\Rightarrow \vec{ds} = \frac{v}{c} \cdot \sin\theta \cdot \frac{\vec{v} \wedge \vec{s}}{v \cdot \sin\theta} = \frac{1}{c} \cdot [(\vec{s} \cdot \vec{s}) \cdot \vec{v} - (\vec{s} \cdot \vec{v}) \cdot \vec{s}]$$

Posición "TOPOCÉNTRICA": OBSERVADA
 "APARENTE": GEOCÉNTRICA
 "VERADERA": HELIOCÉNTRICA

ABERRACIÓN RABETARIA

+ Corrección Por T. LUZ

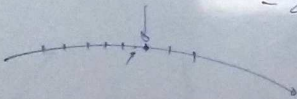
Pos. OBSERVADA = P. TOPOCÉNTRICA

mas
 + α, δ



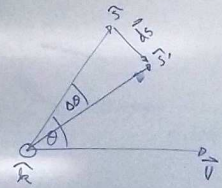
$$-z \cdot \frac{d\alpha}{dt}$$

$$-z \cdot \frac{d\delta}{dt}$$



$$\vec{ds} = \frac{1}{c} [\vec{v} - (\vec{s} \cdot \vec{v}) \cdot \vec{s}]$$

ABERRACIÓN GRAL



$$\Delta\theta = \frac{v}{c} \cdot \sin\theta$$

$$\vec{ds} = \vec{s}' - \vec{s} = \frac{v}{c} \cdot \sin\theta \cdot \hat{k}$$

$$\hat{k} = \vec{s} \wedge \hat{h} \rightarrow \hat{h} = \vec{v} \wedge \hat{s}$$

$$\Rightarrow \vec{ds} = \frac{v}{c} \cdot \sin\theta \cdot \frac{\vec{s} \wedge (\vec{v} \wedge \vec{s})}{\sin\theta} = \frac{1}{c} \cdot [(\vec{s} \cdot \vec{s}) \cdot \vec{v} - (\vec{s} \cdot \vec{v}) \cdot \vec{s}]$$

Posición "TOPOCÉNTRICA": OBSERVADOR
 "APARENTE": GEOCÉNTRICA
 "VERDADERA": HELIOCÉNTRICA

ABERRACIÓN RAYETARIA

+ Corrección Por T. LUZ

Pos. OBSERVADOR = P. TOPOCÉNTRICA

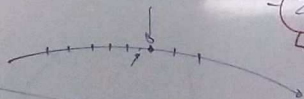
MANDE
 + α, δ



$-z \cdot \alpha$

$-z \cdot \delta$

Lo tiene Luz

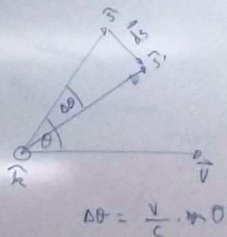


$$\alpha_{OBS} = \alpha_{TOPO} - z \cdot \alpha$$

$$\delta_{OBS} = \delta_{TOPO} - z \cdot \delta$$

$$\vec{ds} = \frac{1}{c} [\vec{v} - (\beta \cdot \vec{v}) \cdot \beta]$$

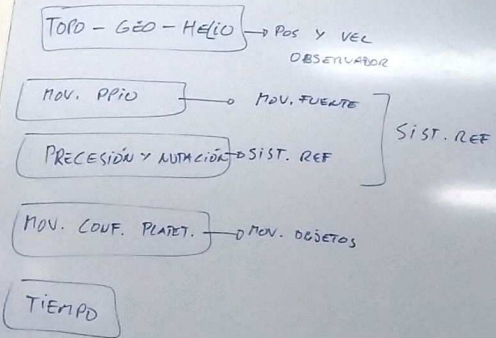
ABERRACIÓN GRAL



$$\vec{ds} = \vec{s}' - \vec{s} = \frac{v}{c} \cdot \sin \theta \cdot \hat{h}$$

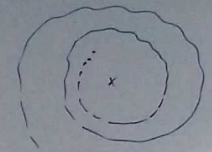
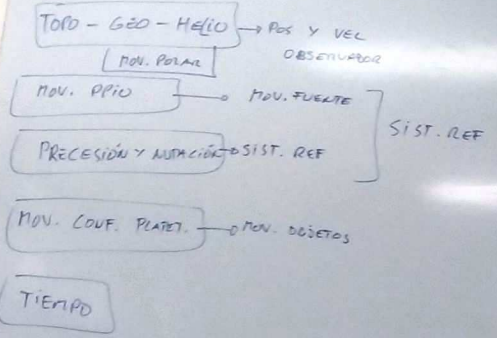
$$\hat{h} = \vec{s} \wedge \hat{k} \rightarrow \hat{h} = \frac{\vec{v} \wedge \vec{s}}{v \cdot \sin \theta}$$

$$\Rightarrow \vec{ds} = \frac{v}{c} \cdot \frac{\vec{s} \wedge (\vec{v} \wedge \vec{s})}{v \cdot \sin \theta} = \frac{1}{c} \cdot [(\vec{\beta} \cdot \vec{s}) \cdot \vec{v} - (\vec{\beta} \cdot \vec{v}) \cdot \vec{s}]$$



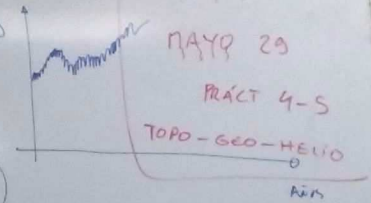
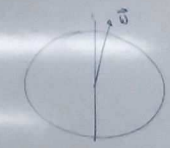
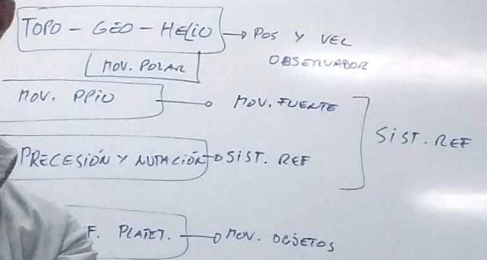
PARCIAL
MAYO 29
PRÁCT 4-5
TOPO - GEO - HELIO

Mov. PROPIO



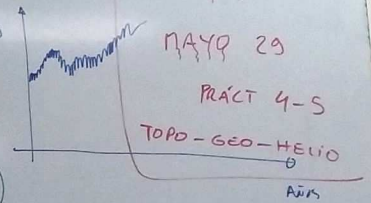
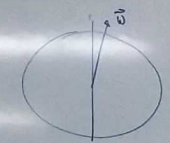
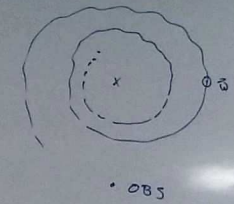
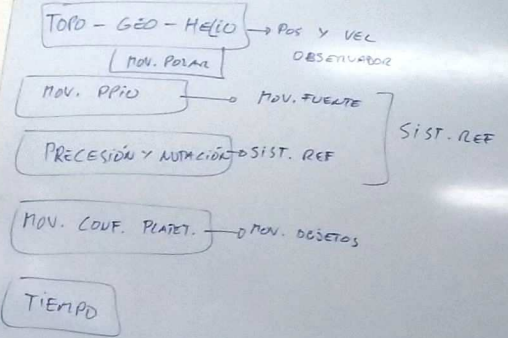
PARCIAL
 MAYO 29
 PRACT 4-5
 TOPO - GEO - HELIO

Mov. PROPIO



$$\vec{L}_{SIST} = \vec{L}_{TERRA}^{ROT} + \vec{L}_{T-L}^{ORB} \quad I \omega$$

Mov. PROPIO



PARCIAL

MAYO 29

PRÁCT 4-5

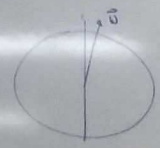
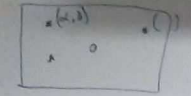
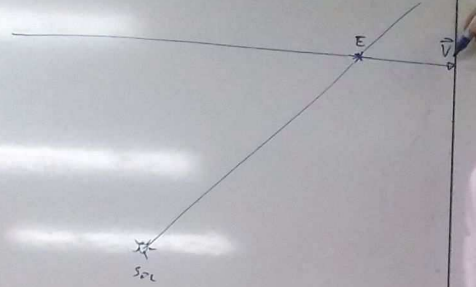
TOPO - GEO - HELIO

AÑOS

$$\vec{L}_{SIST} = \vec{L}_{TIEMPO}^{ROT} + \vec{L}_{T-C}^{ORB} \quad I(\omega)$$

Mov. PROPIO

- TOPO - GEO - HELIO → Pos y VEL OBSERVADOR
- MOV. POLAR
- MOV. PROPIO → MOV. FUENTE
- PRECESIÓN Y NUTACIÓN → SIST. REF
- MOV. CONV. PLANET. → MOV. OBJETOS
- TIEMPO

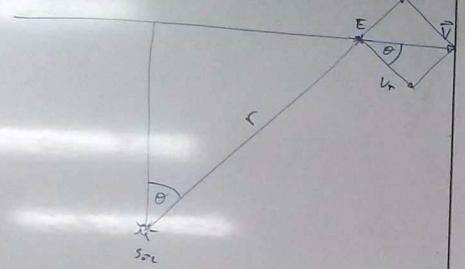
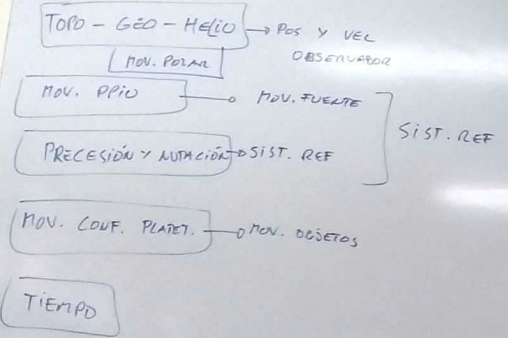


LOD

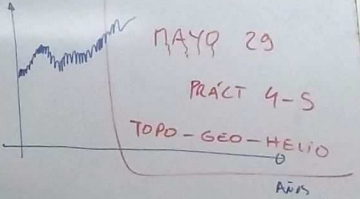
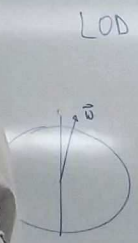
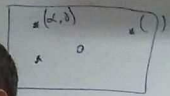


PARCIAL
MAYO 29
PRÁCT 4-5
TOPO - GEO - HELIO

Mov. PROPIO

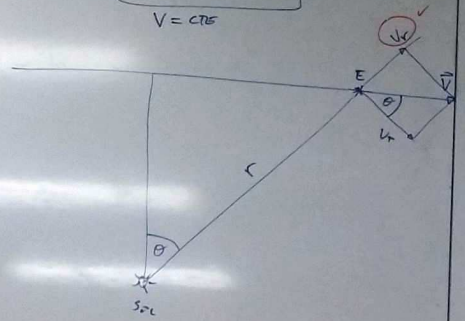


$V_r = V \cdot \sin \theta$
 $V_T = V \cdot \cos \theta$



Mov. PROPIO

$V = c \mu$



$V_r = V \cdot \sin \theta = \frac{dr}{dt}$

$V_t = V \cdot \cos \theta = r \cdot \frac{d\theta}{dt}$ MOV. PROPIO
RMS / AÑO

TOD - GEO - HELIO

→ POS Y VEL OBSERVADOR

(MOV. POLAR)

MOV. PROPIO

→ MOV. FUENTE

PRECESIÓN Y NUTACIÓN

→ SIST. REF

SIST. REF

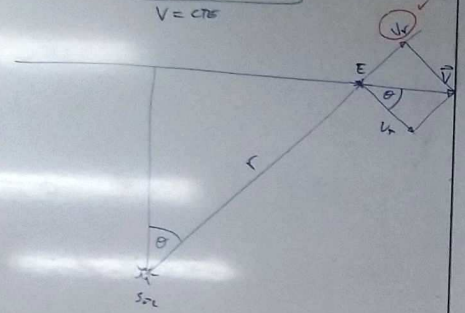
MOV. CONF. PLANET.

→ MOV. OBJETOS

TIEMPO

Mov. PROPIO

$V = c \theta$



$V_r = V \cdot \cos \theta = \frac{dr}{dt}$

$V_t = V \cdot \sin \theta = r \cdot \frac{d\theta}{dt}$ → MOV. PROPIO
RAS/AÑO

$\frac{d\theta}{dt} = \frac{V_t}{r}$

TORO - GEO - HELIO → POS Y VEL OBSERVADOR

Mov. POLAR

Mov. PROPIO → MOV. FUENTE

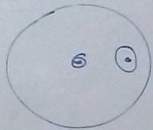
PRECESIÓN Y NUTACIÓN → SIST. REF

Mov. CONF. PLANET. → MOV. OBJETOS

TIEMPO

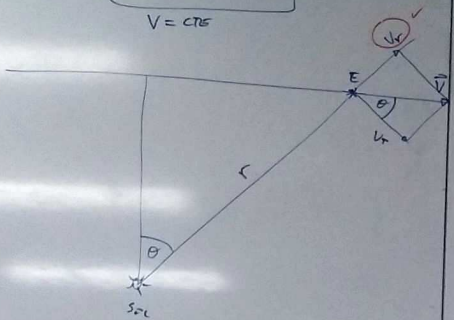


- TORO - GEO - HELIO → Pos y VEL OBSERVADOR
- MOV. POLAR
- MOV. PROPIO → MOV. FUENTE
- PRECESIÓN Y NUTACIÓN → SIST. REF
- MOV. CONF. PLANET. → MOV. OBJETOS
- TIEMPO



MOV. PROPIO

$V = \text{cte}$

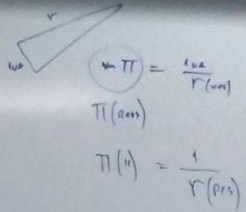


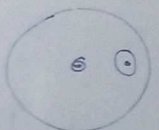
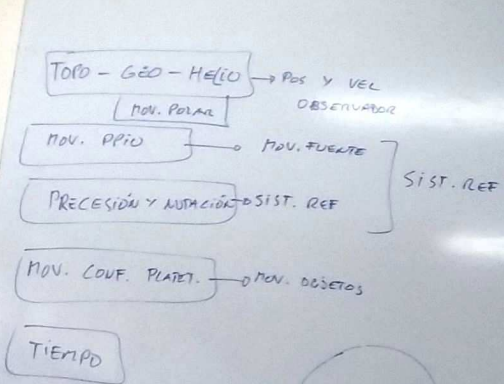
$V_r = V \cdot \cos \theta = \frac{dr}{dt}$

$V_t = V \cdot \sin \theta = r \cdot \frac{d\theta}{dt}$ *MOV. PROPIO*

$\frac{d\theta}{dt} = \frac{V_t}{r} \rightarrow \frac{d\theta}{dt} = \frac{1}{r} \cdot V_t$

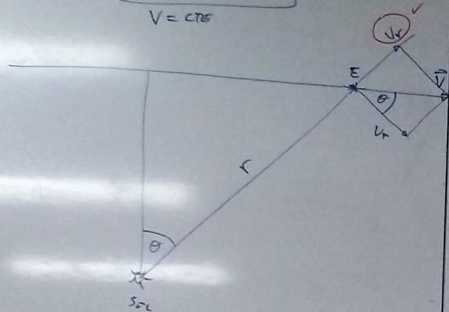
$\Rightarrow V_t = \frac{d\theta}{dt} \cdot r$





Mov. PROPIO $\rightarrow \mu$ ("año)

$V = CTG$



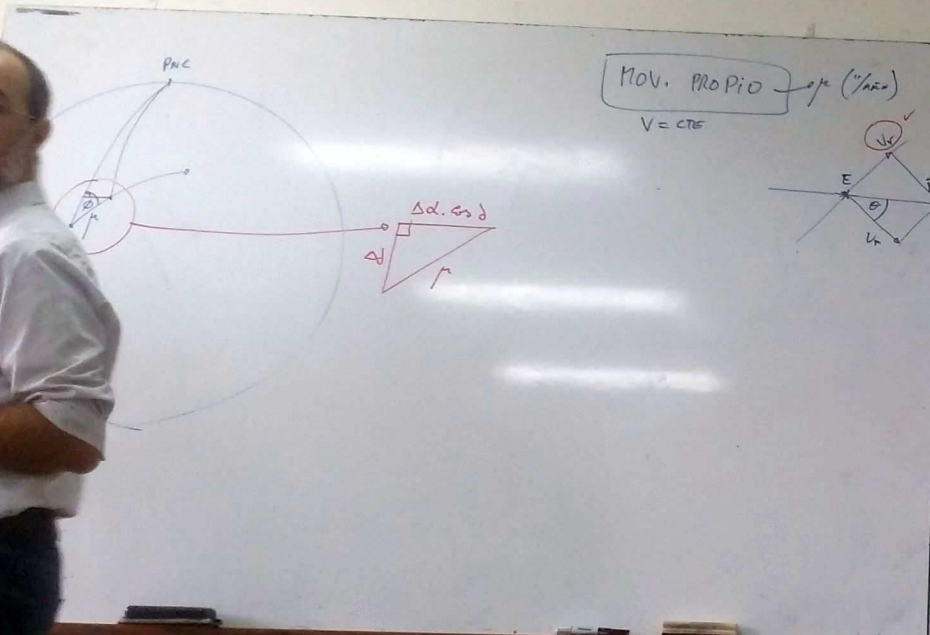
$V_r = V \cdot \cos \theta = \frac{dr}{dt}$

$V_T = V \cdot \sin \theta = r \cdot \frac{d\theta}{dt}$ (MOV. PROPIO) (RAS/AÑO)

$\frac{d\theta}{dt} = \frac{V_T}{r} \rightarrow \frac{d\theta}{dt} = \frac{1}{r} \cdot V_T$

$\Rightarrow V_T = \frac{r}{\pi} = 4.75 \frac{r}{\pi} \text{ (km/seg)}$

$\mu \pi = \frac{1 \mu a}{r (\mu a)}$
 $\pi (\text{arc})$
 $\pi (") = \frac{1}{r (\text{pc})}$



Mov. PROPIO $\rightarrow \mu$ (" / ano)
 $V = CTG$

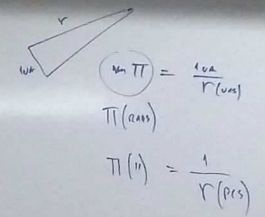
$$V_r = V \cdot \cos \theta = \frac{dr}{dt}$$

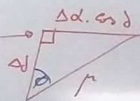
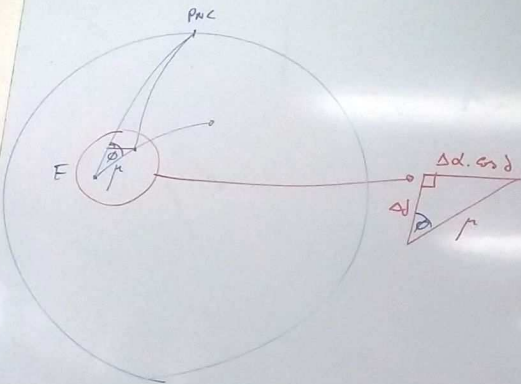
$$V_T = V \cdot \sin \theta = r \cdot \frac{d\theta}{dt}$$

RAAS / AÑO

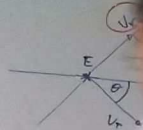
$$\frac{d\theta}{dt} = \frac{V_T}{r} \rightarrow \frac{d\theta}{dt} = \frac{1}{r} \cdot V_T$$

$$\Rightarrow V_T = \frac{r}{\pi} = 4.75 \frac{r}{\pi} \text{ (Km/seg)}$$





Mov. PROPIO $\rightarrow \mu$ ($''/ano$)
 $V = CRTS$



$$\mu_{\alpha} = \frac{1}{15} \cdot \frac{r}{\text{ano}} \cdot \frac{\mu}{\cos \delta}$$

$$\mu_{\delta} = \mu \cdot \cos \delta$$

$$\pi = \frac{r \mu}{r(\mu_{\delta})}$$

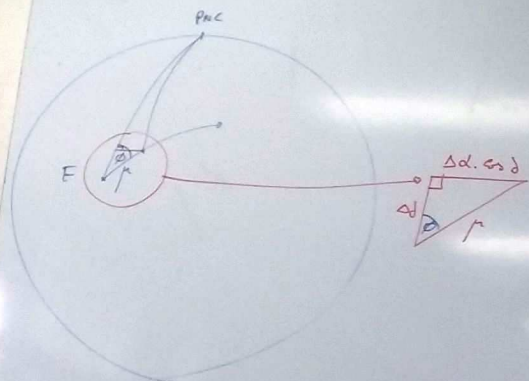
$$\pi(\mu_{\delta}) = \frac{1}{r(\mu_{\delta})}$$

$$\begin{cases} \Delta \alpha \cdot \cos \delta = \mu \cdot \sin \phi \\ \Delta \delta = \mu \cdot \cos \phi \end{cases}$$

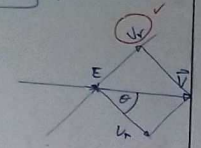
$$\Rightarrow \Delta \alpha = \frac{\mu \cdot \sin \phi}{\cos \delta} \cdot \frac{1}{\mu_{\alpha}}$$

$$\Delta \delta = \mu \cdot \cos \phi \cdot \frac{1}{\mu_{\delta}}$$

LOGO



Mov. PROPIO \rightarrow μ ($''/ano$)
 $V = c \mu$



$$\begin{cases} \Delta d \cdot \cos \phi = r \cdot \sin \phi \\ \Delta d = r \cdot \sin \phi \end{cases}$$

$$\Rightarrow \Delta \alpha = \frac{r \cdot \sin \phi}{\cos \delta} \cdot \frac{1}{15}$$

$$\Delta \delta = r \cdot \cos \phi$$

$$\mu_{\alpha} = \frac{1}{15} \mu \sin \phi / \cos \delta$$

$$\mu_{\delta} = \mu \cdot \cos \phi$$

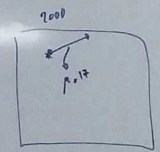
CATALOGO

$$\left. \begin{matrix} \alpha, \mu_{\alpha} \\ \delta, \mu_{\delta} \end{matrix} \right\} \leftrightarrow \left\{ \begin{matrix} \alpha, \mu \\ \delta, \phi \end{matrix} \right.$$



$$\mu \pi = \frac{v_{\alpha}}{r(\text{mas})}$$

$$\pi(\text{mas}) = \frac{1}{r(\text{prs})}$$



ACELERACIÓN DE PERSPECTIVA : $d\mu/dt$

$$V_t = V \cdot \cos \theta = r \cdot \frac{d\theta}{dt}$$

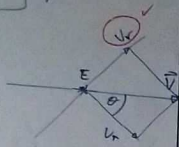
$$-V \cdot \sin \theta \cdot \frac{d\theta}{dt} = \frac{dr}{dt} \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2}$$

$$-2V \cdot \sin \theta \cdot \frac{d\theta}{dt} = r \cdot \frac{d^2\theta}{dt^2}$$

$$V_t = V \cdot \sin \theta$$

Mov. PROPIO $\rightarrow \mu$ ("año)

$$V = c \cos \theta$$



$$\Delta \alpha \cdot \cos \delta = \mu \cdot \sin \phi$$

$$\Delta \delta = \mu \cdot \cos \phi$$

$$\Rightarrow \Delta \alpha = \mu \cdot \sin \phi / \cos \delta \cdot \frac{1}{15}$$

$$\Delta \delta = \mu \cdot \cos \phi$$

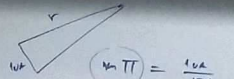


$$\mu_\alpha = \frac{1}{15} \mu \sin \phi / \cos \delta$$

$$\mu_\delta = \mu \cdot \cos \phi$$

CATALOGO

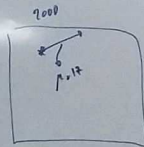
$$\left. \begin{matrix} \alpha, \mu_\alpha \\ \delta, \mu_\delta \end{matrix} \right\} \leftrightarrow \left\{ \begin{matrix} \alpha, \mu \\ \delta, \phi \end{matrix} \right.$$



$$\mu \sin \theta = \frac{1 \mu \alpha}{r(\text{un})}$$

$$\mu(\text{año})$$

$$\mu(\mu) = \frac{1}{r(\text{pc})}$$



2000

$\mu \cdot \sin \phi$

ACELERACIÓN DE PERSPECTIVA : $d\mu/dt$

$$V_T = V \cdot \cos \theta = r \cdot \frac{d\theta}{dt}$$

$$-V \cdot \sin \theta \cdot \frac{d\theta}{dt} = \frac{dr}{dt} \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2}$$

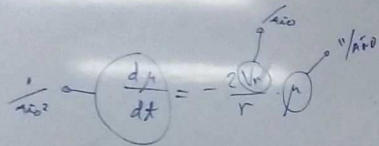
$$V_T = V \cdot \sin \theta$$

$$-2V \cdot \sin \theta \cdot \frac{d\theta}{dt} = r \cdot \frac{d^2\theta}{dt^2}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{2V \cdot \sin \theta}{r} \cdot \frac{d\theta}{dt}$$

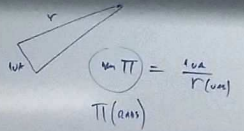
$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{2V_T}{r} \cdot \frac{d\theta}{dt}$$

$d\theta \approx \frac{V_T}{r} dt$



$$\mu_{\alpha} = \frac{1}{15} \frac{V_T}{r} \sin \theta / \cos \delta$$

$$\mu_{\delta} = \mu \cdot \cos \delta$$

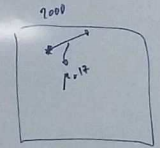


$$\mu \pi = \frac{V_T}{r(\text{mas})}$$

$$\pi(\text{mas}) = \frac{1}{r(\text{prs})}$$

CATALOGO

$$\left. \begin{matrix} \alpha, \mu_{\alpha} \\ \delta, \mu_{\delta} \end{matrix} \right\} \leftrightarrow \left\{ \begin{matrix} \alpha, \mu \\ \delta, \phi \end{matrix} \right.$$



ACELERACIÓN DE PERSPECTIVA : $d\mu/dt$

$$V_T = V \cdot \sin \theta = r \cdot \frac{d\theta}{dt}$$

$$-V \cdot \cos \theta \cdot \frac{d\theta}{dt} = \left(\frac{dr}{dt}\right) \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2}$$

$$V_T = V \cdot \sin \theta$$

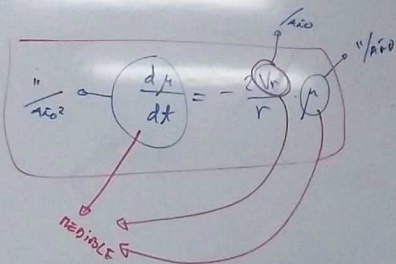
$$-2V \cdot \cos \theta \cdot \frac{d\theta}{dt} = r \cdot \frac{d^2\theta}{dt^2}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{2V \cdot \cos \theta}{r} \cdot \frac{d\theta}{dt}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{2V_T}{r} \cdot \frac{d\theta}{dt}$$

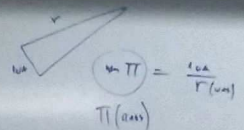
$d\theta$ en "

AFO



$$\mu_\alpha = \frac{1}{15} \frac{V_T}{r} \frac{d\theta}{dt}$$

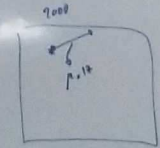
$$\mu_\delta = \mu \cdot \cos \theta$$



$$\pi (\text{arc}) = \frac{1}{r (\text{arc})}$$

CATALOGO

$$\left. \begin{matrix} \alpha, \mu_\alpha \\ \delta, \mu_\delta \end{matrix} \right\} \delta \left\{ \begin{matrix} \alpha, \mu \\ \delta, \mu \end{matrix} \right.$$



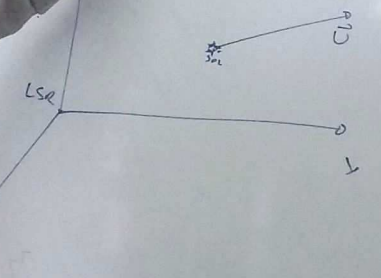
$$\vec{V} = (\vec{V}_* - \vec{V}_{LSR}) - (\vec{V}_0 - \vec{V}_{LSR})$$

PECULIAR

MOV. PARALÁCTICO

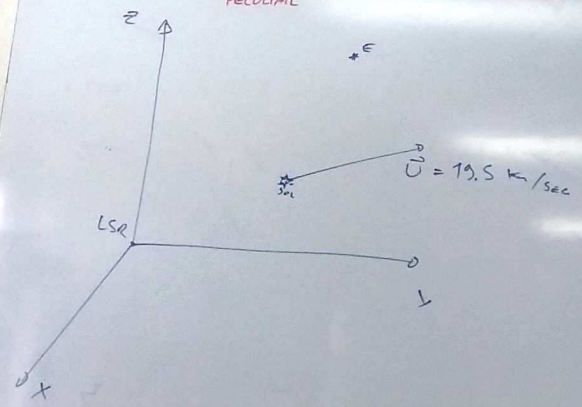
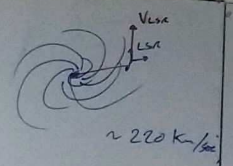
z

* ∈



$$\vec{V} = (\vec{V}_* - \vec{V}_{LSR}) - (\vec{V}_0 - \vec{V}_{LSR})$$

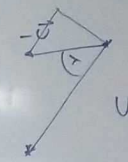
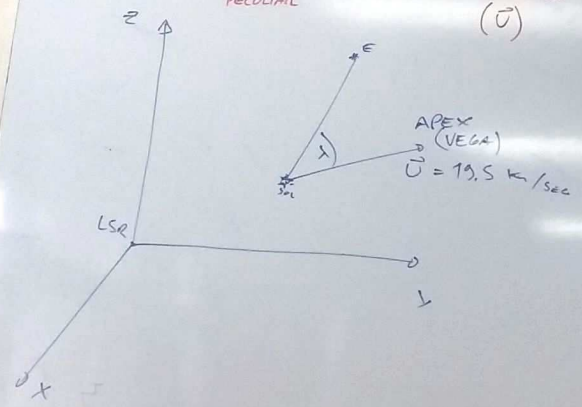
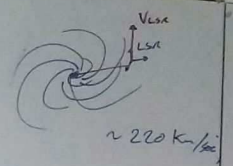
Mov. PECULIAR Mov. PARALÁCTICO



$$\vec{V} = (\vec{V}_* - \vec{V}_{LSR}) - (\vec{V}_0 - \vec{V}_{LSR})$$

Mov. PECULIAR

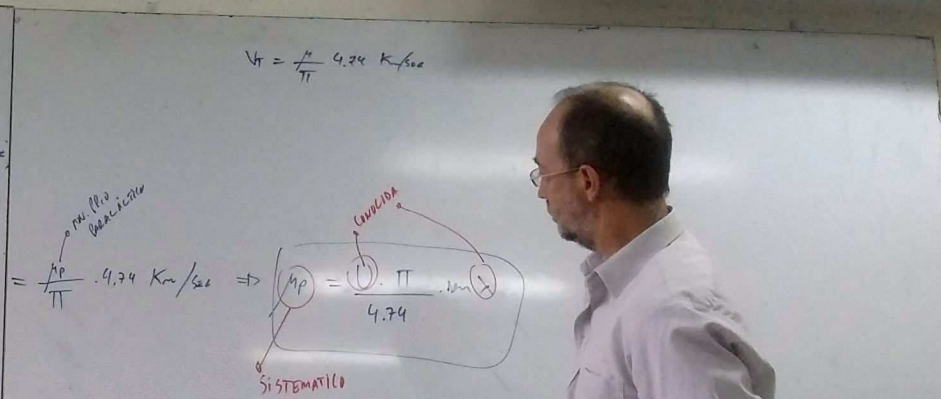
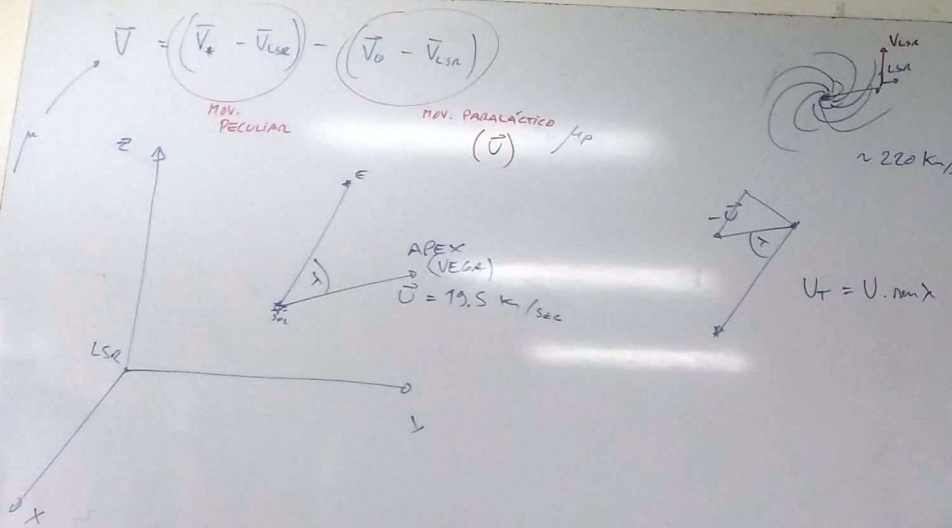
Mov. PARALÁCTICO
(\vec{U})



$$U_T = U \cdot \sin \alpha =$$

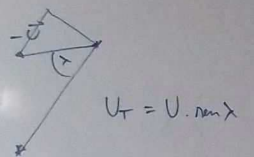
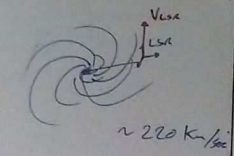
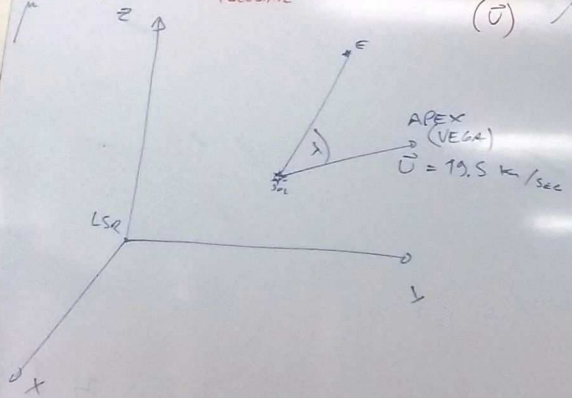
$$U_T =$$



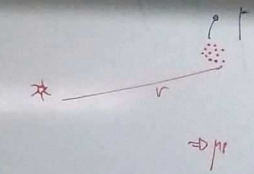


$$\vec{V} = (\vec{V}_* - \vec{V}_{LSR}) - (\vec{V}_0 - \vec{V}_{LSR}) \rightarrow \vec{v} = \vec{U}$$

Mov. PECULIAR (circled)
 Mov. PARALÁCTICO (circled) μ_p

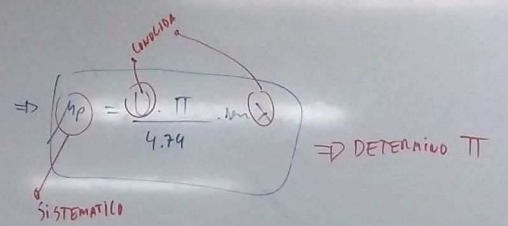


$$V_T = \frac{v}{\sin \lambda} = 4.74 \text{ km/sec}$$



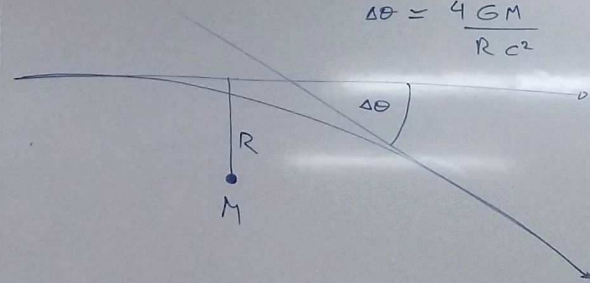
$U_T = U \cdot \sin \lambda$

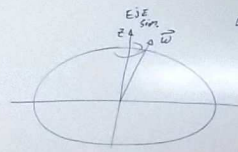
$= \frac{\mu_p}{\pi} \cdot 4.74 \text{ km/sec}$



DESVÍO GRAVITACIONAL

$$\Delta\theta = \frac{4GM}{Rc^2}$$

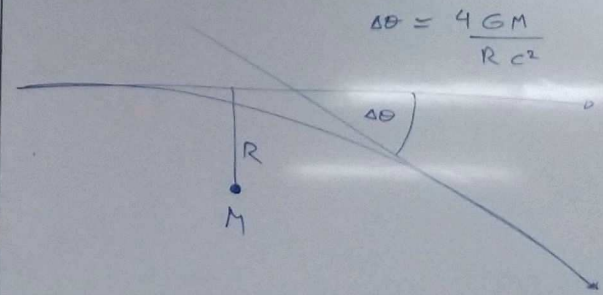




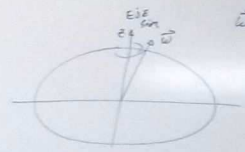
$\vec{\omega}(t) \rightarrow$ MOV. ROTAC.
MOV. LIBRE EULERIANO

$$\frac{d\omega}{dt} = 0$$

DESVIÓ GRAVITACIONAL



$$\Delta\theta = \frac{4GM}{Rc^2}$$

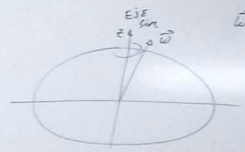


$\vec{\omega}(t) \rightarrow$ MOV. ROTAC.
MOV. LIBRE EULERIANO

$$\frac{d\omega}{dt} < 0$$

TRANSF. DE MOM. ANGULAR

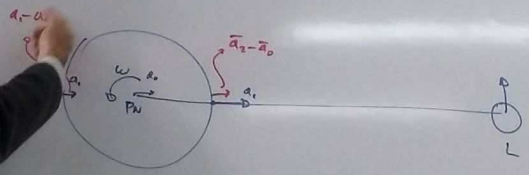


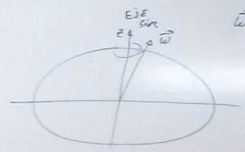


$\vec{\omega}(t) \rightarrow$ MOV. PERIÓDICO
MOV. LIBRE EULERIANO

$$\frac{d\omega}{dt} < 0$$

TRANSF. DE MOM. ANGULAR

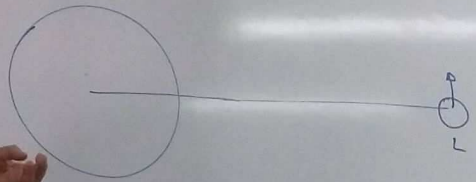


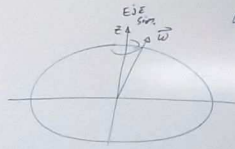


$\vec{\omega}(t) \rightarrow$ MOV. PERIÓDICO
MOV. LIBRE EULERIANO

$$\frac{d\omega}{dt} < 0$$

TRANSF. DE MOM. ANGULAR

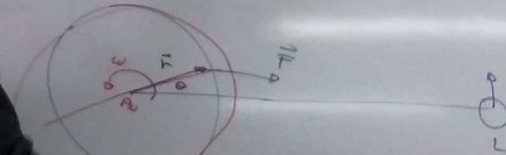


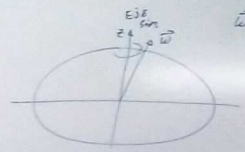


$\vec{\omega}(t) \rightarrow$ MOV. ROTAC.
MOV. LIBRE EULERIANO

$$\frac{d\omega}{dt} < 0$$

TRANSF. DE MOM. ANGULAR





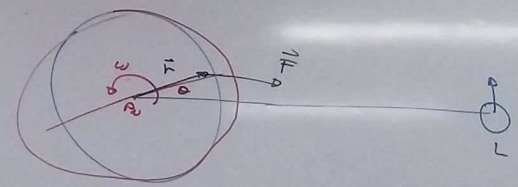
$\vec{\omega}(t) \rightarrow$ MOV. RIGID
 MOV. LIBRE EULERIANO

$\frac{d\omega}{dt} < 0$ — OLLO

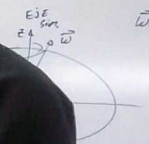
TRANSF. DE MOM. ANGULAR

$\vec{L} \approx \vec{L}_T^{rot} + \vec{L}_{TL}^{orbital} \approx CTE$

$\vec{M} = \vec{r} \wedge \vec{F}$



$\sigma \sim 0$



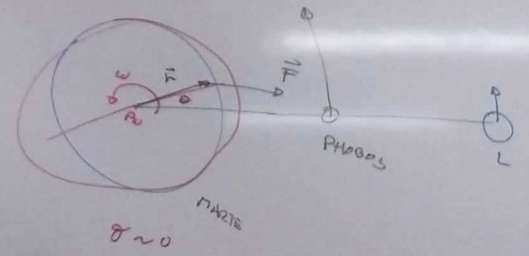
$\vec{\omega}(A) \rightarrow$ mov. rotac
 mov. LIBRE EULERIANO

23:59:55
 :59:60
 00:00:00

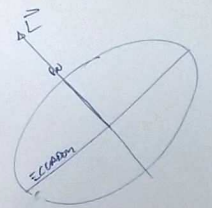
TRANSF. DE MOM. ANGULAR

$$\vec{L} = \vec{L}_1 + \vec{L}_{rel} = CTS$$

$$\vec{M} = \vec{r} \wedge \vec{F}$$



PRECESSION

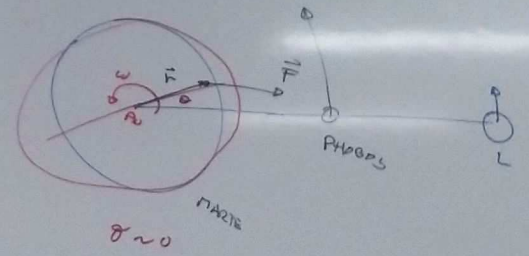


TRANSF. DE MOM. ANGULAR

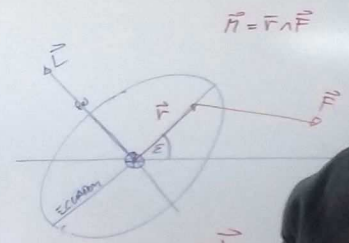
$$\vec{M} = \vec{r} \wedge \vec{F}$$

$$\vec{L} \equiv \vec{L}_1 + \vec{L}_2 = \text{cte}$$

$\frac{M_1 L_1}{M_1 r_1} \quad \frac{M_2 L_2}{M_2 r_2}$



PRECESSION



$$\vec{M} = \vec{r} \wedge \vec{F}$$

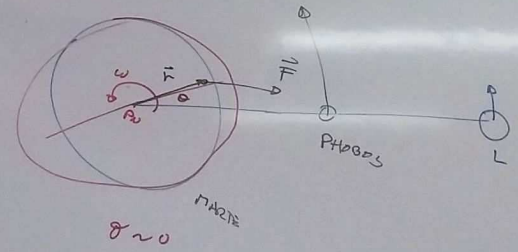
$$\frac{d\vec{L}}{dt}$$



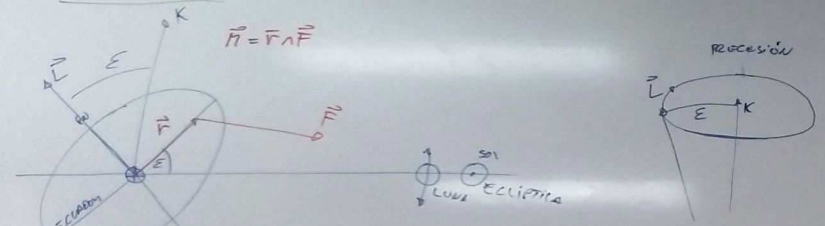
TRANSF. DE MOM. ANGULAR

$$\vec{M} = \vec{r} \wedge \vec{F}$$

$$\vec{L} \approx \vec{L}_T \frac{d\vec{L}}{dt} + \vec{L}_{TL} \frac{d\vec{L}}{dt} \approx \text{cte}$$



PRECESSION



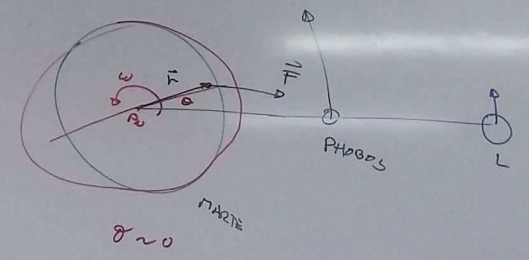
$$\frac{d\vec{L}}{dt} = \vec{N}$$

TRANSF. DE MOM. ANGULAR

$$\vec{N} = \vec{F} \wedge \vec{F}$$

$$\vec{L} = \vec{L}_{rot} + \vec{L}_{trans} \approx CTE$$

$\frac{dL}{dt} \approx 0$
 $\frac{dL}{dt} \approx 0$

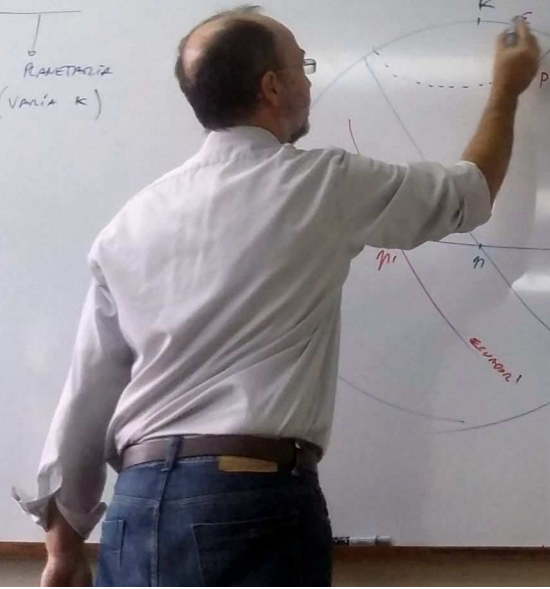
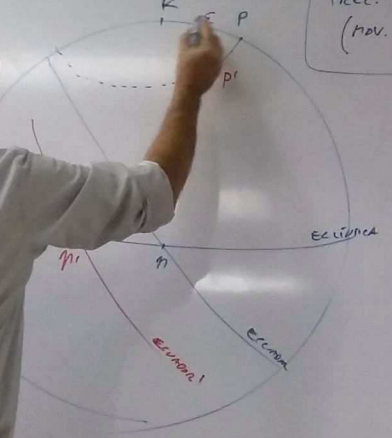


PRECESIÓN Y NUTACIÓN

PLANETARIA
(VARIA k)

→ LUNISOLAR

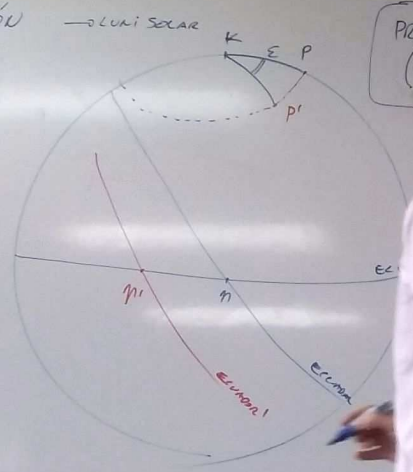
PREC. LUNISOLAR
(MOV. PNC)



PRECESIÓN Y NUTACIÓN

PLANETARIA
(VARIA K)

→ LUNA SOLAR



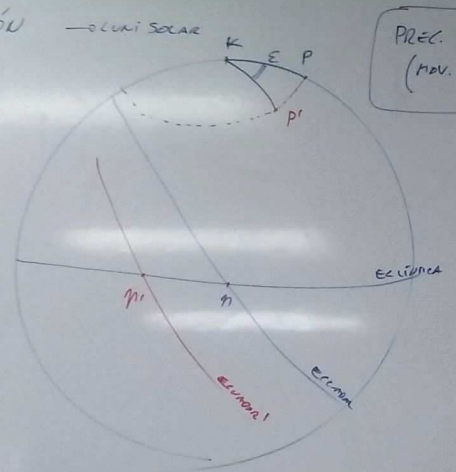
PREC.
(η)

$$\widehat{PKP'} = \widehat{\gamma K \eta'} = \widehat{\eta \eta'}$$



PRECESIÓN Y NUTACIÓN

PLANETARIA
(VARIA κ)



PREC. LUNAR
(MOV. PNC)

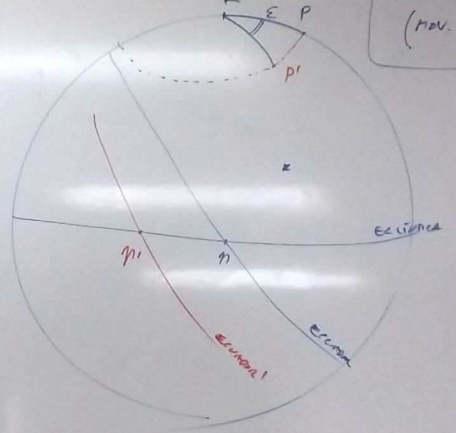
$$\widehat{PKP'} = \widehat{\gamma K \eta'} = \widehat{\eta \eta'} = \psi \cdot \epsilon$$

INTERVALO DE AÑOS
PRECESIÓN LUNAR ANUAL $\psi \approx 50'' \cdot 38 / \text{AÑO}$

PRECESIÓN Y ROTACIÓN

PLANETARIA
(VARIA K)

→ LUNISOLAR



PREC. LUNISOLAR
(MOV. PNC)

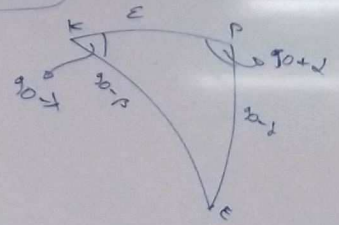
$$\widehat{PKP'} = \widehat{PK\eta'} = \widehat{\eta\eta'} = \psi \cdot \epsilon$$

→ DIFERENCIA DE 120°
PRECESIÓN LUNISOLAR ANUAL $\psi \approx 50'' \cdot 38 / \text{AÑO}$

$$d\beta = 0$$

$$d\alpha = \psi \cdot \epsilon$$

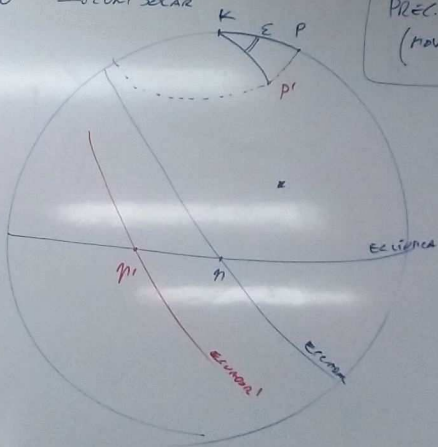
→ $d\alpha \cdot d\delta >$



PRECESIÓN Y ROTACIÓN

PLANETARIA
(VARIA κ)

→ LUNISOLAR



PREC. LUNISOLAR
(MOV. PERI)

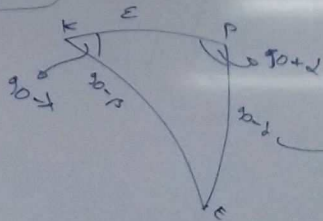
$$\widehat{PKP'} = \widehat{PKP''} = \widehat{PP''} = \psi \cdot \epsilon$$

→ ROTACIÓN EN LOS ASES
PRECESIÓN LUNISOLAR ANUAL $\psi \approx 50'' \cdot 360 / \text{año}$

$$d\beta = 0$$

$$d\lambda = \psi \cdot \epsilon$$

→ $d\alpha, d\delta$



$$\cos(90 - \delta) = \cos \epsilon \cdot \cos(90 - \beta) + \sin \epsilon \cdot \sin(90 - \beta) \cdot \cos(90 + \lambda)$$

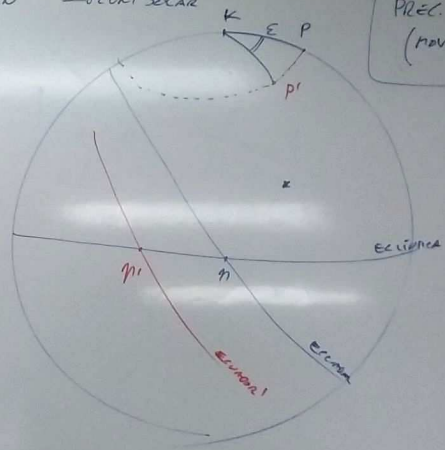
$$\sin \delta = \cos \epsilon \cdot \sin \beta + \sin \epsilon \cdot \cos \beta \cdot \sin \lambda$$

$$\cos \delta \cdot d\delta = 0 + \sin \epsilon \left[\cancel{\cos \beta \cdot d\beta \cdot \sin \lambda} + \cos \beta \cdot \cos \lambda \cdot d\lambda \right]$$

$$\sin \delta \cdot d\delta = \sin \epsilon \cdot \cos \beta \cdot \cos \lambda \cdot d\lambda$$

PRECESIÓN Y ROTACIÓN → LUNISOLAR

PREC. LUNISOLAR (MOV. PNC)



$$\frac{\sin(90-\beta)}{\sin(90+\alpha)} = \frac{\sin(90-\delta)}{\sin(90-\lambda)}$$

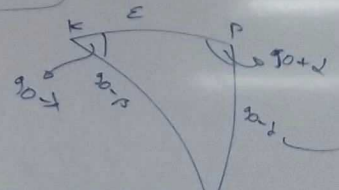
$$\Rightarrow \frac{\cos \beta}{\cos \alpha} = \frac{\cos \delta}{\cos \lambda}$$

$$\Rightarrow \cos \beta \cdot \cos \lambda = \cos \delta \cdot \cos \alpha$$

$$\widehat{PKP'} = \widehat{PKn'} = \widehat{nPn'} = \psi \cdot \epsilon$$

PRECESIÓN LUNISOLAR ANUAL $\psi \approx 50'' \cdot 38 / \text{año}$

$$\begin{aligned} d\beta &= 0 \\ d\lambda &= \psi \cdot z \end{aligned} \Rightarrow \begin{cases} d\alpha \\ d\delta \end{cases}$$



$$\Rightarrow d\delta = \sin \epsilon \cdot \cos \delta \cdot \psi \cdot z$$

$$\cos(90-\delta) = \cos \epsilon \cdot \cos(90-\beta) + \sin \epsilon \cdot \sin(90-\beta) \cdot \cos(90-\lambda)$$

$$\sin \delta = \cos \epsilon \cdot \sin \beta + \sin \epsilon \cdot (\cos \beta \cdot \sin \lambda)$$

$$\cos \delta \cdot d\delta = 0 + \sin \epsilon \left[(-\sin \beta) d\beta \cdot \sin \lambda + \cos \beta \cdot \cos \lambda \cdot d\lambda \right]$$

$$\cos \delta \cdot d\delta = \sin \epsilon \cdot \cos \beta \cdot \cos \lambda \cdot d\lambda$$

POVEN EN P. 65 d. d

PRECESIÓN Y ROTACIÓN → LUNAR

PREC. LUNAR (MOV. PNC)

$$\frac{\sin(90-\beta)}{\sin(90+\delta)} = \frac{\sin(90-\beta)}{\sin(90-\delta)}$$

DEMOSTRAR:

$$d\delta = (\cos \epsilon + \sin \epsilon \cdot \sin \delta \cdot \frac{1}{\sin \delta}) \psi \cdot z$$

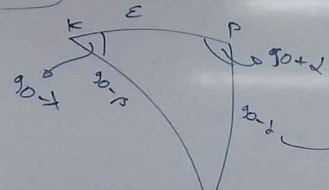
$$\Rightarrow \frac{\cos \beta}{\sin \delta} = \frac{\cos \delta}{\sin \lambda}$$

$$\Rightarrow \cos \beta \cdot \sin \lambda = \sin \delta \cdot \sin \delta$$

$$\widehat{PKP'} = \widehat{PK\eta'} = \widehat{\eta\eta'} = \psi \cdot z$$

INTERVALO EN AÑOS
PRECESIÓN LUNAR ANUAL $\psi \approx 50'' \cdot 38 / \text{año}$

$$\begin{aligned} d\beta &= 0 \\ d\lambda &= \psi \cdot z \end{aligned} \Rightarrow \{d\lambda, d\delta\}$$



$$\Rightarrow d\delta = \sin \epsilon \cdot \cos \delta \cdot \psi \cdot z$$

$$\cos(90-\delta) = \cos \epsilon \cdot \cos(90-\beta) + \sin \epsilon \cdot \sin(90-\beta) \cdot \cos(90-\lambda)$$

$$\sin \delta = \cos \epsilon \cdot \sin \beta + \sin \epsilon \cdot (\cos \beta) \cdot \sin \lambda$$

$$\cos \delta \cdot d\delta = 0 + \sin \epsilon \cdot [\cancel{\sin \beta \cdot d\beta} + \cos \beta \cdot \cos \lambda \cdot d\lambda]$$

$$\cos \delta \cdot d\delta = \sin \epsilon \cdot \cos \beta \cdot \sin \lambda \cdot d\lambda$$

POWER EN P. DE δ, λ

PRECESIÓN Y ROTACIÓN → LUNAR

$$\frac{m(90-\beta)}{m(90+\alpha)} = \frac{m(90-\delta)}{m(90-\lambda)}$$

$$\Rightarrow \frac{\cos \beta}{\cos \alpha} = \frac{\cos \delta}{\cos \lambda}$$

$$\Rightarrow \cos \beta \cdot \cos \lambda = \cos \delta \cdot \cos \alpha$$

DEMOSTRAR:

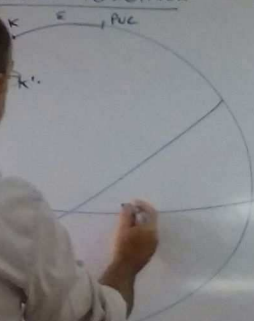
$$d\lambda = \left(\cos E + \sin E \cdot \sin \alpha \cdot \frac{1}{\cos \delta} \right) \psi \cdot z$$

$$\psi = 50''.3878 + 0''.0049T$$

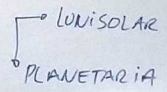
$$T = \left(\frac{t}{100} - 2000 \right) / 100$$

PREC. LUNAR
(MOV. PNC)

PRECESIÓN PLANETARIA



PRECESIÓN



PRECESIÓN

↳ LUNISOLAR
($d\beta=0$) $\rightarrow \begin{cases} d\alpha = \psi \cdot z (G_E + M_E m_2 \frac{1}{a} \delta) \\ d\delta = \end{cases}$

↳ PLANETARIA
($d\delta=0$) $\rightarrow \begin{cases} d\alpha = -\lambda' \cdot z \\ d\delta = 0 \end{cases}$

PRECESIÓN

- ↳ LUNISOLAR
($d\beta=0$) $\rightarrow \begin{cases} d\alpha = \chi \cdot z \cdot (\cos \epsilon + m \cdot \sin \epsilon \cdot \sin \delta) \\ d\delta = \chi \cdot z \cdot m \cdot \sin \epsilon \cdot \cos \delta \end{cases}$
- ↳ PLANETARIA
($d\delta=0$) $\rightarrow \begin{cases} d\alpha = -\lambda' \cdot z \\ d\delta = 0 \end{cases}$

↳ GENERAL : LUNIS + PLANETARIA

$$\begin{cases} d\alpha = \textcircled{m} \cdot z + \textcircled{m} \cdot z \cdot \sin \epsilon \cdot \sin \delta \\ d\delta = \textcircled{m} \cdot z \cdot \cos \delta \end{cases}$$

\downarrow
 CTES
 PRECESIONALES

PRECESIÓN

- ↳ LUNISOLAR
($d\beta=0$) $\rightarrow \begin{cases} d\alpha = \psi \cdot z (\cos \epsilon + m \cdot \epsilon \cdot \sin \alpha \sin \delta) \\ d\delta = \psi \cdot z \cdot m \cdot \epsilon \cdot \cos \alpha \end{cases}$
- ↳ PLANETARIA
($d\delta=0$) $\rightarrow \begin{cases} d\alpha = -\chi' \cdot z \\ d\delta = 0 \end{cases}$

↳ GENERAL : LUNIS + PLANETARIA

$$\begin{cases} d\alpha = m \cdot z + m \cdot z \cdot \epsilon \cdot \sin \alpha \sin \delta \\ d\delta = m \cdot z \cdot \cos \alpha \end{cases}$$

"CTES"
PRECESIONALES

$$\begin{aligned} m &= \psi \cdot \cos \epsilon - \chi' \\ m &= \psi \cdot \sin \epsilon \end{aligned}$$

PRECESIÓN

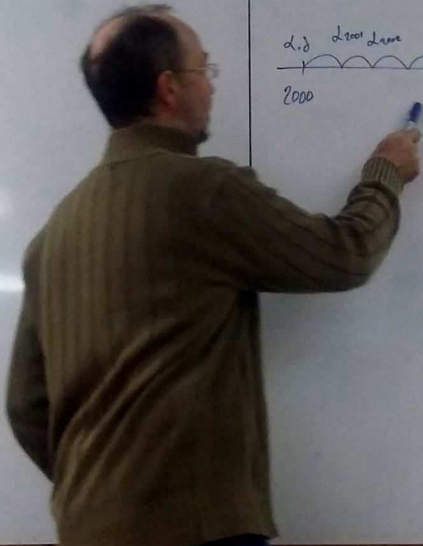
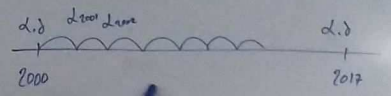
LUNISOLAR ($d\beta=0$) \rightarrow $\begin{cases} d\alpha = \psi \cdot z (\cos \epsilon + m \cdot \epsilon \cdot \sin \alpha \sin \delta) \\ d\delta = \psi \cdot z \cdot m \cdot \epsilon \cdot \cos \alpha \end{cases}$
 PLANETARIA ($d\delta=0$) \rightarrow $\begin{cases} d\alpha = -\lambda' \cdot z \\ d\delta = 0 \end{cases}$

GENERAL : LUNIS + PLANETARIA

$$\begin{cases} d\alpha = m \cdot z + m \cdot z \cdot \epsilon \cdot \sin \alpha \sin \delta \\ d\delta = m \cdot z \cdot \cos \alpha \end{cases}$$

"CTES" PRECESIONALES

$$\begin{aligned} m &= \psi \cdot \cos \epsilon - \lambda' \\ m &= \psi \cdot \cos \epsilon \end{aligned}$$



PRECESIÓN

- LUNISOLAR ($d\beta=0$)

$$\begin{cases} d\alpha = \psi \cdot z (\cos \epsilon + m \sin \epsilon \sin \delta) \\ d\delta = \psi \cdot z \sin \epsilon \cos \delta \end{cases}$$
- PLANETARIA ($d\delta=0$)

$$\begin{cases} d\alpha = -\lambda' \cdot z \\ d\delta = 0 \end{cases}$$

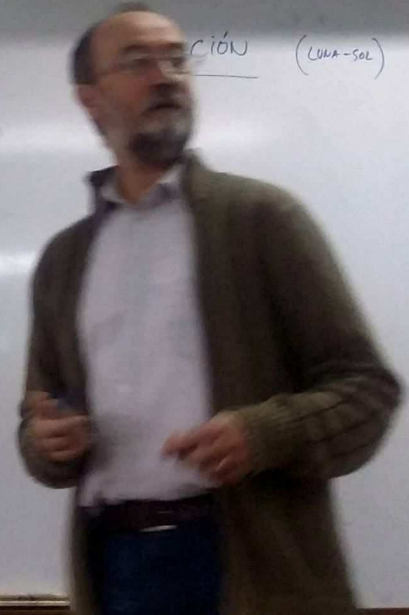
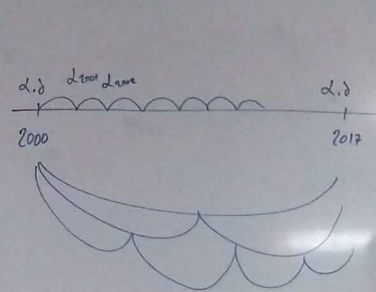
GENERAL : LUNIS + PLANETARIA

$$\begin{cases} d\alpha = m \cdot z + m' \cdot z \cdot \sin \epsilon \sin \delta \\ d\delta = m \cdot z \cdot \cos \delta \end{cases}$$

"CTES"
PRECESIONALES

$$m = \psi \cdot \cos \epsilon - \lambda'$$

$$m' = \psi \cdot \sin \epsilon$$



PRECESIÓN

LUNISOLAR $(d\beta=0)$ $\rightarrow \begin{cases} d\alpha = \psi \cdot z (\cos \epsilon + m \cdot \epsilon \cdot \sin \alpha) \\ d\delta = \psi \cdot z \cdot m \cdot \epsilon \cdot \cos \alpha \end{cases}$
 PLANETARIA $(d\beta=0)$ $\rightarrow \begin{cases} d\alpha = -\chi' \cdot z \\ d\delta = 0 \end{cases}$

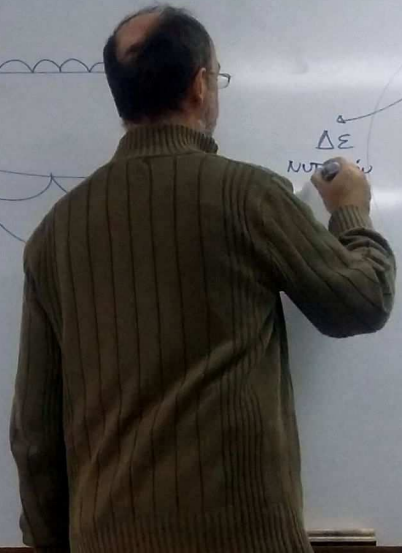
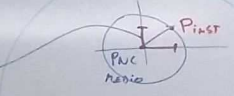
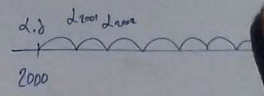
GENERAL : LUNIS + PLANETARIA

$$\begin{cases} d\alpha = m \cdot z + m \cdot z \cdot m \cdot \epsilon \cdot \sin \alpha \\ d\delta = m \cdot z \cdot \epsilon \cdot \cos \alpha \end{cases}$$

"CTES" PRECESIONALES

$$\begin{aligned} m &= \psi \cdot \cos \epsilon - \chi' \\ m &= \psi \cdot m_2 \cdot \epsilon \end{aligned}$$

NUTACIÓN (LUNA-SOL)



$$\begin{cases} \text{LUNISOLAR} \\ (d\beta=0) \end{cases} \rightarrow \begin{cases} d\alpha = \psi \cdot z (\cos \epsilon + m \cdot \epsilon \cdot \sin \alpha \frac{1}{\cos \delta}) \\ d\delta = \psi \cdot z \cdot m \cdot \epsilon \cdot \sin \alpha \end{cases}$$

$$\begin{cases} \text{PLANETARIA} \\ (d\delta=0) \end{cases} \rightarrow \begin{cases} d\alpha = -\lambda' \cdot z \\ d\delta = 0 \end{cases}$$

: LUNIS + PLANETARIO

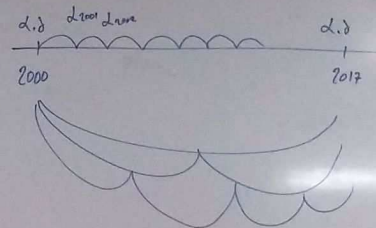
$$= (\overset{m}{m}) z + (\overset{m}{m}) z \cdot m \cdot \epsilon \cdot \sin \alpha \frac{1}{\cos \delta}$$

$$= (\overset{m}{m}) z \cdot \cos \alpha$$

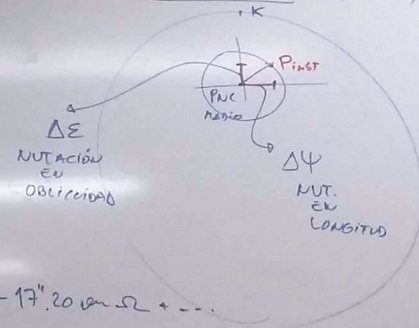
RESIDUALES

$$m = \psi \cdot \cos \epsilon - \lambda'$$

$$m = \psi \cdot m_2 \cdot \epsilon$$



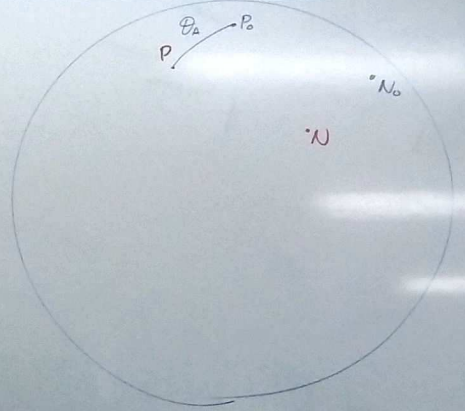
NUTACIÓN (LUNA-SOL)



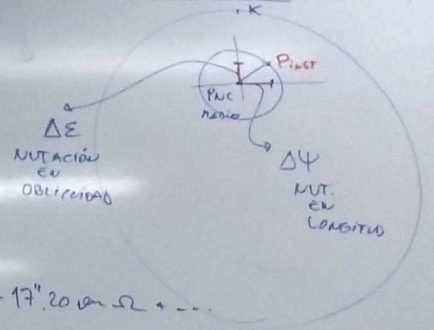
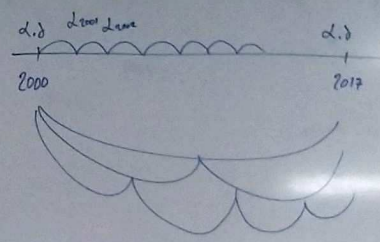
$$\Delta \psi = -17'' \cdot 20 \sin \Omega + \dots$$

$$\Delta \epsilon = 9'' \cdot 2 \cdot \cos \Omega + \dots$$

Fórm. Rigurosas Prec. Gral



NUTACIÓN (LUNA-SOL)



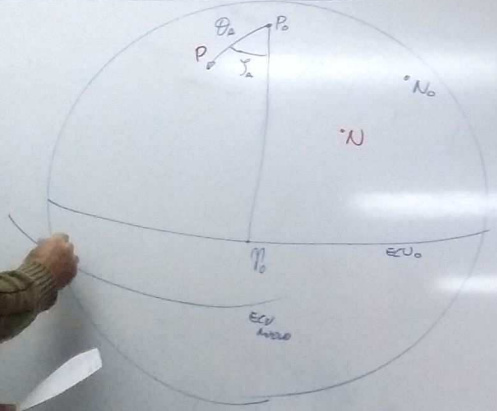
ARIES MEDIO \rightarrow PRECESIÓN
 " INSTANTÁNEO \rightarrow P + NUTACIÓN

$$\Delta\psi = -17'' \cdot 20 \sin \Omega + \dots$$

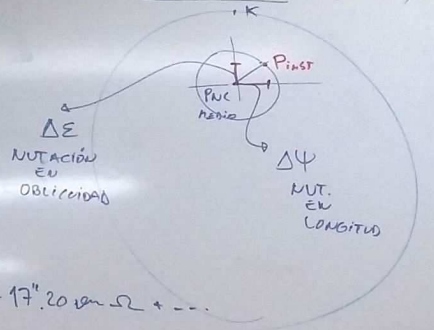
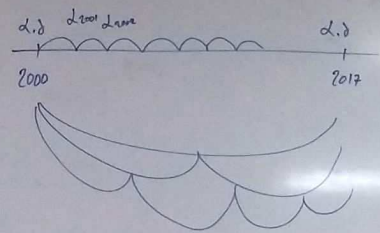
$$\Delta\epsilon = 9'' \cdot 2 \cdot \cos \Omega + \dots$$

FÓRM. RIGUROSAS PREC. GRAL

$\theta_n =$
 $\Sigma_n =$



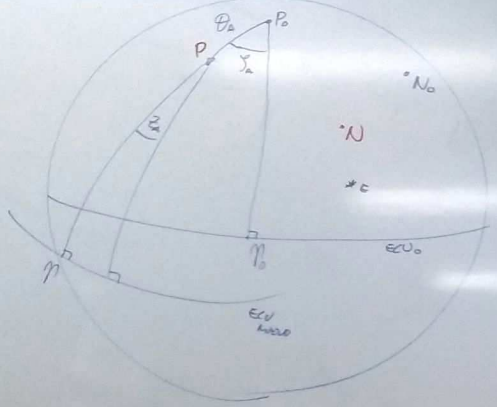
NUTACIÓN (LUNA-SOL)



ARIES MEDIO \rightarrow PRECESIÓN
 " INSTANTÁNEO \rightarrow P + NUTACIÓN

$\Delta\psi = -17''.20 \sin \Omega + \dots$
 $\Delta\epsilon = 9''.2 \cos \Omega + \dots$

FÓRM. RIGUROSAS PREC. GRAL

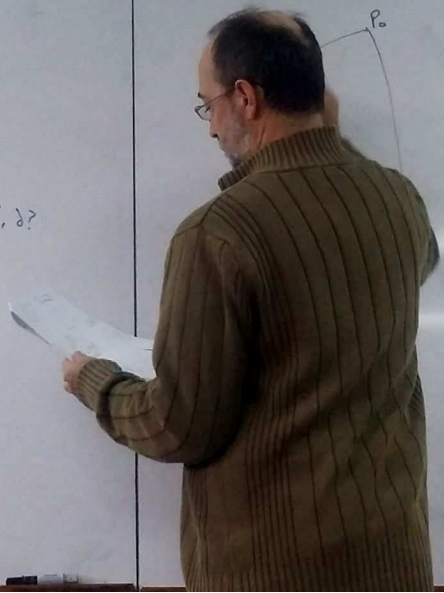


$$\theta_A =$$

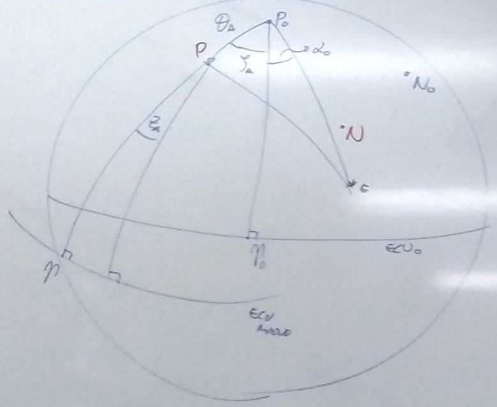
$$\delta_A =$$

$$z_A =$$

$$z_c, \delta_0 \rightarrow z, \delta?$$



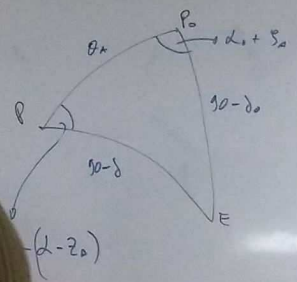
FÖRM. RIGUROSAS PREC. GRAL



$\theta_A =$
 $\gamma_A =$
 $z_A =$

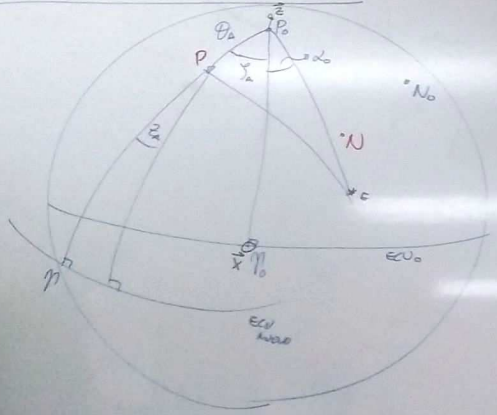
$(\alpha, \delta) \rightarrow i, \alpha, \delta$

$X_0 = \cos \delta_0 \cdot \cos \alpha_0$
 $Y_0 = \cos \delta_0 \cdot \sin \alpha_0$
 $Z_0 = \sin \delta_0$



F. COSAU } ⇒ (α, δ) EN FUNCIÓN DE (α₀, δ₀)
 SENU

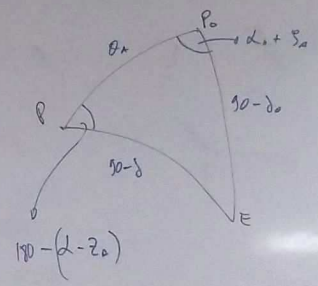
FÓRM. RIGUROSAS PREC. GERAL



$\theta_A =$
 $\gamma_A =$
 $z_A =$

$(\alpha_0, \delta_0) \rightarrow j \alpha, \delta?$

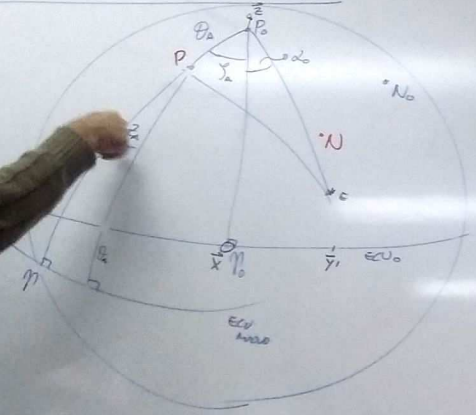
$x_0 = \cos \delta_0 \cdot \cos \alpha_0$
 $y_0 = \cos \delta_0 \cdot \sin \alpha_0$
 $z_0 = \sin \delta_0$



F. (cos ec) } $\Rightarrow (\alpha, \delta)$ EN FUNCIÓN DE (α_0, δ_0)

$R_z(-\gamma_A) \cdot \bar{x}_0$

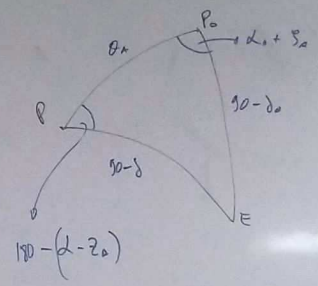
FÖRM. RIGUROSAS PREC. GRAL



$\theta_A =$
 $\gamma_A =$
 $z_A =$

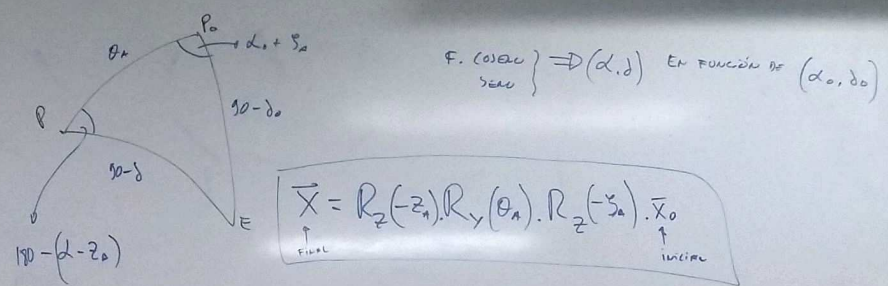
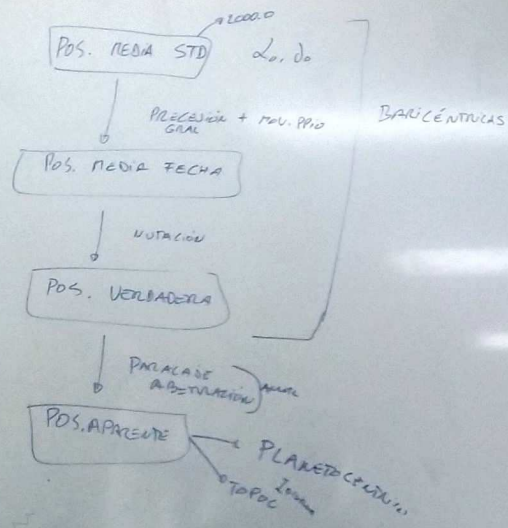
$(\alpha, \delta) \rightarrow \gamma, \delta?$

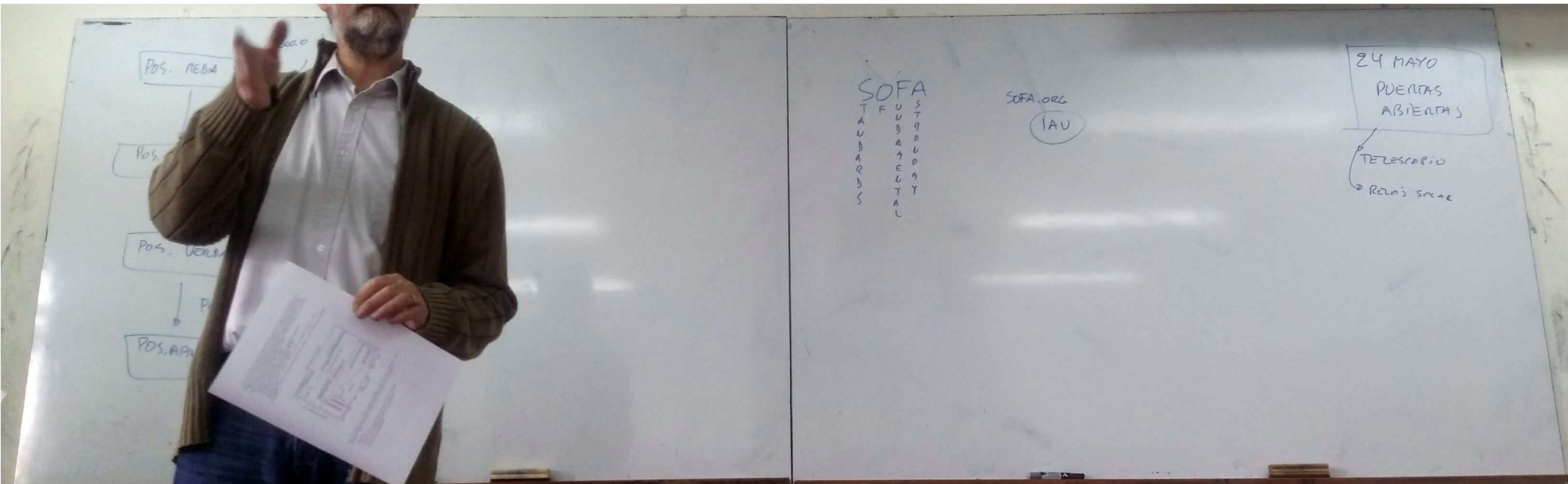
$x_0 = \cos \delta_0 \cdot \cos \alpha_0$
 $y_0 = \cos \delta_0 \cdot \sin \alpha_0$
 $z_0 = \sin \delta_0$



F. COS ECU } $\Rightarrow (\alpha, \delta)$ EN FUNCIÓN DE (α_0, δ_0)
 SENA

$R_z(-z_A) \cdot R_y(\theta_A) \cdot R_z(-\gamma_A) \cdot \bar{x}_0$





POS. MEDIA

POS.

POS. VERNA

POS. ARA

S
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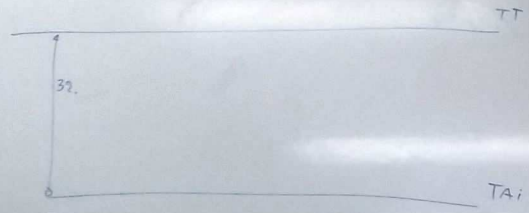
T
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SOFA.ORG

IAU

24 MAYO
PUERTAS
ABIERTAS

TELESCOPIO
RELOJ SOLAR



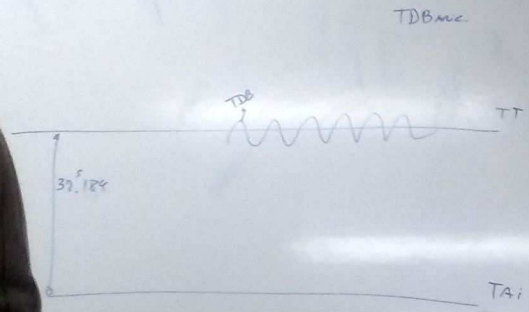
SOFA
STANDARD
FUNDAMENTAL
T A U
S

SOFA.ORG

IAU

24 MAYO
PUERTAS
ABIERTAS

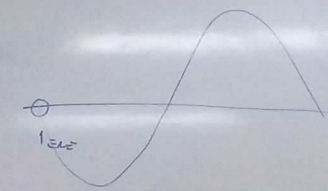
TELESCOPIO
RELOJ SOLAR



SOFA
STANDARD
FUNDAMENTAL
S

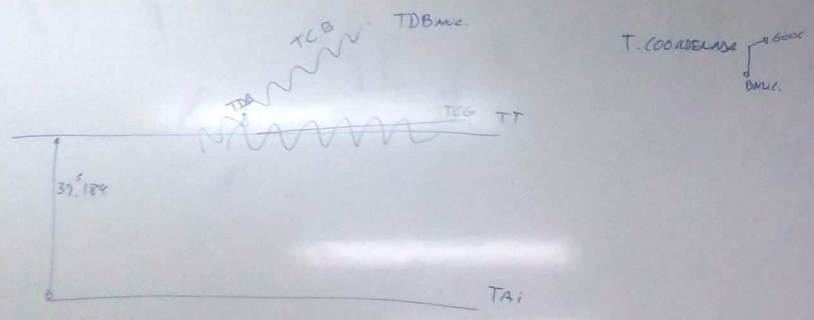
SOFA.ORG

IAU



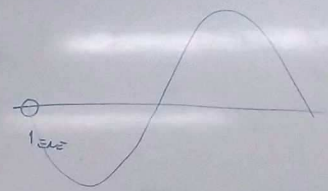
24 MAYO
PUERTAS
ABIERTAS

- TELESCOPIO
- RELOJ SOLAR



SOFA
T F U N D A M E N T A L
S T A N D A R D S

SOFA.ORG
IAU



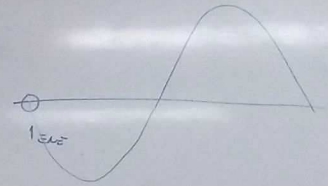
24 MAYO
PUERTAS
ABIERTAS

TELESCOPIO
Reloj solar



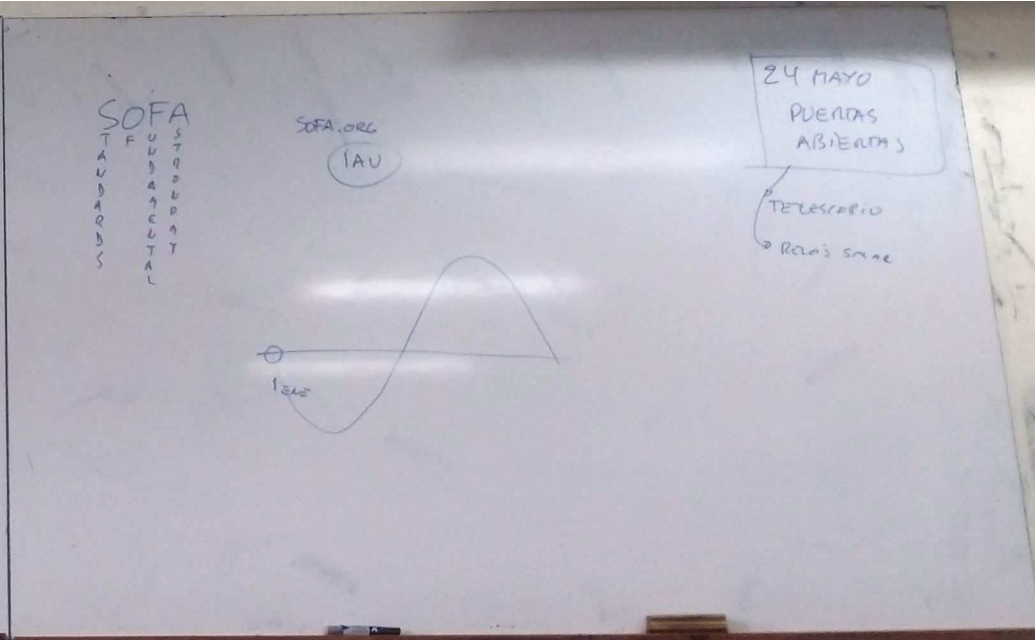
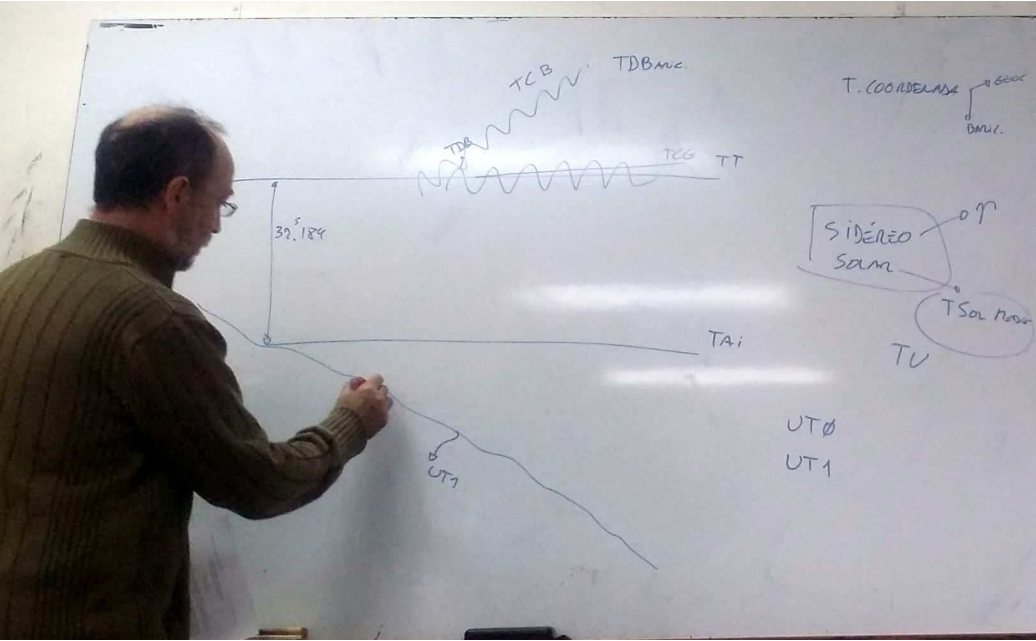
SOFA
 T F U S
 A U D S
 V D A T
 A R D S
 S T Q D U P H Y
 F U N D A M E N T A L

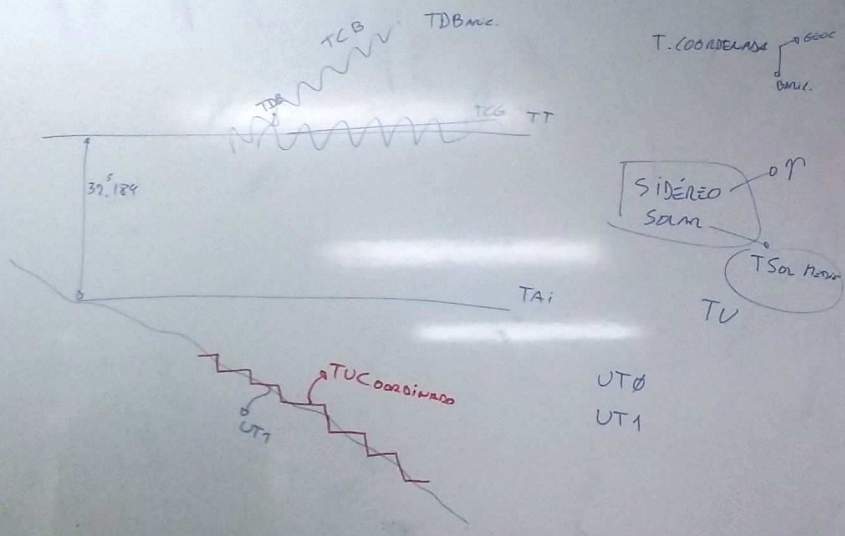
SOFA.ORG
 IAU



24 MAYO
 PUERTAS
 ABIERTAS

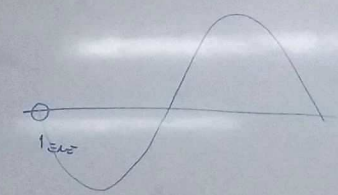
TELESCOPIO
 ROJO SOLAR





SOFA
 STANDARDS
 FUNDAMENTAL

SOFA.ORG
 IAU



24 MAYO
 PUERTAS
 ABIERTAS

- TELESCOPIO
- Relo's solar

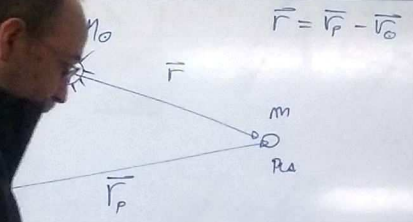
MOV. Y CONFIG. PLANETARIAS



Fijo
○
x

m
○
Ra

MOV. Y CONFIG. PLANETARIAS

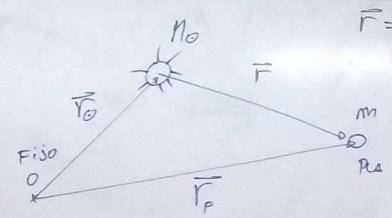


$$\vec{r} = \vec{r}_P - \vec{r}_0$$

$$m \ddot{\vec{r}}_P = - \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

MOV. Y CONFIG. PLANETARIAS



$$\vec{r} = \vec{r}_P - \vec{r}_0$$

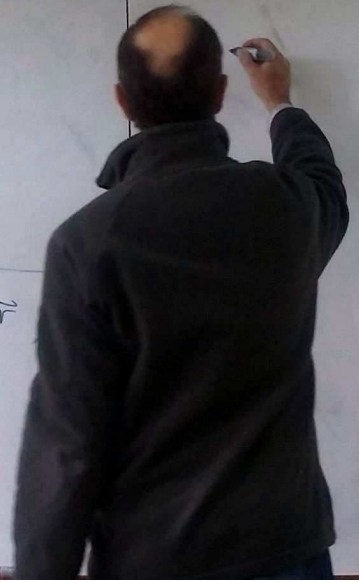
$$m \ddot{\vec{r}}_P = - \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

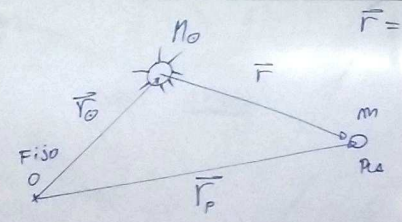
RESTO

$$\ddot{\vec{r}} (\ddot{\vec{r}}_P - \ddot{\vec{r}}_0) = - \frac{G(M_0 + m)}{r^3} \vec{r}$$

$$\Rightarrow \ddot{\vec{r}} = - \frac{G(M_0 + m)}{r^3} \vec{r}$$



MOV. Y CONFIG. PLANETARIAS



$$\vec{r} = \vec{r}_p - \vec{r}_0$$

$$m \ddot{\vec{r}}_p = - \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

RESID

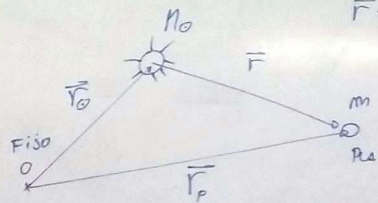
$$\ddot{\vec{r}} = \ddot{\vec{r}}_p - \ddot{\vec{r}}_0 = - \frac{G(M_0 + m)}{r^3} \vec{r}$$

$$\Rightarrow \ddot{\vec{r}} = - \frac{G(M_0 + m)}{r^3} \vec{r} \Rightarrow \ddot{\vec{r}} = - \frac{\mu}{r^3} \vec{r}$$

$$\mu = G(M_0 + m) \approx GM_0 = k^2 = (0.01720209895)^2$$

GAUSS
UA
DIA
M₀

MOV. Y CONFIG. PLANETARIAS



$$\vec{r} = \vec{r}_p - \vec{r}_0$$

$$M m \ddot{\vec{r}}_p = - \frac{G M_0 m}{r^2} \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{G M_0 m}{r^2} \frac{\vec{r}}{r}$$

RESTO

$$\ddot{\vec{r}} = - \frac{G (M_0 + m)}{r^3} \vec{r}$$

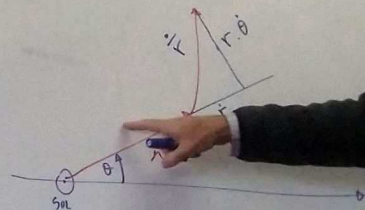
$$\Rightarrow \ddot{\vec{r}} = - \frac{G (M_0 + m)}{r^3} \vec{r} \Rightarrow \ddot{\vec{r}} = - \frac{\mu}{r^3} \vec{r}$$

$$\mu = G (M_0 + m) \approx G M_0 = k^2 = (0.01720209895)^2$$

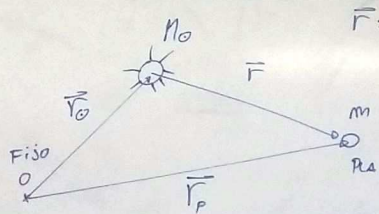
GAUSS
UA
DIA
M₀

$$\vec{r} \wedge \ddot{\vec{r}} = 0 \rightarrow \vec{r} \wedge \dot{\vec{r}} = c \vec{e}_z = \vec{h}$$

~~$\dot{\vec{r}} \wedge \vec{r} + \vec{r} \wedge \ddot{\vec{r}} = 0$~~



MOV. Y CONFIG. PLANETARIAS



$$\vec{r} = \vec{r}_p - \vec{r}_0$$

$$M_0 \ddot{\vec{r}}_p = - \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

RESTO

$$\ddot{\vec{r}} = - \frac{G(M_0 + m)}{r^2} \frac{\vec{r}}{r}$$

$$\Rightarrow \ddot{\vec{r}} = - \frac{G(M_0 + m)}{r^3} \vec{r} \Rightarrow \ddot{\vec{r}} = - \frac{\mu}{r^3} \vec{r}$$

$$\mu = G(M_0 + m) \approx GM_0 = k^2 = (0.01720209895)^2$$

GRAVIT
UA
DIA
M₀

$$\vec{r} \wedge \ddot{\vec{r}} = 0 \xrightarrow{\text{INTEGRAR}} \vec{r} \wedge \dot{\vec{r}} = \vec{c} = \frac{d}{dt} \vec{h} = \vec{r} \wedge \dot{\vec{r}} = \vec{h}$$

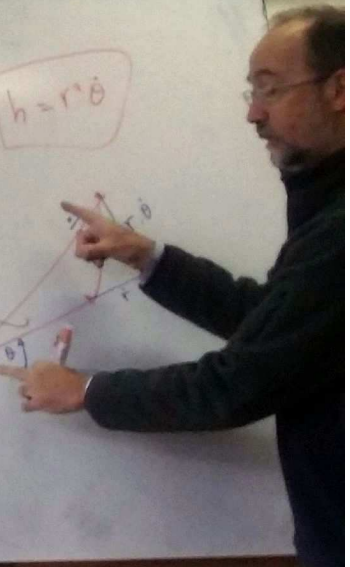
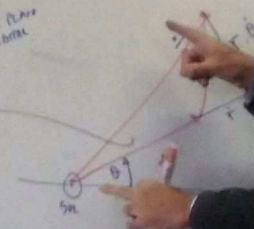
INTEGRA

$$h = r^2 \dot{\theta}$$

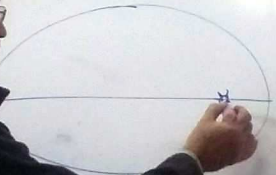
⊥ al plano orbital

$$\frac{dA}{dt} = \frac{r \cdot r \dot{\theta}}{2} = \frac{h}{2}$$

AREA



MOV. Y CONFIG. PLANETARIAS



$$M_P \ddot{\vec{r}}_P = - \frac{GM_0 m}{r^2} \cdot \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

RESTO

$$\ddot{\vec{r}}_P - \ddot{\vec{r}}_0 = - \frac{G(M_0 + m)}{r^3} \vec{r}$$

$$\Rightarrow \ddot{\vec{r}} = - \frac{G(M_0 + m)}{r^3} \vec{r} \Rightarrow \ddot{\vec{r}} = - \frac{\mu}{r^3} \vec{r}$$

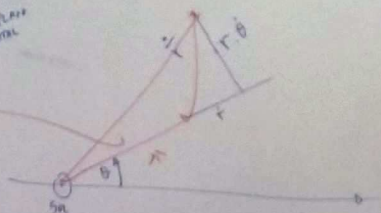
$$\mu = G(M_0 + m) \approx GM_0 = k^2 = (0.01720209895)^2$$

SAVES (pointing to k^2)
LA DÍA M_0 (pointing to M_0)

$$\vec{r} \wedge \ddot{\vec{r}} = 0 \xrightarrow{\text{INTEGRAL}} \vec{r} \wedge \dot{\vec{r}} = \vec{c} = \vec{h} = r \cdot r \cdot \dot{\theta} \hat{z}$$

LA PLANA ORBITAL (pointing to \hat{z})

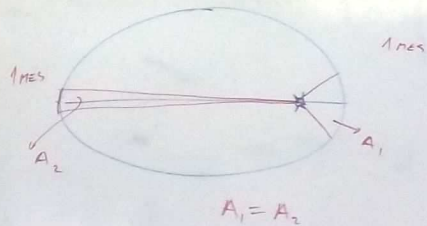
$$h = r^2 \dot{\theta}$$



$$\frac{dA}{dt} = \frac{r \cdot r \cdot \dot{\theta}}{2} = \frac{h}{2}$$

2ª LEY KEPLER

MOV. Y CONFIG. PLANETARIAS



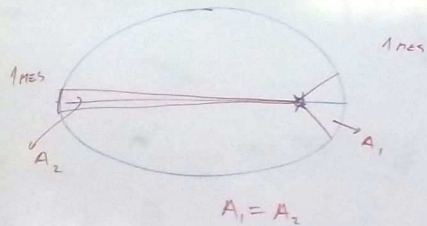
$$M_P \ddot{\vec{r}}_P = - \frac{GM_0 m}{r^2} \cdot \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

$$\text{RESID} \quad \ddot{\vec{r}}_P - \ddot{\vec{r}}_0 = - \frac{G(M_0 + m)}{r^3} \vec{r}$$

$$\Rightarrow \ddot{\vec{r}} = - \frac{G(M_0 + m)}{r^3} \vec{r} \Rightarrow \ddot{\vec{r}} = - \frac{\mu}{r^3} \vec{r}$$

MOV. Y CONFIG. PLANETARIAS



$$M_P \ddot{\vec{r}}_P = - \frac{GM_0 m}{r^2} \cdot \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

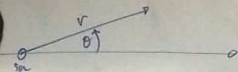
$$\text{RESID} \quad \ddot{\vec{r}}_P - \ddot{\vec{r}}_0 = - \frac{G(M_0 + m)}{r^3} \vec{r}$$

$$\Rightarrow \ddot{\vec{r}} = - \frac{G(M_0 + m)}{r^3} \vec{r} \Rightarrow \ddot{\vec{r}} = - \frac{\mu}{r^3} \vec{r}$$

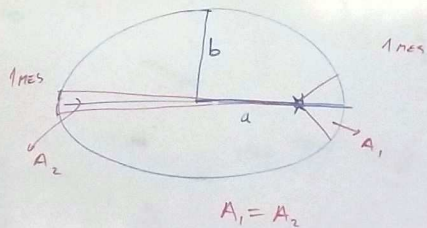
SOLUCION

$$r = \frac{h^2/\mu}{1 + e \cdot \cos \theta}$$

Cónica



MOV. Y CONFIG. PLANETARIAS



$A_1 = A_2$

$$M_P \ddot{\vec{r}}_P = - \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{GM_0 m}{r^2} \frac{\vec{r}}{r}$$

RESTO

$$\ddot{\vec{r}} = - \frac{G(M_0 + m)}{r^3} \vec{r}$$

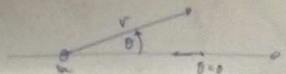
$$\Rightarrow \ddot{\vec{r}} = - \frac{G(M_0 + m)}{r^3} \vec{r} \Rightarrow \ddot{\vec{r}} = - \frac{\mu}{r^3} \vec{r}$$

Solución

$$r = \frac{h^2/\mu}{1 + e \cos \theta}$$

Cónica

e - EXCENTRICIDAD



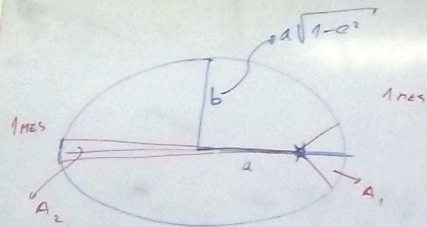
Perihelio

$$r_{\min} = \frac{a(1-e)}{1+e} = \frac{a(1-e)}{1+e}$$

$$\frac{h^2}{\mu} = a(1-e^2)$$

$$r = \frac{a(1-e^2)}{1 + e \cos \theta}$$

MOV. Y CONFIG. PLANETARIAS



$$\frac{b}{a} = \sqrt{1-e^2}$$

$A_1 = A_2$

$$M_P \ddot{\vec{r}}_P = - \frac{GM_0 M_P}{r^2} \cdot \frac{\vec{r}}{r}$$

$$M_0 \ddot{\vec{r}}_0 = + \frac{GM_0 M_P}{r^2} \frac{\vec{r}}{r}$$

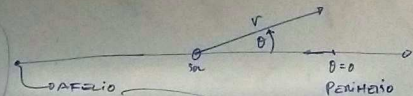
$$\text{RESID} \quad \ddot{\vec{r}}_P - \ddot{\vec{r}}_0 = - \frac{G(M_0 + M_P)}{r^3} \vec{r}$$

$$\Rightarrow \ddot{\vec{r}} = - \frac{G(M_0 + M_P)}{r^3} \vec{r} \Rightarrow \ddot{\vec{r}} = - \frac{\mu}{r^3} \vec{r}$$

SOLUCION

$$r = \frac{h^2/\mu}{1 + e \cdot \cos \theta}$$

Cónica



$$\frac{h^2}{\mu} = a(1-e^2)$$

$$r = \frac{a(1-e^2)}{1 + e \cdot \cos \theta}$$

$$r_{\min} = \frac{a(1-e)}{1+e} = \frac{a(1-e)}{1+e}$$

$$r_{\max} = \frac{a(1-e)}{1-e} = \frac{a(1+e)}{1-e}$$

Y CONFIG. PLANETARIAS



1 mes

$$2 \frac{dA}{dt} = h = \sqrt{\mu a (1-e^2)}$$

$$\frac{\pi a b}{T} \Rightarrow \frac{2\pi a \sqrt{1-e^2}}{T} = \sqrt{\mu a (1-e^2)}$$

PERÍODO ORB.

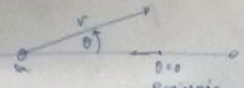
SOLUCION

$$r = \frac{h^2/\mu}{1 + e \cos \theta}$$

Cónica

$$\frac{h^2}{\mu} = a(1-e^2)$$

$$r = \frac{a(1-e^2)}{1 + e \cos \theta}$$



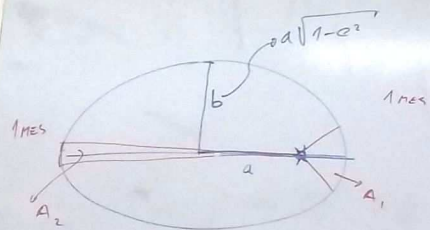
PERÍODO
E - EXCENTRICIDAD

$$r_{\min} = \frac{a(1-e)}{1+e} = \frac{a(1-e)}{1+e}$$

PERÍODO

$$r_{\max} = \frac{a(1-e)}{1-e} = \frac{a(1+e)}{1-e}$$

MOV. Y CONFIG. PLANETARIAS



$\frac{b}{a} = \sqrt{1-e^2}$

$A_1 = A_2$

1 mes

$2 \frac{dA}{dt} = h = \sqrt{4a(1-e^2)}$

$\frac{\pi a b}{T} \Rightarrow \frac{2\pi a \sqrt{1-e^2}}{T}$

PERIODO ORB.

$\Rightarrow \frac{2\pi}{T} = \frac{\sqrt{1-e^2}}{a}$

$\left(\frac{2\pi}{T}\right)^2 = \frac{1}{a^3}$

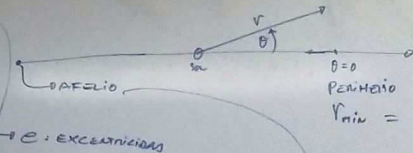
$M = \frac{2\pi}{T}$

MOV. MEDIO
RADS / DÍA

$M = \sqrt{1/a^3}$

SOLUCION

$r = \frac{h^2/\mu}{1 + e \cdot \cos \theta}$



e : EXCENTRICIDAD

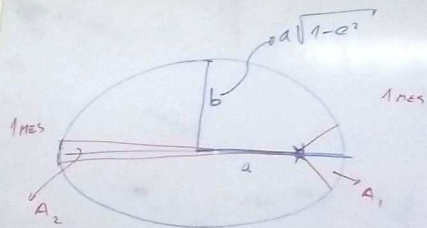
$r = \frac{a(1-e^2)}{1 + e \cdot \cos \theta}$

$r_{min} = \frac{a(1-e)}{1+e} = \frac{a(1-e)}{1+e}$

$r_{max} = \frac{a(1-e)}{1-e} = \frac{a(1+e)}{1-e}$

PERIHELIO Q

MOV. Y CONFIG. PLANETARIAS



$\frac{b}{a} = \sqrt{1-e^2}$

$A_1 = A_2$

$M = \sqrt{\mu/a^3}$

MOV. MEDIO
RADS/OIA

$M = \frac{2\pi}{T}$

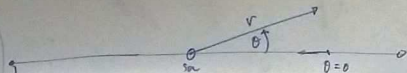
$\Rightarrow \frac{2\pi}{T} = \frac{\sqrt{\mu a}}{a^2}$

$\left(\frac{2\pi}{T}\right)^2 = \frac{\mu}{a^3}$

$\frac{2dA}{dt} = h = \sqrt{\mu a(1-e^2)}$
 $\frac{\pi ab}{T} \Rightarrow \frac{2\pi a \sqrt{1-e^2}}{T} = \sqrt{\mu a(1-e^2)}$

SOLUCION

$r = \frac{h^2/\mu}{1+e \cdot \cos \theta}$



e: EXCENTRICIDAD

$\frac{T^2}{a^3} = \frac{(2\pi)^2}{\mu} \rightarrow G(M_0 + m)$

$r = \frac{a(1-e^2)}{1+e \cdot \cos \theta}$

$a_T = 1 \text{ ua}$

$T_T = 1 \text{ a} \Rightarrow T = a^{3/2}$

$r_{\min} = \frac{a(1-e)}{1+e} = \frac{a(1-e)}{1+e}$

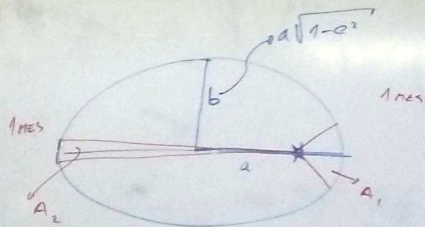
$r_{\max} = \frac{a(1-e)}{1-e} = \frac{a(1+e)}{1-e}$

PERIHELIO

Q

Q

MOV. Y CONFIG. PLANETARIAS



$\frac{b}{a} = \sqrt{1-e^2}$

$A_1 = A_2$

$M = \sqrt{\mu/a^3}$

$M = \frac{2\pi}{T}$
MOV. MEDIO
RADS/OIA

$\frac{2dA}{dt} = h = \sqrt{4a(1-e^2)}$

$\frac{\pi ab}{T} \Rightarrow \frac{2\pi a \sqrt{1-e^2}}{T} = \sqrt{4a(1-e^2)}$

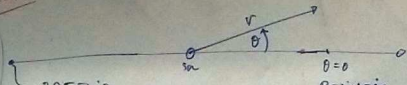
$\Rightarrow \frac{2\pi}{T} = \frac{\sqrt{\mu a}}{a^2}$

$\left(\frac{2\pi}{T}\right)^2 = \frac{\mu}{a^3}$

SOLUCION

$r = \frac{h^2/\mu}{1+e \cdot \cos \theta}$

1ª LEY



e : EXCENTRICIDAD

$r_{min} = \frac{a(1-e)}{1+e} = \frac{a(1-e)}{2}$

PERIHELIO

3ª LEY KEPLER

$\frac{T^2}{a^3} = \frac{(2\pi)^2}{\mu} \rightarrow G(M_0+m)$

$r = \frac{a(1-e^2)}{1+e \cdot \cos \theta}$

$r_{max} = \frac{a(1-e)}{1-e} = \frac{a(1+e)}{2}$

Q

$a_T = 102$

$T_T = 1.2 \text{ años}$

$T = a^{3/2}$

DUAS
AÑOS

JUPITER $a_J = 5.2$

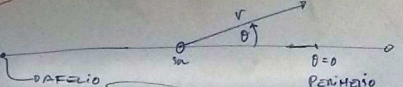
$T_J =$

$$\vec{r} = -\frac{\mu}{r^3} \vec{r}$$

$$\ddot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^3} \dot{\vec{r}} \cdot \dot{\vec{r}}$$

SOLUCION 1ª LEY

$$r = \frac{h^2/\mu}{1 + e \cdot \cos \theta}$$



AFELIO
e: EXCENTRICIDAD

PERIHELIO

$$r_{\min} = \frac{a(1-e)}{1+e} = a(1-e) \quad q$$

$$r_{\max} = \frac{a(1-e)}{1-e} = a(1+e) \quad Q$$

3ª LEY KEPLER

$$\frac{T^2}{a^3} = \frac{(2\pi)^2}{\mu} \rightarrow G(M_0 + m)$$

$$r = \frac{a(1-e^2)}{1 + e \cdot \cos \theta}$$

$a_T = 1 \text{ ua}$
 $T_T = 1 \text{ año}$

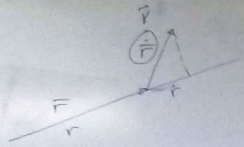
$$T = a^{3/2}$$

→ años

JUPITER $a_J = 5.2$
 $T_J =$

$$\vec{r} = -\frac{\mu}{r^3} \vec{r}$$

$$\ddot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^3} \dot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^2} \dot{r}$$



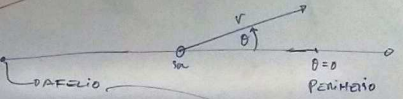
INTEGRAR:

$$\frac{\dot{r}^2}{2} = +\frac{\mu}{r} + CTE$$

$$\frac{1}{2} v^2 - \frac{\mu}{r} = CTE$$

SOLUCION 1ª LEY

$$r = \frac{h^2/\mu}{1 + e \cdot \cos \theta}$$



e: EXCENTRICIDAD

PERIHELIO

$$r_{min} = \frac{a(1-e)}{1+e} = a(1-e)$$

3ª LEY KEPLER

$$\frac{T^2}{a^3} = \frac{(2\pi)^2}{\mu} \rightarrow G(M_0 + m)$$

$$r = \frac{a(1-e^2)}{1 + e \cdot \cos \theta}$$

APHELIO

$$r_{max} = \frac{a(1-e)}{1-e} = a(1+e)$$

$a_T = 104$
 $T_T = 11.86$

$$T = a^{3/2}$$

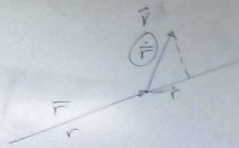
DUAS
 AÑOS

JUPITER $a_J = 5.2$
 $T_J =$



$$\vec{r} = -\frac{\mu}{r^3} \vec{r}$$

$$\ddot{\vec{r}} \cdot \vec{r} = -\frac{\mu}{r^3} \vec{r} \cdot \vec{r} = -\frac{\mu}{r^2}$$



INTEGRAR:

$$\frac{\dot{r}^2}{2} = +\frac{\mu}{r} + CTE$$

$$\frac{1}{2} v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$\frac{1}{2} v^2 - \frac{\mu}{r} = CTE$$

$$E_c + E_{pot} = E_{tot} = -\frac{\mu}{2a}$$

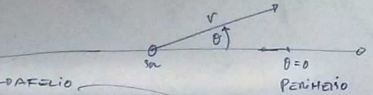
SE PRESERVA

$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

SOLUCION

$$r = \frac{h^2/\mu}{1 + e \cdot \cos \theta}$$

1ª LEY



AFELIO
e: EXCENTRICIDAD

PERIHELIO

$$r_{min} = \frac{a(1-e)}{1+e} = a(1-e)$$

9

$$r_{max} = \frac{a(1-e)}{1-e} = a(1+e)$$

Q

3ª LEY KEPLER

$$\frac{T^2}{a^3} = \frac{(2\pi)^2}{\mu} \rightarrow G(M_0 + m)$$

$$r = \frac{a(1-e^2)}{1 + e \cdot \cos \theta}$$

$$a_T = 104$$

$$T_T = 1.2 \text{ años}$$

$$T = a^{3/2}$$

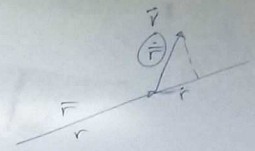
0 años
DUAS

JUPITER $a_J = 5.2$

$$T_J =$$

$$\ddot{\vec{r}} = -\frac{\mu}{r^3} \vec{r}$$

$$\ddot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^3} (\dot{\vec{r}} \cdot \dot{\vec{r}}) = -\frac{\mu}{r^2} \dot{r}$$



INTEGRADO:

$$\frac{\dot{r}^2}{2} = +\frac{\mu}{r} + CTE$$

$$\frac{1}{2} \dot{r}^2 - \frac{\mu}{r} = CTE$$

$$E_C + E_{Pot} = E_{Tot} = -\frac{\mu}{2a}$$

SE PUEDE

$$\frac{1}{2} v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$$

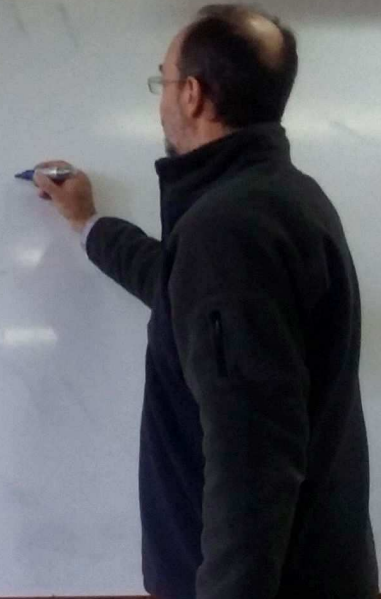
$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

si $r = a$
v. circular

"ENERGIA" TOTAL

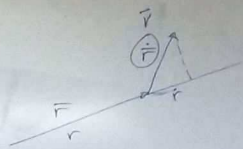
$$E = \frac{1}{2} v^2 - \frac{\mu}{r}$$

* si $E < 0 \Rightarrow r < \infty$



$$\ddot{\vec{r}} = -\frac{\mu}{r^3} \vec{r}$$

$$\ddot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^3} (\dot{\vec{r}} \cdot \dot{\vec{r}}) = -\frac{\mu}{r^2} \dot{r}$$



INTEGRAR:

$$\frac{\dot{r}^2}{2} = +\frac{\mu}{r} + CTE$$

$$\frac{1}{2} \dot{r}^2 - \frac{\mu}{r} = CTE$$

$$E_C + E_{Pot} = E_{Tot} = -\frac{\mu}{2a}$$

SE PROBEA

$$\frac{1}{2} \dot{r}^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$$

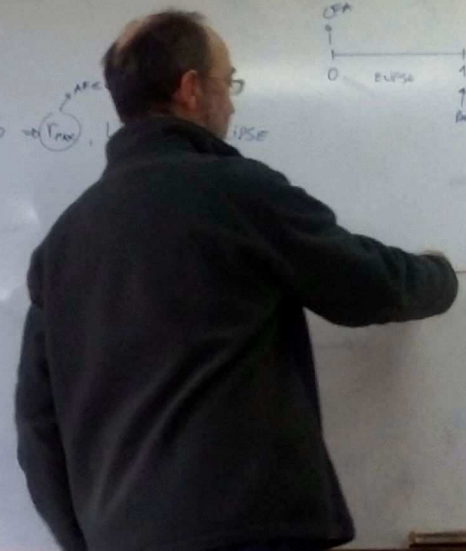
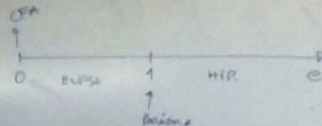
$$\dot{r}^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

si $r = a$
 $\dot{r}^2 = \frac{\mu}{a}$
 v. circular

"ENERGIA" TOTAL

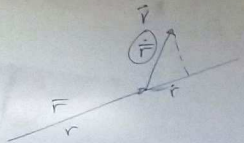
$$E = \frac{1}{2} \dot{r}^2 - \frac{\mu}{r}$$

* si $E < 0 \Rightarrow r < \infty \Rightarrow r_{max}$



$$\ddot{\vec{r}} = -\frac{\mu}{r^3} \vec{r}$$

$$\ddot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^3} \dot{\vec{r}} \cdot \vec{r} = -\frac{\mu}{r^2}$$



INTEGRADO:

$$\frac{\dot{r}^2}{2} = +\frac{\mu}{r} + CTE$$

$$\frac{1}{2} v^2 - \frac{\mu}{r} = CTE$$

$$E_c + E_{pot} = E_{tot} = -\frac{\mu}{2a}$$

SE PRESERVA

$$\frac{1}{2} v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$$

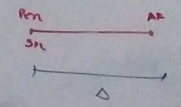
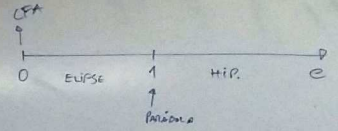
$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

si $r \equiv a$
v. circular
 $v^2 = \frac{\mu}{a}$

"ENERGÍA" TOTAL

$$E = \frac{1}{2} v^2 - \frac{\mu}{r}$$

* si $E < 0 \Rightarrow r < \infty \Rightarrow r_{max}$, LIBADO, ELIPSE

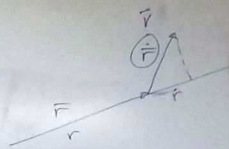


$$T_{orbit} = \frac{2\pi a^{3/2}}{\mu}$$

$$D = \frac{384,000}{150,106}$$

$$\ddot{\vec{r}} = -\frac{\mu}{r^3} \vec{r}$$

$$\ddot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^3} \dot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^2} \dot{r}$$



INTEGRADO:

$$\frac{\dot{r}^2}{2} = +\frac{\mu}{r} + CTE$$

$$\frac{1}{2} \dot{r}^2 - \frac{\mu}{r} = CTE$$

$$E_C + E_{Pot} = E_{Tot} = -\frac{\mu}{2a}$$

SE PROEVA

$$\frac{1}{2} \dot{r}^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$\dot{r}^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

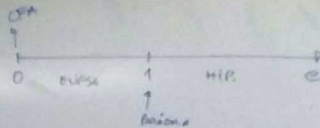
"ENERGIA" TOTAL

$$E = \frac{1}{2} \dot{r}^2 - \frac{\mu}{r}$$

* si $E < 0 \Rightarrow r < \infty \Rightarrow r_{max}$, LIBADO, ELIPSE

* si $E > 0 \Rightarrow r$ AUMENTA HASTA $\infty \Rightarrow$ NO LIBADO, HIPÉRBOLA ($\dot{r}(r=\infty) \neq 0$)

* si $E = 0 \Rightarrow r$ AUMENTA HASTA ∞ , PERO $\dot{r}(r=\infty) = 0 \Rightarrow$ PARÁBOLA



$$\vec{r} = -\frac{\mu}{r^2} \hat{r}$$

$$\ddot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^3} \dot{\vec{r}} \cdot \dot{\vec{r}} = -\frac{\mu}{r^2} \dot{r}$$

INTEGRADO:

$$\frac{\dot{r}^2}{2} = +\frac{\mu}{r} + CTE$$

$$\frac{1}{2} v^2 - \frac{\mu}{r} = CTE$$

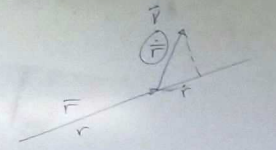
$$E_C + E_{Pot} = E_{Tot} = -\frac{\mu}{2a}$$

se conserva

$$\frac{1}{2} v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

si $r = a$
 $v^2 = \frac{\mu}{a}$
 v. circular



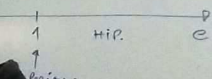
"ENERGÍA" TOTAL

$$E = \frac{1}{2} v^2 - \frac{\mu}{r}$$

* si $E < 0 \Rightarrow r < \infty \Rightarrow r_{max}$, LIBADO, ELIPSE

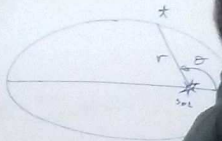
* si $E > 0 \Rightarrow r$ puede ser $\infty \Rightarrow$ NO LIBADO, HIPERBOLA

* si $E = 0 \Rightarrow r$ puede ser ∞ , PERO $v(r=\infty) =$

$$E = -\frac{\mu}{2a}$$


DADO PLANETA m, a, e

$$r^2 \dot{\theta} = h$$



$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

"ENERGÍA" TOTAL

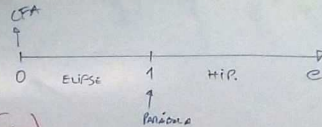
$$\mathcal{E} = \frac{1}{2} v^2 - \frac{\mu}{r}$$

* si $\mathcal{E} < 0 \Rightarrow r < \infty \Rightarrow r_{\text{max}}$, **LIGADO**, **ELIPSE** ($a > 0$)

* si $\mathcal{E} > 0 \Rightarrow r$ puede ser $\infty \Rightarrow$ **NO LIGADO**, **HIPIERABOLA** ($N(r=\infty) \neq 0$) ($a < 0$)

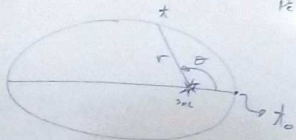
* si $\mathcal{E} = 0 \Rightarrow r$ puede ser ∞ , pero $N(r=\infty) = 0 \Rightarrow$ **PARABOLA** ($a = \infty$)

$$\mathcal{E} = -\frac{\mu}{2a}$$



DADO PLANETA m, a, e

$t_0 = \text{INST. PASAJE PERIHELIO}$



$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

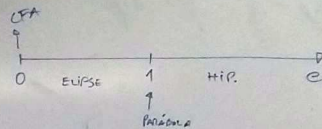
$$r^2 \dot{\theta} = h \rightarrow \theta(t)$$

SOLUCIÓN

$$\theta = M + 2e \cos M + \dots$$

AU. MEDIA

$$M = \frac{2\pi}{P} (t - t_0)$$

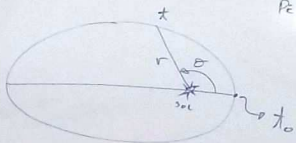


ELIPSE $(r(r=\infty) \neq 0)$ $(a < \infty)$

$(r=\infty) = 0 \Rightarrow \text{Parábola}$ $(a = \infty)$

DADO PLANETA m, a, e

$t_0 = \text{INST. PASAJE PERIHELIO}$



$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

$$r^2 \dot{\theta} = h \rightarrow \theta(t)$$

SOLUCIÓN

$$\theta = M + 2e \sin M + \dots$$

AN. MEDIA

NUMERICA

$$M = m \cdot (t - t_0)$$

MÁS

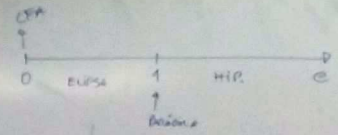
$$M = E - e \sin E$$

FORMULA

EC. KEPLER

→ E → OBTEN

r =

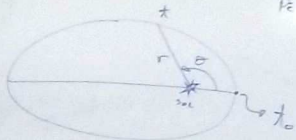


ELIPSE $(\sqrt{r=0} \neq 0)$ $(a < 0)$

$(\infty) = 0 \Rightarrow \text{PARABOLA}$ $(a = \infty)$

DADO PLANETA m, a, e

$t_0 = \text{inst. PASAJE PERIHELIO}$



$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

$$r^2 \dot{\theta} = h \rightarrow \theta(t)$$

SOLUCIÓN

$$\theta = M + 2e \sin M + \dots$$

AU. MEDIA

$$M = m \cdot (t - t_0)$$

NUMÉRICA

$$M = E - e \sin E$$

EXCÉNTRICA

EC. KEPLER

$\rightarrow E \rightarrow$ OBTENGO r DE:

$$r = a(1 - e \cos E)$$

DADO $t \rightarrow$ OBSTACLO θ, r
 \downarrow
 r, θ

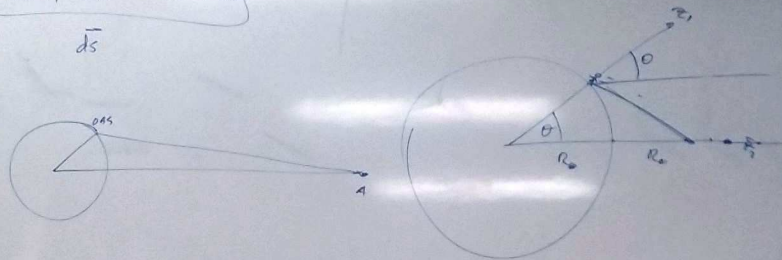
POSICIÓN (r, θ) EN SU ÓRBITA

¿POSICIÓN EN EL ESPACIO?

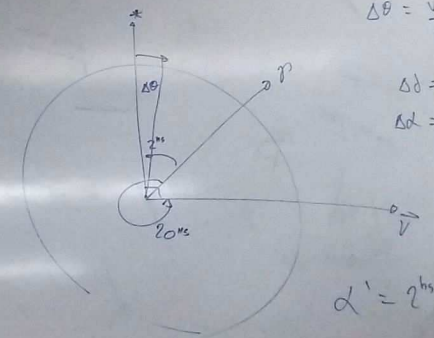
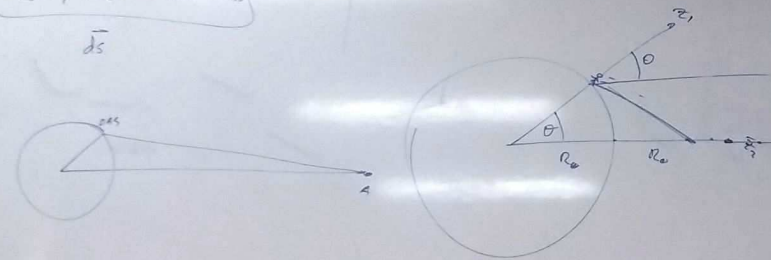
F. O*

$\Delta\alpha, \Delta\delta$ $P_{M} + \Delta S$

\bar{dS}



F. O'F
 $\Delta\alpha, \Delta\delta$ Par + AS
 \vec{ds}



$$\Delta\theta = \frac{v}{c} \sin\theta$$

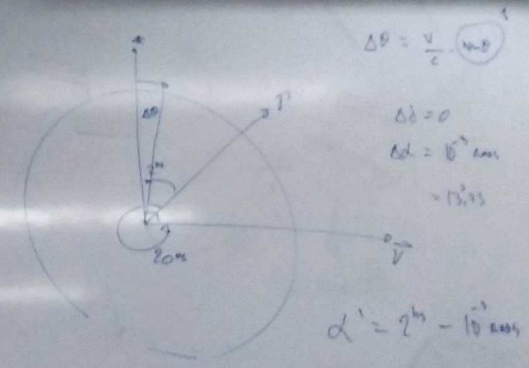
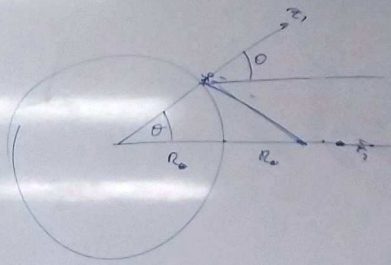
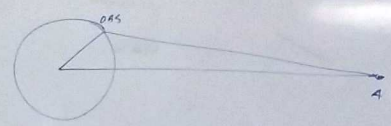
$$\Delta\delta = 0$$

$$\Delta\alpha = \dots$$

$$\alpha' = 2^{\text{hs}} - 10^{-3} \text{ rads}$$

$$10^{-3}$$

F. O.F.
 $\Delta\alpha, \Delta\delta$ Par + A.S.
 \bar{ds}



$$\Delta\delta = \frac{v}{c} \sin\theta$$

$$\Delta\delta = 0$$

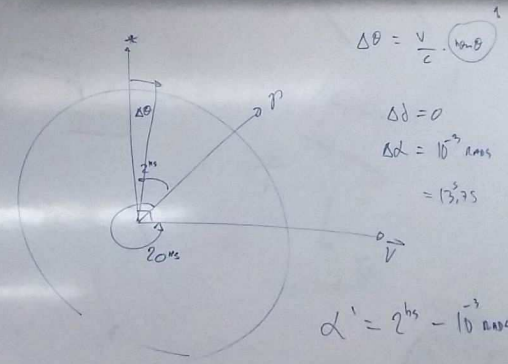
$$\Delta\alpha = 10^{\circ} \sin\theta$$

$$= 13,75$$

$$\alpha' = 2^{\text{h}} - 10^{\circ} \sin\theta$$

$$\alpha' = 1^{\text{h}} 59^{\text{m}} 46,25^{\text{s}}$$

$$10^{\circ} \sin\theta = 10 \cdot \frac{180}{\pi} \cdot \frac{1}{15} \sin\theta$$



$$\Delta\theta = \frac{v}{c} \cdot \sin\alpha$$

$$\Delta\theta = 0$$

$$\Delta\alpha = 10^{-3} \text{ rads}$$

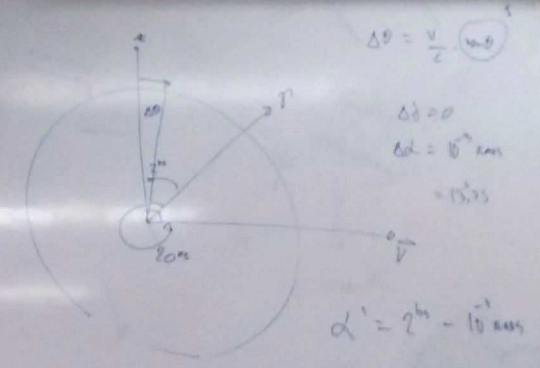
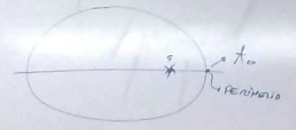
$$= 13,35$$

$$\alpha' = 2^{\text{hs}} - 10^{-3} \text{ rads}$$

$$\alpha' = 1^{\text{h}} 59^{\text{m}} 46,25^{\text{s}} \quad 10^{-3} \text{ rads} = \frac{10^{-3} \cdot 180}{\pi} \cdot \frac{1}{15} \text{ arcsec}$$

a, e, t_0

DADO t :



$$\Delta \theta = \frac{v}{r} \Delta t$$

$$\Delta \theta = 0$$

$$\Delta t = 10^2 \text{ ans}$$

$$= 15,75$$

$$\alpha' = 2^{\text{hs}} - 10^{\text{ans}}$$

$$\alpha' = 1^{\text{hs}} 59^{\text{m}} 46,25^{\text{s}}$$

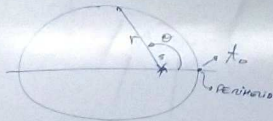
$$10^2 \text{ ans} = \frac{10^2 \cdot 180}{\pi} \frac{1}{15} \text{ ans}$$

$$\mu = \frac{GM}{a^3} (M_0 + M_1)$$

a, e, t_0, m

DADO t :

MOV. MEDIO $M = \sqrt{\frac{\mu}{a^3}} t$

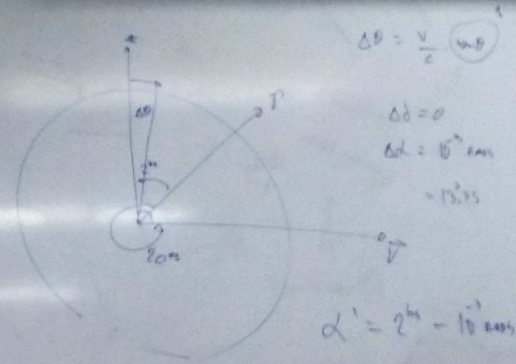


ANOM. MEDIA: $M = m(t - t_0)$

$$\theta = M + 2e \sin M + \dots$$

$$r = \frac{a(1-e^2)}{1+e \cos \theta}$$

$$M = E - e \sin E \Rightarrow E \Rightarrow r = a(1 - e \cos E)$$



$$\Delta \theta = \frac{v}{c} \sin \theta$$

$$\Delta \theta = 0$$

$$\Delta \alpha = 10^\circ \text{ ems}$$

$$= 15,75$$

$$\alpha' = 2^\circ - 10^\circ \text{ ems}$$

$$\alpha' = 1^\circ 50' 46,75''$$

$$10^\circ \text{ ems} = \frac{10 \cdot 180}{\pi} \cdot \frac{1}{15} \text{ emms}$$

$\mu = \frac{G}{h^2} (M_1 + M_2) \approx h^2 \mu_{10} = h^2$

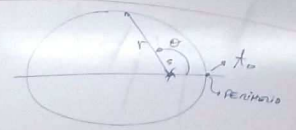
a, e, t_0, m

$h^2 = G$ ou $\frac{G}{M_1}$

DADO t :

MOV. MEDIO $M = \sqrt{\mu} \frac{t}{a^3}$

ANOM. MEDIA: $M = m(t - t_0)$

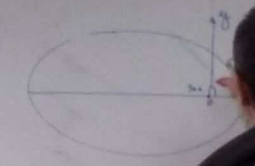


OGTENCIOS
 $r(t)$
 $\theta(t)$

$\theta = M + 2e \sin M + \dots$

$r = \frac{a(1-e^2)}{1+e \cos \theta}$

$M = E - e \sin E \Rightarrow E \Rightarrow r = a(1 - e \cos E)$



$\mu = \frac{G}{k^2} (M_0 + M_1) \approx \frac{G}{k^2} M_0 = k^2$

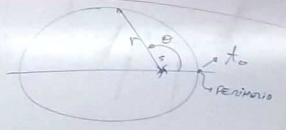
$k^2 = G$ ou
dias
 M_0

a, e, t_0, m

DADO t :

MOV. MEDIO $M = \sqrt{\mu} t/a^3$

ADDM. MEDIA: $M = m(t - t_0)$

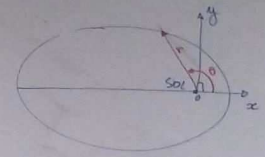


OBTENEMOS
 $r(t)$
 $\theta(t)$

$\theta = M + 2e \sin M + \dots$

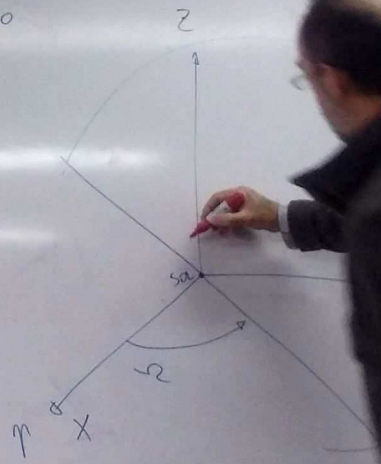
$M = E - e \cos E \Rightarrow E \Rightarrow r = a(1 - e \cos E)$

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$



$x = r \cos \theta$
 $y = r \sin \theta$
 $z = 0$

XY \equiv PLANO
ECLIPTICO



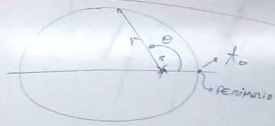
$\mu = \frac{G}{k^2} (M_0 + m) \approx k^2 \mu_0 = k^2$
 $k^2 = G$ ua días M_0

a, e, t_0, m

DADO t :

MOV. MEDIO $M = \sqrt{\mu} t/a^3$

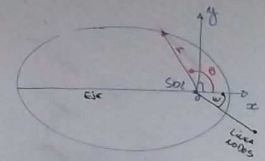
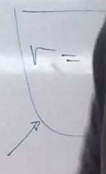
ANOM. MEDIA: $M = m(t - t_0)$



$r(t)$
 $\theta(t)$

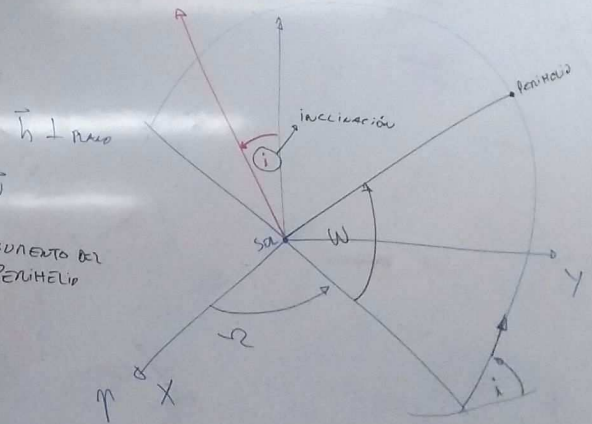
$\theta = M + 2e \sin M + \dots$

$M = E - e \cos E \Rightarrow E \Rightarrow r = a(1 - e \cos E)$



$x = r \cos \theta$
 $y = r \sin \theta$
 $z = 0$

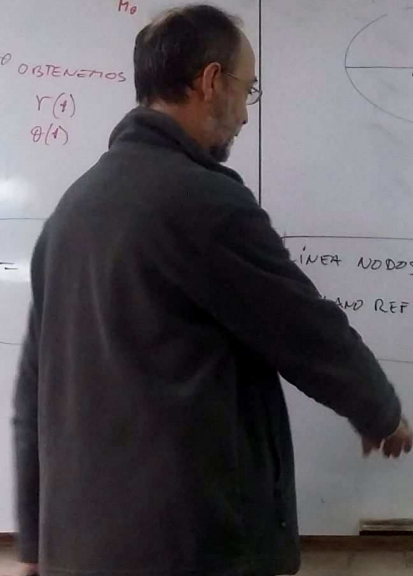
$XY \equiv$ PLANO ECLIPTICO
 $\Omega =$ LONGITUD DEL NUDO ASCENDENTE



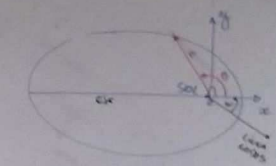
$\vec{h} = \vec{r} \wedge \vec{v}$

LINEA NODOS \equiv
 PLANO REF \cap PLANO ORBITAL

W = ARGUMENTO DEL PERHELIO



PASAJE DE $(\alpha, \delta, 0) \rightarrow (X, Y, Z)$
 ↑ ↑
 DRAGON EQUINOX

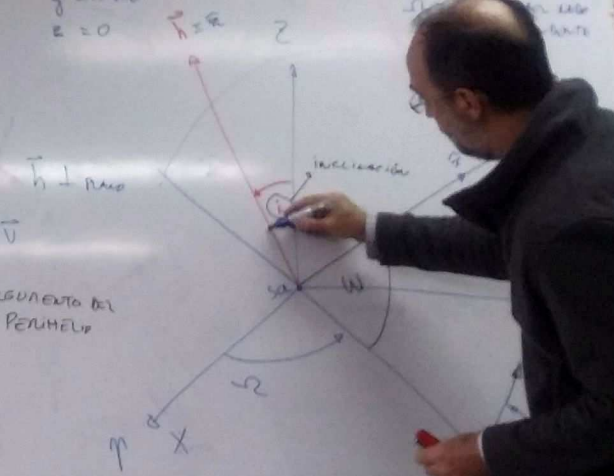


$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = 0$$

XY = PLANO
 ECLIPTICO



$$\vec{h} = \vec{r} \wedge \vec{v}$$

LÍNEA NODOS =
 PLANO REF \cap PLANO ORBITAL

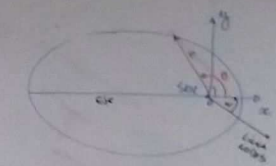
ω = ARGUMENTO DEL PERHELIO



PASAJE DE $(z, y, 0) \rightarrow (x, y, z)$
 ↑ ↑
 ORBITAL ECCLIPTICA

$$R_x(-i) \cdot R_z(-w) \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

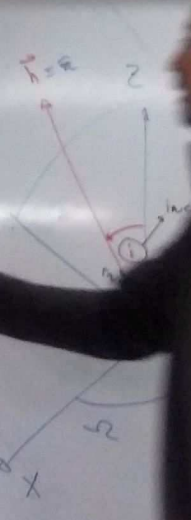
↑
ω



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = 0$$



$$\vec{h} \perp \text{PLANO}$$

$$\vec{h} = \vec{r} \wedge \vec{v}$$

LÍNEA NODOS =
 PLANO REF \cap PLANO ORBITAL

ω = ARGUMENTO DEL PERIHELIO

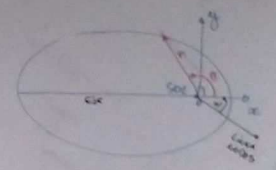
= PLANO ECCLIPTICA
 SISTEMA SOLAR
 ASCENDENTE

EFEMÉRIDES

(x, y, z)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_z(-\omega) \cdot R_x(-i) \cdot R_z(-\Omega) \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

COORD. RECT. ECLIPTICAS HELIOCÉNTRICAS
 $\hat{X} \equiv \Upsilon$



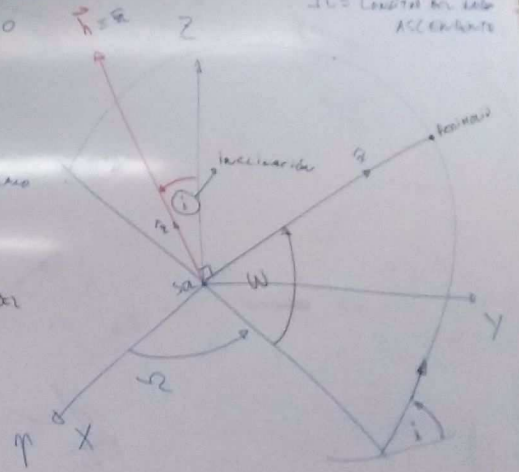
$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \\ z &= 0 \end{aligned}$$

XY = PLANO ECLÍPTICA
 Ω = LONGITUD DEL NUDO ASCENDENTE

LINEA NODOS =
 PLANO REF \cap PLANO ORBITAL

$$\vec{h} = \vec{r} \wedge \vec{v}$$

W = ARGUMENTO DEL PERHELIO



EFEMÉRIDES

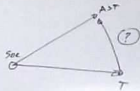
(X, Y, Z) ASTEROIDE

(X_T, Y_T, Z_T) TIERRA

$(X - X_T, Y - Y_T, Z - Z_T)$

COORD. GEOCÉNTRICAS

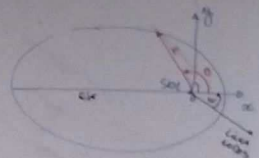
$\rightarrow \alpha, \delta$



$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R_z(-\Omega) \cdot R_x(-i) \cdot R_z(-\omega) \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

COORD. RECT. ECLÍPTICAS HELIOCÉNTRICAS

$$\hat{X} \equiv \uparrow$$



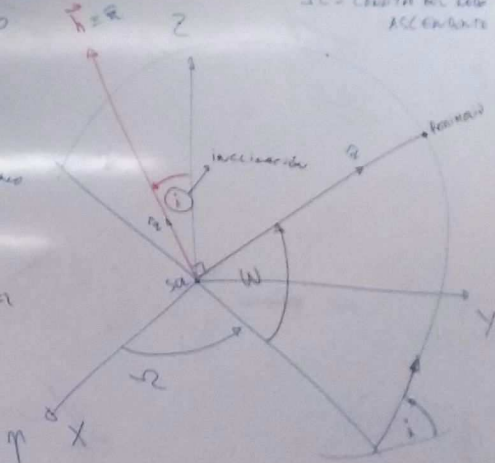
$$\begin{aligned} x &= r \cos u \\ y &= r \sin u \\ z &= 0 \end{aligned}$$

XY = Plano eclíptico
 Ω = Longitude del nodo ASCENDENTE

LINEA NODOS =
 PLANO REF \cap PLANO ORBITAL

$$\vec{h} = \vec{r} \wedge \vec{v}$$

W = ARGUMENTO DEL PERIHELIO



EFEMÉRIDES

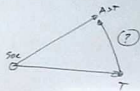
(X, Y, Z) ASTEROIDE

(X_T, Y_T, Z_T) TIERRA

$(X - X_T, Y - Y_T, Z - Z_T)$

COORD. GEOCÉNTRICAS

$\rightarrow \alpha, \beta \text{ e } \alpha', \delta$

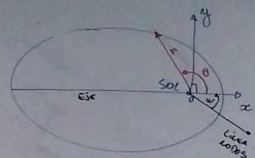
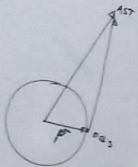


$$= R_z(-\Omega) \cdot R_x(-i) \cdot R_z(-\omega) \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix}$$

(X, Y, Z)

COORD. RECT. ECLÍPTICAS HELIOCÉNTRICAS

$\hat{X} \equiv \Upsilon$



$x = r \cos \theta$

$y = r \sin \theta$

$z = 0$

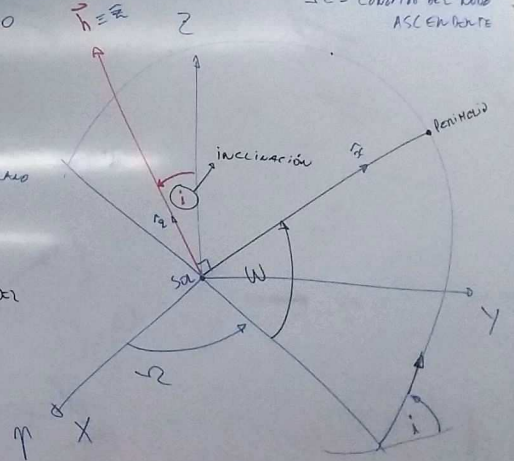
$XY \equiv$ PLANO ECLÍPTICO
 $\Omega =$ LONGITUD DEL NUDO ASCENDENTE

$\vec{h} \perp$ PLANO

$\vec{h} = \vec{r} \wedge \vec{v}$

LÍNEA NODOS \equiv
 PLANO REF \cap PLANO ORBITAL

$\omega =$ ARGUMENTO DEL PERHELIO



EFEMÉRIDES

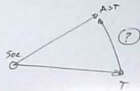
(X, Y, Z) ASTEROIDE

(X_T, Y_T, Z_T) TIERRA

$(X - X_T, Y - Y_T, Z - Z_T)$

COORD. GEOCÉNTRICAS

β ó α, δ

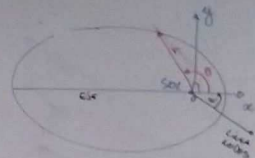
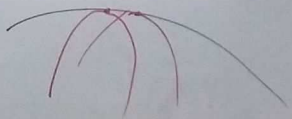
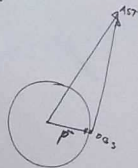


$$= R_z(-\Omega) \cdot R_x(-i) \cdot R_z(-\omega) \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

(X, Y, Z)

COORD. RECT. ECLÍPTICAS HELIOCÉNTRICAS

$$\hat{X} \equiv \tau$$

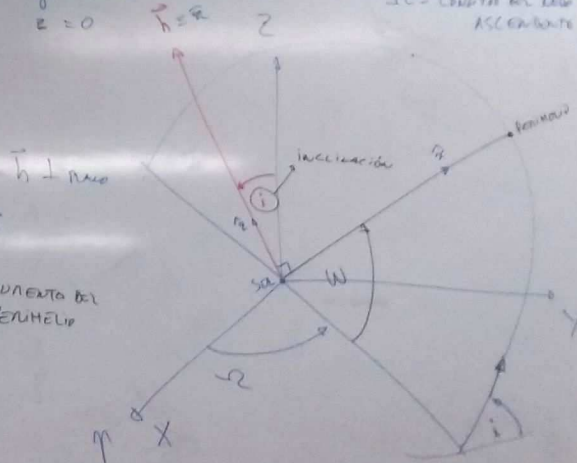


$$x = r \cos \omega$$

$$y = r \sin \omega$$

$$z = 0$$

XY = PLANO ECLÍPTICO
 Ω = LONGITUD DEL NUDO ASCENDENTE



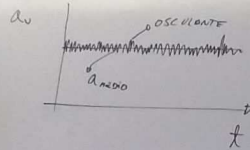
LINEA NODOS =
 PLANO REF \cap PLANO ORBITAL

$$\vec{h} = \vec{r} \wedge \vec{v}$$

W = ARGUMENTO DEL PERHELIO

ELEMENTOS ORBITALES → OSCULANTES (INSTANTÁNEOS PARA CIERTA FECHA)
NODOS
 $a(t), e(t), \dots$

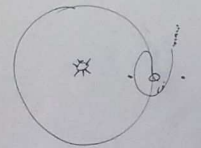
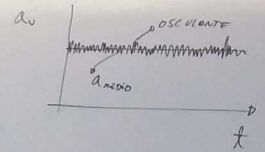
$a, e, i, \Omega, \omega, t_0$ → \vec{r}, \vec{v}
EPOCA J0



ELEMENTOS ORBITALES → OSCULANTES (INSTANTÁNEOS PARA CIERTA FECHA)
 → NODOS
 $a(t), e(t), \dots$

$a, e, i, \Omega, \omega, t_0$ → \vec{r}, \vec{v}
 época J0

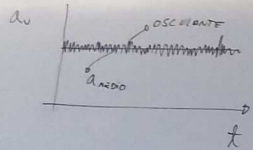
- $a_H = 0.39 \text{ ua}$
- $a_V = 0.72 \text{ ua}$
- $a_T = 1 \text{ ua}$
- $a_M = 1.52 \text{ ua}$



AÑO TRÓPICO : 2 PASAJES POR ARIES

ELEMENTOS ORBITALES → OSCULANTES (INSTANTÁNEOS PARA CIERTA FECHA)
 MEDOS

- $a_M = 0.39 \text{ ua}$
- $a_V = 0.72 \text{ ua}$
- $a_T = 1 \text{ ua}$
- $a_M = 1.52 \text{ ua}$



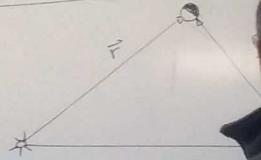
$$\frac{T^2}{a^3} = \text{cte}$$

$a, e, i, \Omega, \omega, t_0$ → r, \dot{r}, \dot{v}
 Época JD

- AÑO TRÓPICO: 2 PASAJES POR ARIES
365.2422 días
- AÑO SIDERO: 365.2564
- AÑO ANOMALÍSTICO: PERIHELIO A PERIHELIO
365.2556
- AÑO CIVIL: 365 o 366
Promedio: 365.2425

DRUSIAS POR 4 →
 11 EUNAS 100 →
 11 EUNAS 400 →

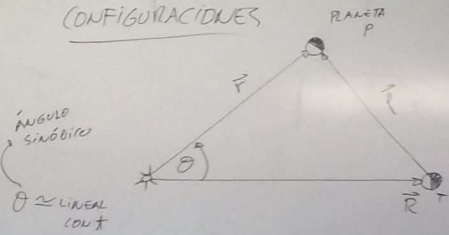
CONFIGURACIONES



- 9000
- Divisible por 4 → bis
 - 1. Entre 100 → no bis.
 - 2. Entre 400 → sí es bisesto

CALENDARIO

CONFIGURACIONES



ÁNGULO SINÓDICO

$\theta \approx$ LINEAL
COUT

$$\dot{\theta} = m_p - m_t = \frac{2\pi}{T_p} - \frac{2\pi}{T_t}$$

$$\frac{2\pi}{S}$$

SINÓDICO

$$\Rightarrow \frac{1}{S} = \frac{1}{T_p} - \frac{1}{T_t}$$

2000

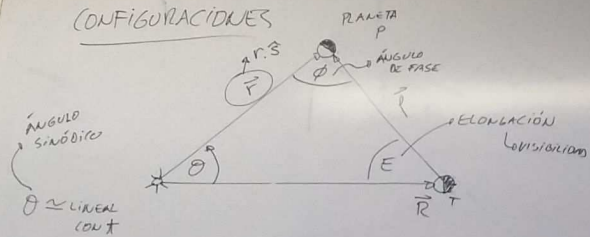
Divisible por 4 \rightarrow bis

1. ENTRE 100 \rightarrow NO BIS.

2. ENTRE 400 \rightarrow SÍ es BISSEXTO

CALENDARIO

CONFIGURACIONES



$$\dot{\theta} = \dot{M}_P - \dot{M}_T = \frac{2\pi}{T_P} - \frac{2\pi}{T_T}$$

$$\frac{2\pi}{S} \Rightarrow \frac{1}{S} = \frac{1}{T_P} - \frac{1}{T_T}$$

lúdico

División de la Tierra → 61.5
 1. CANTIDAD → 42.5
 2. CANTIDAD → 51.5
 CALENDARIO

CONFIGURACIONES



$$\vec{P} = \vec{r} - \vec{R}$$

$$\hat{s} \cdot \hat{r} = \cos \phi$$

$$\cos \phi = \hat{s} \cdot \frac{\vec{r}}{r}$$

$$-\frac{\vec{R}}{R} \cdot \hat{s} = \cos E$$

ANGULO SINODICO
 $\theta \approx$ LINEAL
 CONT

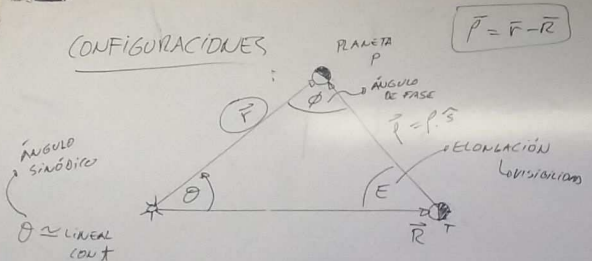
$$\dot{\theta} = \dot{M}_P$$

$$\frac{2\pi}{S}$$

ADICO

- Busque en el \rightarrow SIS
1. entre 100 \rightarrow 120 \rightarrow 130
 2. entre 140 \rightarrow 150 \rightarrow 160
- CALENDARIO

CONFIGURACIONES



$$\vec{P} = \vec{r} - \vec{R}$$

$$\hat{z} \cdot \hat{r} = \cos \phi$$

$$\cos \phi = \hat{z} \cdot \frac{\vec{r}}{r}$$

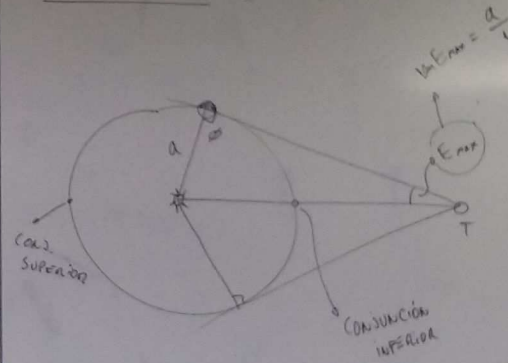
$$-\frac{\vec{R}}{R} \cdot \hat{s} = \cos E$$

$$\dot{\theta} = M_P - M_T = \frac{2\pi}{T_P} - \frac{2\pi}{T_T}$$

$$\frac{2\pi}{S} \Rightarrow \frac{1}{S} = \frac{1}{T_P} - \frac{1}{T_T}$$

MODULO

PLANETA INFERIOR ($a_p < a_T$)



- Busque en 4 \rightarrow 61.5
 1. 40.7100 \rightarrow 40.71.1
 2. 40.71.40 \rightarrow 40.71.40.00

CALENDARIO

VELOS: $a = 0.72$

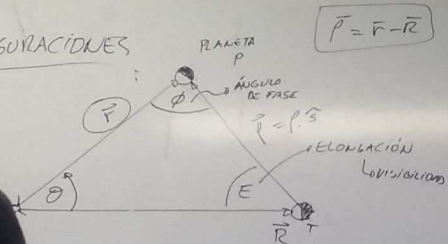
$E_{max} = 46^\circ$

$$\frac{1}{S} = \frac{1}{T_P} - \frac{1}{1}$$

$T^2 = a^3$

$T = a^{3/2}$

CONFIGURACIONES



$$\vec{p} = \vec{r} - \vec{R}$$

$$\zeta \cdot \hat{p} = \cos \phi$$

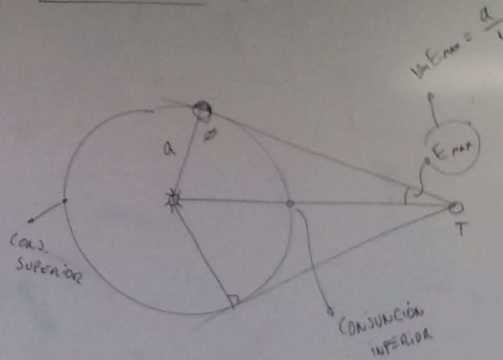
$$\cos \phi = \zeta \cdot \frac{r}{r}$$

$$-\frac{\vec{R}}{R} \cdot \hat{S} = \cos E$$

$$= \frac{2\pi}{T_P} - \frac{2\pi}{T_T}$$

$$\delta = \frac{1}{T_P} - \frac{1}{T_T}$$

PLANETA INFERIOR ($a < a_T$)



- División de 4 → 0.25
1. 0.25 x 100 → 25.00
 2. 0.25 x 100 → 25.00

CALENDARIO

VELOC: $a = 0.71$

$E_{max} = 46^\circ$

$$\frac{1}{S} = \frac{1}{T_V} - \frac{1}{1} = 0.64 \text{ a\u00f1}^{-1}$$

$$\Rightarrow S = 1.57 \text{ a\u00f1os}$$

$$T^2 = a^3$$

$$T = a^{3/2} = 0.61 \text{ a\u00f1os}$$

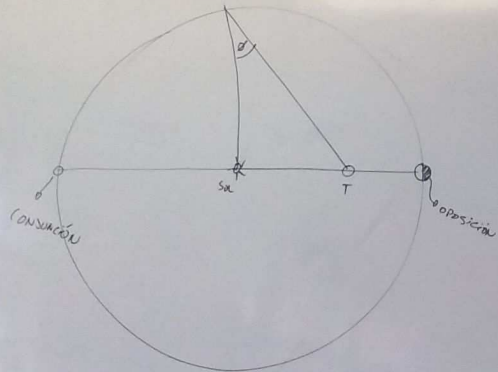
PLANETA SUPERIOR ($a > a_T$)

$$\vec{p} = \vec{r} - \vec{R}$$

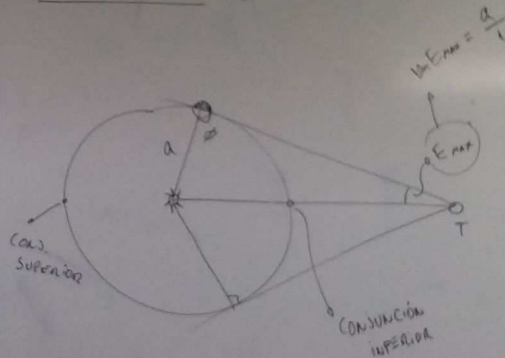
$$\hat{s} \cdot \hat{r} = \cos \phi$$

$$\cos \phi = \hat{s} \cdot \frac{\vec{r}}{r}$$

$$-\frac{\vec{R}}{R} \cdot \hat{s} = \cos E$$



PLANETA INFERIOR ($a < a_T$)



División de 4 → 4/5

1. CONJUNCIÓN → 4/5

2. OPOSICIÓN → 4/5

CALENDARIO

VELOC: $a = 0.72$

$E_{max} = 46^\circ$

$$\frac{1}{S} = \frac{1}{T_p} - \frac{1}{1} = 0.64 \text{ a}^{-1} \Rightarrow S = 1.57 \text{ años}$$

$$T^2 = a^3$$

$$T = a^{3/2} = 0.61 \text{ años}$$

PLANETA SUPERIOR ($a > a_T$)

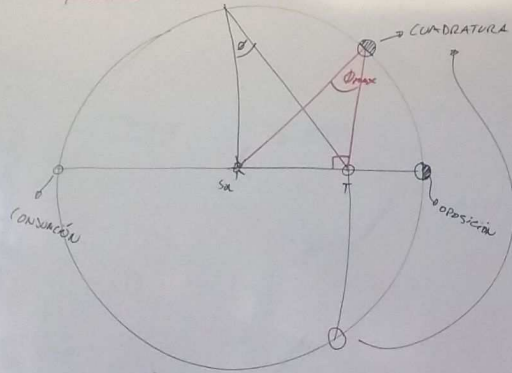
$$\vec{P} = \vec{r} - \vec{R}$$

$$\hat{s} \cdot \hat{r} = \cos \phi$$

$$\cos \phi = \hat{s} \cdot \frac{\vec{r}}{r}$$

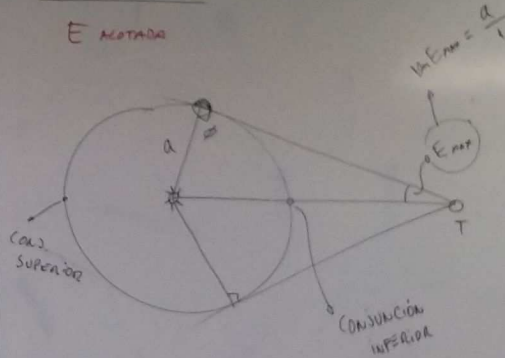
$$-\frac{\vec{R}}{R} \cdot \hat{s} = \cos E$$

ϕ ACOTADO



PLANETA INFERIOR ($a_p < a_T$)

E ACUTADO



Distancia An 4 → 0.15

1. entre 100 → 142 An.

2. entre 400 → 450 An.

CALENDARIO

VELUS: $a = 0.72$

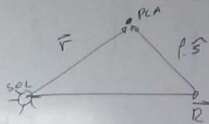
$E_{max} = 46^\circ$

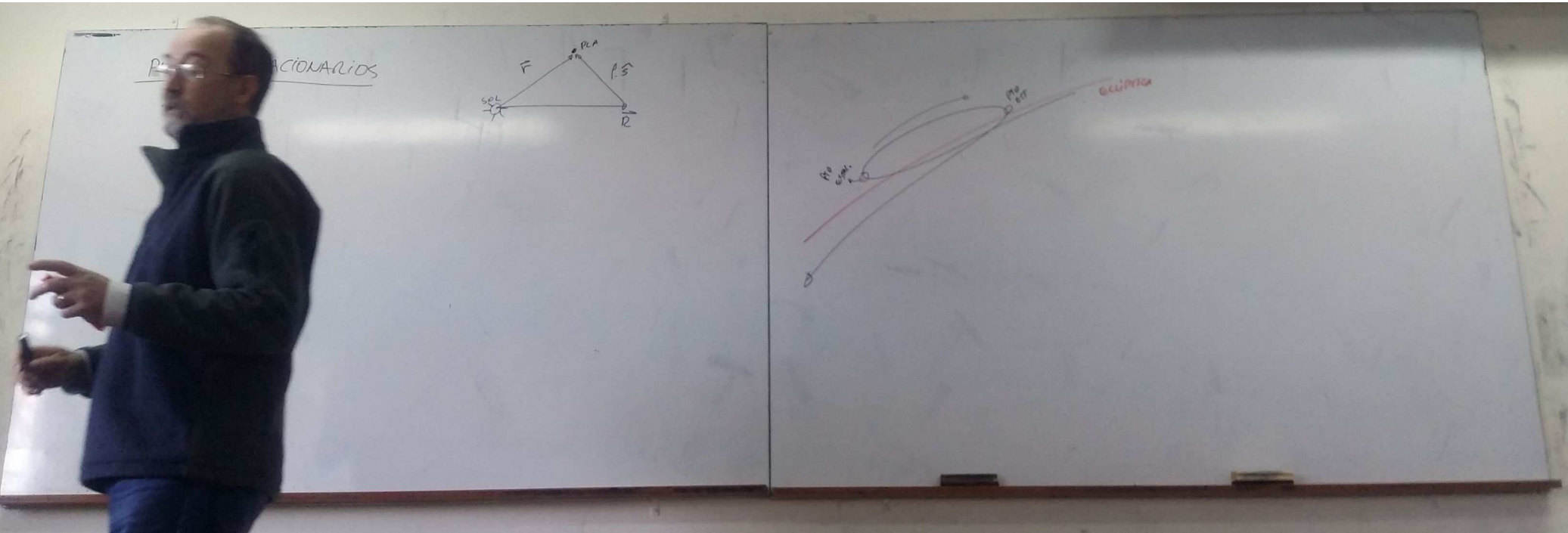
$$\frac{1}{S} = \frac{1}{T_p} - \frac{1}{1} = 0.64 \text{ a}^{-1} \Rightarrow S = 1.57 \text{ años}$$

$$T^2 = a^3$$

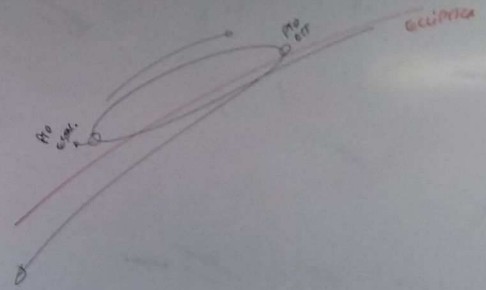
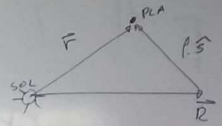
$$T = a^{3/2} = 0.61 \text{ años}$$

ESTACIONARIOS





PRINCIPALES
ACIONARIOS



PUNTOS ESTACIONARIOS

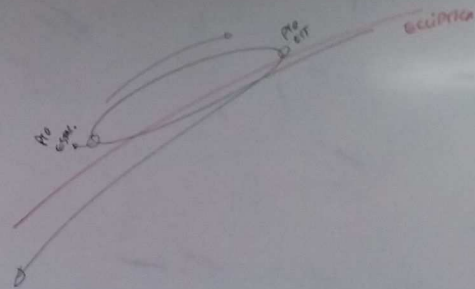
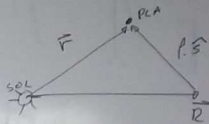
$$\dot{\hat{s}} \cong 0$$

$$\rho \hat{s} = \vec{r} - \vec{r}_0$$

$$\rho \dot{\hat{s}} + \rho \dot{\hat{s}} = \dot{\vec{r}} - \dot{\vec{r}}_0$$

$$\hat{s} \wedge \dot{\hat{s}} = \hat{s} \wedge (\dot{\vec{r}} - \dot{\vec{r}}_0) = 0 \Rightarrow$$

|| 0



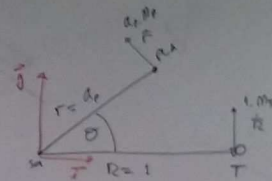
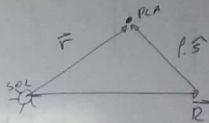
PUNTOS ESTACIONARIOS

$$\dot{\hat{s}} \cong 0$$

$$\rho \hat{s} = \vec{r} - \vec{r}_2$$

$$\dot{\rho} \hat{s} + \rho \dot{\hat{s}} = \dot{\vec{r}} - \dot{\vec{r}}_2$$

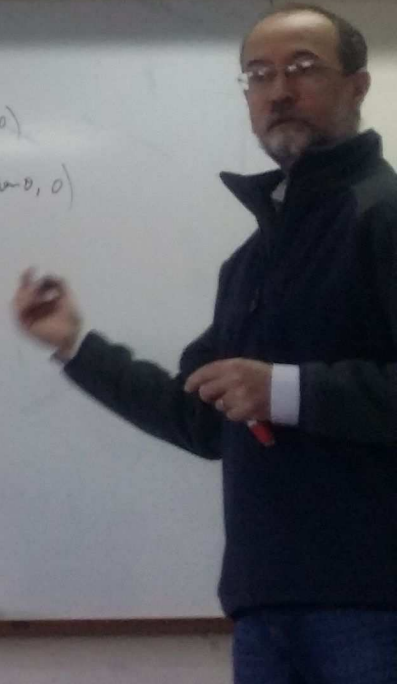
$$\hat{s} \wedge \dot{\rho} \hat{s} = \hat{s} \wedge (\dot{\vec{r}} - \dot{\vec{r}}_2) = 0 \Rightarrow (\vec{r} - \vec{r}_2) \wedge (\dot{\vec{r}} - \dot{\vec{r}}_2) = 0$$



$$\vec{r} = (1, 0, 0)$$

$$\dot{\vec{r}} = (0, m_r, 0)$$

$$\vec{r} = a_p (\cos \theta, \sin \theta, 0)$$



PUNTOS ESTACIONARIOS

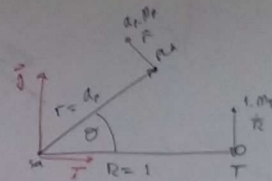
$$\dot{\hat{s}} \cong 0$$

$$P\hat{s} = \vec{r} - \vec{r}_0$$

$$P\dot{\hat{s}} + P\dot{\hat{s}} = \dot{\vec{r}} - \dot{\vec{r}}_0$$

$$\hat{s} \wedge \dot{\hat{s}} = \hat{s} \wedge (\dot{\vec{r}} - \dot{\vec{r}}_0) = 0 \Rightarrow (\vec{r} - \vec{r}_0) \wedge \dot{\vec{r}} = 0$$

PTOS.



$$\vec{r} = (1, 0, 0)$$

$$\dot{\vec{r}} = (0, \omega r, 0)$$

$$\vec{r} = a_p (\cos \theta, \sin \theta, 0)$$

$$\dot{\vec{r}} = a_p \omega (-\sin \theta, \cos \theta, 0)$$

$$(\vec{r} - \vec{r}_0) = (a_p \cos \theta - 1, a_p \sin \theta, 0)$$

$$(\dot{\vec{r}} - \dot{\vec{r}}_0) = (-a_p \omega \sin \theta, a_p \omega \cos \theta - \omega r, 0)$$

i	j	k
$a_p \cos \theta - 1$	$a_p \sin \theta$	0
$-a_p \omega \sin \theta$	$a_p \omega \cos \theta - \omega r$	0

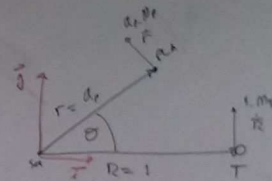
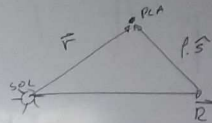
PUNTOS ESTACIONARIOS

$$\dot{\hat{S}} \cong 0$$

$$(a_p \cos \theta - 1)(a_p m_p \cos \theta - m_T) + a_p^2 m_p \sin^2 \theta = 0$$

$$a_p^2 m_p \sin^2 \theta - m_T a_p \cos \theta - a_p m_p \cos \theta + m_T + 0 = 0$$

$$\Rightarrow a_p^2 m_p - a_p \cos \theta (m_T + m_p) + m_T = 0$$



i	j	k
$a_p \cos \theta - 1$	$a_p \sin \theta$	0
$-a_p m_p \sin \theta$	$a_p m_p \cos \theta - m_T$	0

$$\bar{r} = (1, 0, 0)$$

$$\dot{\bar{r}} = (0, m_T, 0)$$

$$\bar{r} = a_p (\cos \theta, \sin \theta, 0)$$

$$\dot{\bar{r}} = a_p m_p (-\sin \theta, \cos \theta, 0)$$

$$(\bar{r} - \bar{r}_T) = (a_p \cos \theta - 1, a_p \sin \theta, 0)$$

$$(\dot{\bar{r}} - \dot{\bar{r}}_T) = (-a_p m_p \sin \theta, a_p m_p \cos \theta - m_T, 0)$$

PUNTOS ESTACIONARIOS

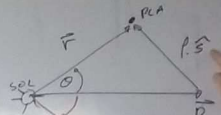
$$\dot{\hat{S}} \cong 0$$

$$(a_p \cos \theta - 1)(a_p m_p \cos \theta - m_T) + a_p^2 m_p \sin^2 \theta = 0$$

$$a_p^2 m_p \cos^2 \theta - m_T a_p \cos \theta - a_p m_p \cos \theta + m_T + a_p^2 m_p \sin^2 \theta = 0$$

$$\Rightarrow a_p^2 m_p - a_p \cos \theta (m_T + m_p) + m_T = 0$$

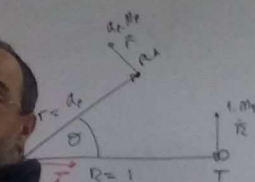
$$\Rightarrow \frac{a_p^2}{a_p^{3/2}} - a_p \cos \theta \left(1 + a_p^{-3/2}\right) + 1 = 0 \Rightarrow \cos \theta = \frac{a_p^{1/2} + 1}{(a_p + a_p^{-1/2})}$$



$$m = \sqrt{\frac{\mu}{a^3}}$$

$$m_p = \sqrt{\mu} \cdot a_p^{-3/2}$$

$$m_T = \sqrt{\mu} \cdot 1$$



$$\vec{R} = (1, 0, 0)$$

$$\dot{\vec{R}} = (0, m_T, 0)$$

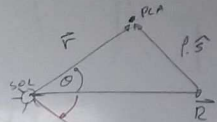
$$\vec{F} = a_p (\cos \theta, \sin \theta, 0)$$

$$\dot{\vec{F}} = a_p m_p (-\sin \theta, \cos \theta, 0)$$

$$(\vec{F} - \ddot{\vec{R}}) = (a_p \cos \theta - 1, a_p \sin \theta, 0)$$

$$(\dot{\vec{F}} - \dot{\vec{R}}) = (-a_p m_p \sin \theta, a_p m_p \cos \theta - m_T, 0)$$

PUNTOS ESTACIONARIOS



$$-a_p m_p \cos \theta + a_p^2 m_p \sin^2 \theta = 0$$

$$-a_p m_p \cos \theta + m_T = 0$$

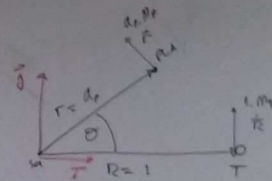
$$+m_p + m_T = 0$$

$$m = \sqrt{\frac{M}{a^3}}$$

$$m_p = \sqrt{M} \cdot a_p^{-3/2}$$

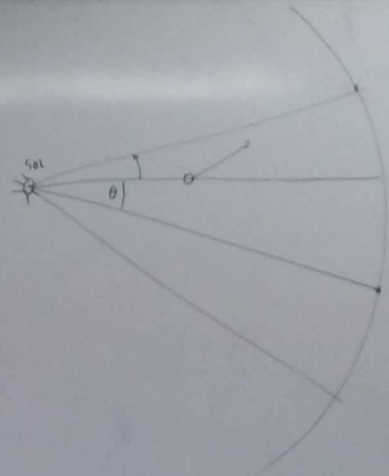
$$m_T = \sqrt{M} \cdot 1$$

$$\theta = \frac{a_p^{3/2} + 1}{(a_p + a_p^{-1/2})} > 0 \Rightarrow \theta < 90^\circ$$

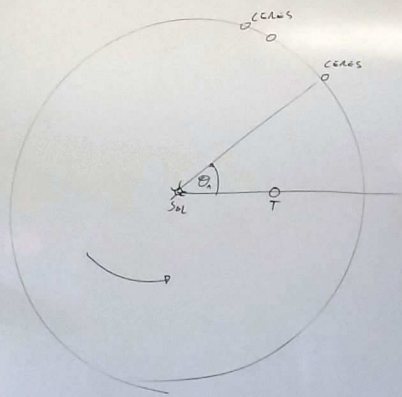


i	j	k
$a_p \cos \theta - 1$	$a_p \sin \theta$	0
$-a_p m_p \sin \theta$	$a_p m_p \cos \theta - m_T$	0

EJEMPLO: MARTE
 $a_p = 1.52 \Rightarrow \theta = 16.7^\circ$



CERES



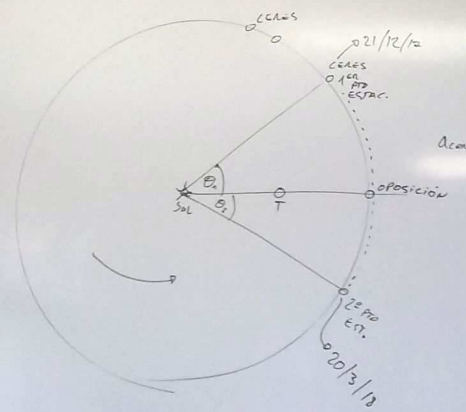
$$T^2 = a^3$$

\uparrow años \uparrow UAS

$$\cos \theta_{\text{err}} = \frac{1 + a^{1/2}}{a + a^{1/2}}$$

$a_{\text{CERES}} =$

CERES

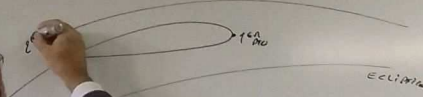


$$T^2 = a^3$$

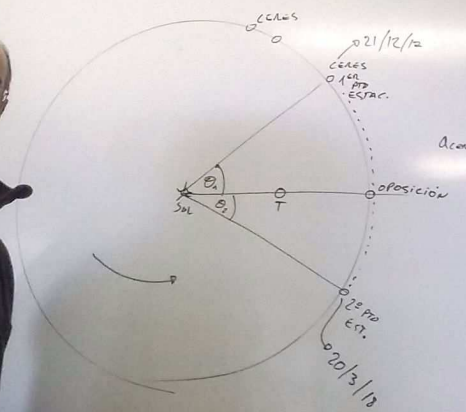
\uparrow años \uparrow UAS

$$\cos \theta_{\text{ex}} = \frac{1 + a^{3/2}}{a + a^{-1/2}} = 0,73 \Rightarrow \theta_{\text{ex}} = 37,8$$

$a_{\text{ceres}} = 2,768 \text{ ua}$



CERES

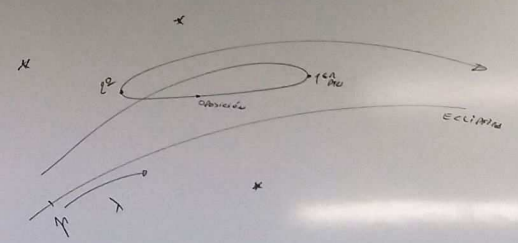


$$T^2 = a^3$$

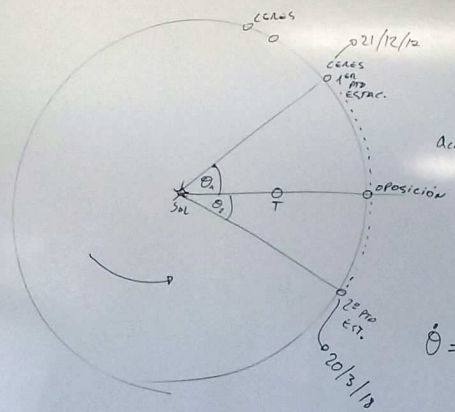
\uparrow años \uparrow UAS

$$\cos \theta_{\text{EY}} = \frac{1 + a^{1/2}}{a + a^{-1/2}} = 0,79 \Rightarrow \theta_{\text{EY}} = 37,8$$

$a_{\text{CERES}} = 2,768 \text{ ua}$



CERES



$$T^2 = a^3$$

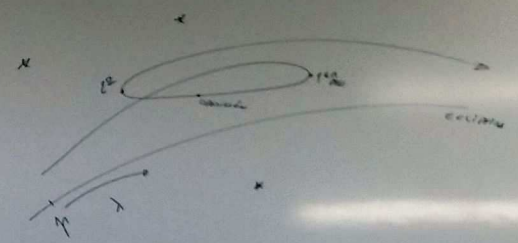
↑ años ↑ UAS

$$\cos \theta_{\text{err}} = \frac{1 + a^{1/2}}{a + a^{-1/2}} = 0,79 \Rightarrow \theta_{\text{err}} = 37,8$$

$$a_{\text{ceres}} = 2,768 \text{ ua}$$

$$\frac{1}{S} = \frac{1}{T_{\text{ceres}}} - 1 = a^{-3/2} - 1$$

$$\theta = \text{CIE} \Rightarrow S = 1,28 \text{ años}$$

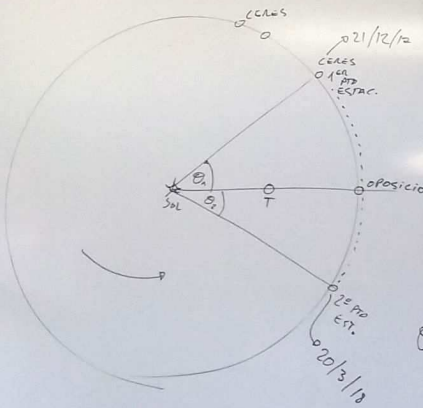


$$\Delta \theta$$

$$360^\circ \rightarrow S$$

$$2,8 \theta_{\text{err}} \rightarrow X = \frac{2 \times 37,8 \cdot S}{360^\circ} \approx 98 \text{ días}$$

CERES



$$T^2 = a^3$$

↑ años ↑ UAS

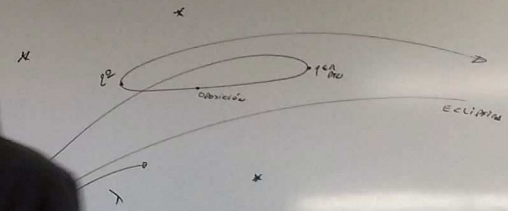
$$\cos \theta_{\text{ext}} = \frac{1 + a}{a + a'} = 0,79 \Rightarrow \theta_{\text{ext}} = 37^\circ$$

$$a_{\text{ceres}} = 2,768 \text{ ua}$$

$$\frac{1}{S} = \frac{1}{T_{\text{ceres}}} - 1 =$$

$$\dot{\theta} = \text{CIE} \Rightarrow S = 1$$

P. S. móvil



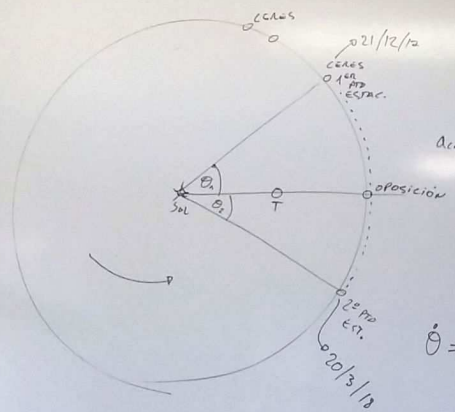
MÓV. APARENTE EN OPOSICIÓN :

$$\rho \dot{\delta} = \dot{r} - \dot{r}_E$$

$$\dot{\rho} \delta + \rho \dot{(\delta)} = \dot{r} - \dot{r}_E$$

$$X = \frac{2 \times 37,8 \cdot S}{360^\circ} \approx 98 \text{ días}$$

CERES



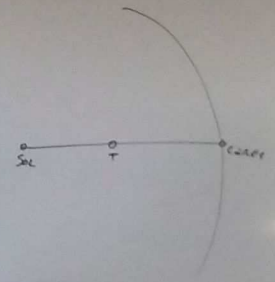
$T^2 = a^3$

$\cos \theta_{est} = 0,79 \Rightarrow \theta_{est} = 37,8^\circ$

$a_{ceres} = a^{-3/2} - 1$

$\dot{\theta} = \dots$

28 años



$m = \sqrt{\frac{a^3}{k^2}}$

$m = k a^{-3/2}$

$|\dot{r}| = m \cos \theta^a = k a^{-3/2} a$

$|\dot{r}| = m r \sin \theta = k$

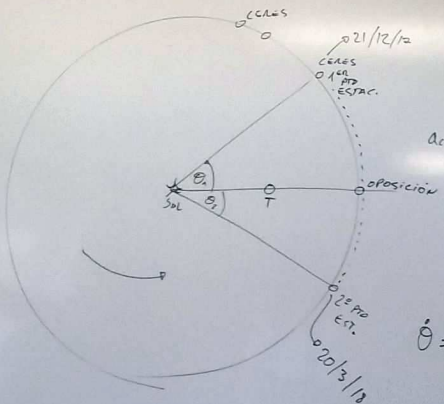
MÓV. APARENTE EN OPOSICIÓN

$\rho \dot{s} = \dot{r} - \dot{r}$

$\dot{s} \cdot \rho(\dot{s}) = \dot{r} - \dot{r}$

$\rho \dot{s} = \dot{r} - \dot{r} - \rho(\dot{s}) \approx 0$

CERES



$$T^2 = a^3$$

↑ años ↑ años

$$\cos \theta_{\text{op}} = \frac{1 + a^{1/2}}{a + a^{-1/2}} = 0,79 \Rightarrow \theta_{\text{op}} = 37,8$$

$$a_{\text{ceres}} = 2,768 \text{ ua}$$

$$\frac{1}{S} = \frac{1}{T_{\text{ceres}}} - 1 = a^{-3/2} - 1$$

$$\theta = \text{cte} \Rightarrow S = 1,28 \text{ años}$$

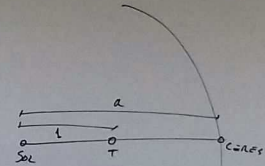
P. S. simbólico

$$M = \sqrt{\frac{r^3}{a^3}} \sim k^2$$

$$M \equiv k a^{-3/2}$$

$$|\dot{r}| = M_{\text{ceres}} \dot{r}^a = k a^{-3/2} a$$

$$|\dot{r}| = M_{\text{T.}} \dot{r} = k$$



$$\text{CERES: } |\dot{S}| \approx 3,5 \times 10^3 \text{ kmos/Día}$$

Oposición años/Día

$$\sim 0,22 \text{ día}$$

MÓV. APARENTE EN OPOSICIÓN:

$$P \hat{S} = \vec{r} - \vec{R}$$

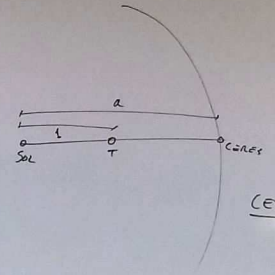
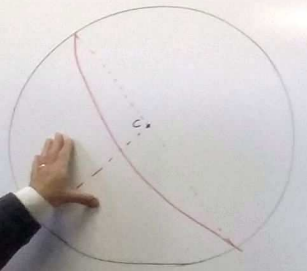
$$\dot{P} \hat{S} + P \dot{\hat{S}} = \dot{r} - \dot{R}$$

$$P \dot{\hat{S}} = \dot{r} - \dot{R} - \dot{P} \hat{S} \sim 0$$

$$P \dot{\hat{S}} = k (a^{-1/2} - 1)$$

$$|\dot{\hat{S}}| = \frac{k (a^{-1/2} - 1)}{a - 1}$$

FASES Y BRILLO



$$m = \sqrt{\frac{R^3}{a^3}} \sim k^2$$

$$m \equiv k a^{-3/2}$$

$$|\dot{r}| = m_{\text{ceres}} \dot{r}^a = k \cdot a^{-3/2} \cdot a$$

$$|\dot{r}| = m_{\text{T. l. un.}} = k$$

CERES: $|\dot{s}| \sim 3,9 \times 10^3$
 oposición RAOS/Dip
 ~ 0.22 dia

MÖV. APARENTE EN OPOSICIÓN :

$$P\dot{s} = \dot{r} - \dot{R}$$

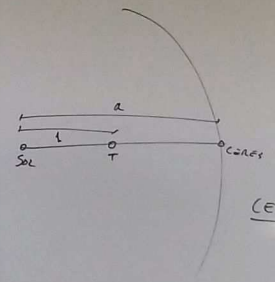
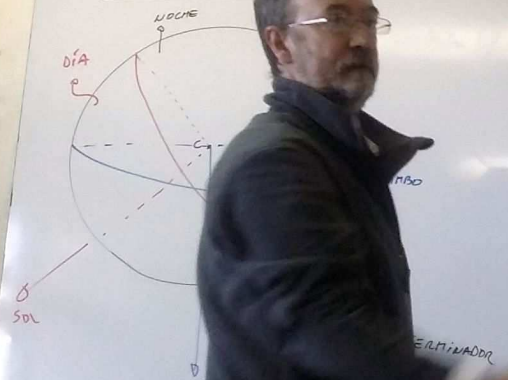
$$P\dot{s} + P(\dot{s}) = \dot{r} - \dot{R}$$

$$P\dot{s} = \dot{r} - \dot{R} - \dot{s}$$

$$P\dot{s} = k(a^{-1/2} - 1)$$

$$|\dot{s}| = \frac{k \cdot (a^{-1/2} - 1)}{a - 1}$$

FASES Y BRILLO



$$M = \sqrt{\frac{GM}{a^3}} \sim k^2$$

$$m \equiv k a^{-3/2}$$

$$|\dot{\vec{r}}| = m_{\text{ceres}} \dot{\theta}^a = k \cdot a^{-3/2} \cdot a$$

$$|\dot{\vec{r}}| = M_{\text{tr.}} \cdot u_{\text{ca}} = k$$

CERES: $|\dot{\vec{s}}| \approx 3,5 \times 10^3$
 Dposición $\frac{\text{RAOS}}{\text{Día}}$
 $\sim 0,22 \text{ / día}$

MÓV. APARENTE EN OPOSICIÓN:

$$P\dot{\vec{s}} = \dot{\vec{r}} - \dot{\vec{r}}_T$$

$$\dot{P}\dot{\vec{s}} + P(\dot{\dot{\vec{s}}}) = \dot{\dot{\vec{r}}} - \dot{\dot{\vec{r}}}_T$$

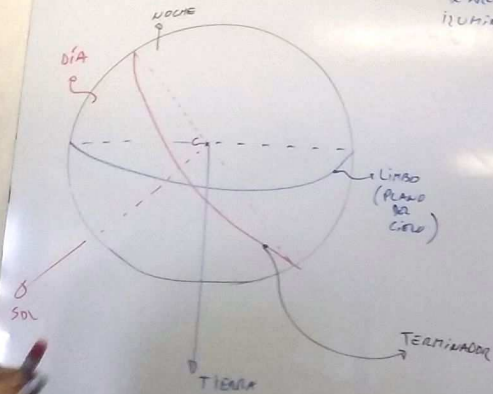
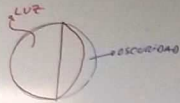
$$P \cdot \dot{\dot{\vec{s}}} = \dot{\dot{\vec{r}}} - \dot{\dot{\vec{r}}}_T - \dot{P}\dot{\vec{s}} \approx 0$$

$$P \dot{\dot{\vec{s}}} = k (a^{-1/2} - 1)$$

$$|\dot{\dot{\vec{s}}}| = \frac{k \cdot (a^{-1/2} - 1)}{a - 1}$$

FASES Y BRILLO

FRACCIÓN DE ÁREA ILLUMINADA

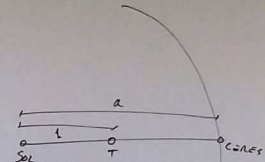


$$M = \sqrt{\frac{GM}{a^3}} \sim k^2$$

$$m \equiv k a^{-3/2}$$

$$|\dot{\vec{r}}| = m_{\text{ceres}} \dot{r}^a = k \cdot a^{3/2} \cdot a$$

$$|\dot{\vec{r}}| = m_{\text{tr.}} \cdot v_{\text{orb}} = k$$



CERES: $|\dot{\hat{s}}| \sim 3,5 \times 10^3$ (arcos/día)

OPOSICIÓN

$\sim 0,22$ día

MÓV. APARENTE EN OPOSICIÓN:

$$\dot{P}\hat{s} = \dot{\vec{r}} - \dot{\vec{R}}$$

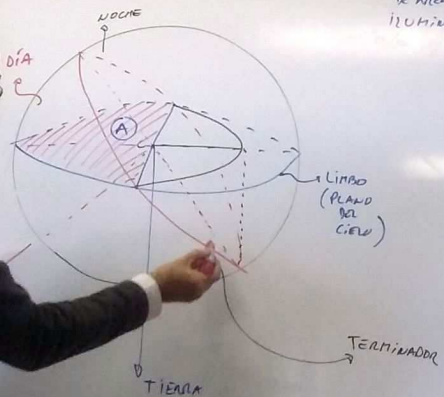
$$\dot{P}\hat{s} + P(\dot{\hat{s}}) = \dot{\vec{r}} - \dot{\vec{R}}$$

$$P \cdot \dot{\hat{s}} = \dot{\vec{r}} - \dot{\vec{R}} - \dot{P}\hat{s} \sim 0$$

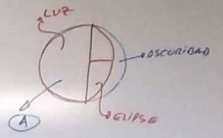
$$P \cdot \dot{\hat{s}} = k(a^{-1/2} - 1)$$

$$|\dot{\hat{s}}| = k \cdot \frac{(a^{-1/2} - 1)}{a - 1}$$

FASES Y BRILLO



FRACCIÓN DE ÁREA ILUMINADA

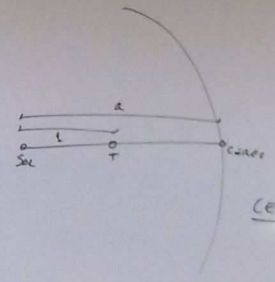


$$m = \sqrt{\frac{a^3}{a^3}} \sim k^2$$

$$m = k a^{-1/2}$$

$$|\dot{r}| = m \cos \theta = k a^{-1/2} a$$

$$|\dot{r}| = m_r \cdot \sin \theta = k$$



ceros: $|\dot{s}| \approx 3.5 \cdot 10^3$
 oposición años/día
 ≈ 0.22 día

MÓV. APARENTE EN OPOSICIÓN

$$P\dot{s} = \dot{r} - \dot{R}$$

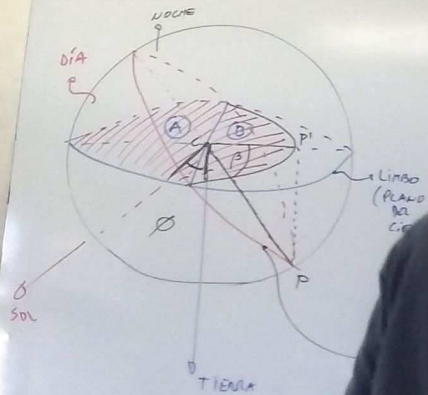
$$P\dot{s} \cdot P\dot{s} = \dot{r} - \dot{R}$$

$$P\dot{s} = \dot{r} - \dot{R} - \underbrace{P\dot{s}}_{\approx 0}$$

$$P\dot{s} = k(a^{-1/2} - 1)$$

$$|\dot{s}| = \frac{k \cdot (a^{-1/2} - 1)}{a - 1}$$

FASES Y BRILLO



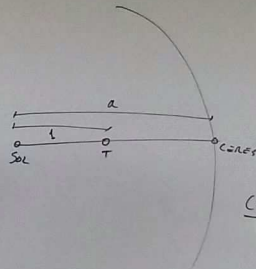
FRACCIÓN DE ÁREA ILUMINADA

$$CP' = C \cos \phi$$

ELIPSE = $\frac{\pi R_p \cdot CP'}{2}$

RAIO PLANETA $R_p \cdot \cos \phi$

$$= \frac{\pi}{2} R_p^2 \cos \phi$$



$$M = \sqrt{\frac{GM}{a^3}} \sim k^2$$

$$M \equiv k a^{-3/2}$$

$$|\dot{\vec{r}}| = M_{\text{CERES}} \dot{r} = k \cdot a^{-3/2} \cdot a$$

$$|\dot{\vec{r}}| = M_{\text{TERRA}} \cdot \omega = k$$

CERES: $|\dot{\vec{s}}| \approx 3,5 \times 10^3$
OPOSICIÓN KMS/DÍA

$$\sim 0,22 \text{ día}$$

$$a - l$$

$$|\dot{\vec{s}}| = k \cdot \frac{(a^{-1/2} - 1)}{a - l}$$

MÓV. APARENTE EN OPOSICIÓN:

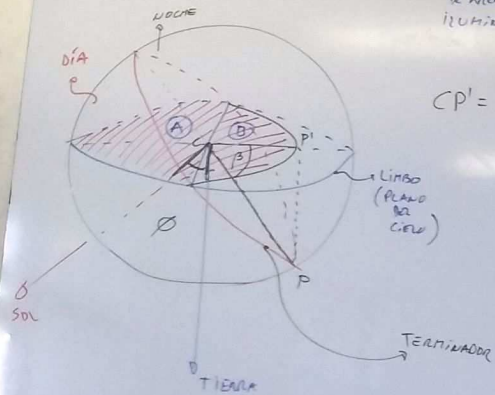
$$\dot{P} \hat{s} = \dot{\vec{r}} - \dot{\vec{R}}$$

$$\dot{P} \hat{s} + P(\dot{\hat{s}}) = \dot{\vec{r}} - \dot{\vec{R}}$$

$$P \cdot \dot{\hat{s}} = \dot{\vec{r}} - \dot{\vec{R}} - \dot{P} \hat{s} \sim 0$$

$$P \dot{\hat{s}} = k(a^{-1/2} - 1)$$

FASES Y BRILLO



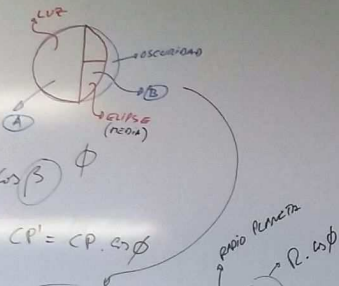
FRACCIÓN DE ÁREA ILUMINADA

$$CP' = CP \cdot \cos \phi$$

$$CP' = CP \cdot \cos \phi$$

$$\text{ÁREA ELIPSE (A+B)} = \frac{\pi R \cdot CP'}{2} = \frac{\pi}{2} R^2 \cos \phi$$

$$\text{ÁREA (A)} : \frac{\pi R^2}{2}$$



ÁREA TOTAL ILUMINADA: $(A) + (B) = \frac{\pi R^2}{2} (1 + \cos \phi)$

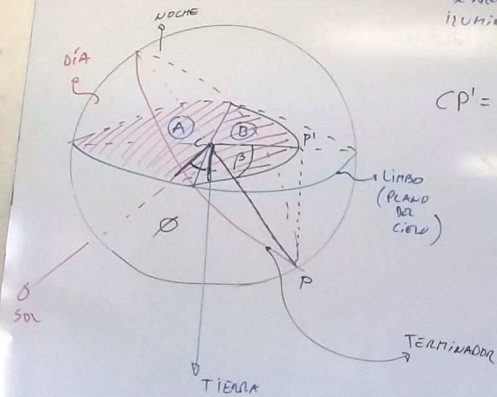
ÁREA PLANETA: πR^2

$$\text{FASE} = \frac{\text{ÁREA ILUMINADA}}{\text{ÁREA PLANETA}} = \frac{1}{2} (1 + \cos \phi) = \text{"FASE"}$$

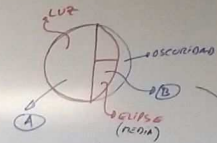
(0, 1)



FASES Y BRILLO



FRACCIÓN DE ÁREA ILUMINADA



$$CP' = CP \cdot \cos \beta \cdot \phi$$

$$CP' = CP \cdot \cos \phi$$

$$\text{ÁREA ECLIPSE (NEOIA)} = \frac{\pi R \cdot CP'}{2} = \frac{\pi}{2} R^2 \cos \phi$$

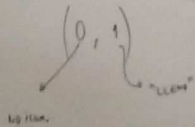
$$\text{ÁREA (A)} : \frac{\pi R^2}{2}$$

ÁREA TOTAL ILUMINADA: $(A) + (B) = \frac{\pi R^2}{2} (1 + \cos \phi)$

ÁREA PLANETA: πR^2

$$E = \frac{\text{ÁREA ILUMINADA}}{\text{ÁREA PLANETA}} = \frac{1}{2} (1 + \cos \phi) = \text{"FASE"}$$

BRILLO:



FASE Y BRILLO



FRACCIÓN DE ÁREA ILUMINADA

$CP' = CP \cdot \cos \beta$
 $CP' = CP \cdot \cos \phi$
 $\Rightarrow \text{ÁREA ELIPSE (zona B)} = \frac{\pi R \cdot CP'}{2} = \frac{\pi}{2} R^2 \cos \phi$
 $\text{ÁREA (A)} : \frac{\pi R^2}{2}$

RAYO PLANETA
 $R \cdot \cos \phi$
 LÍMBO (PLANO DEL CIELO)
 TERMINADOR

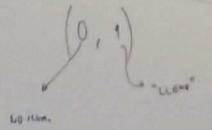
ÁREA TOTAL ILUMINADA: $(A) + (B) = \frac{\pi R^2}{2} (1 + \cos \phi)$

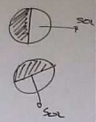
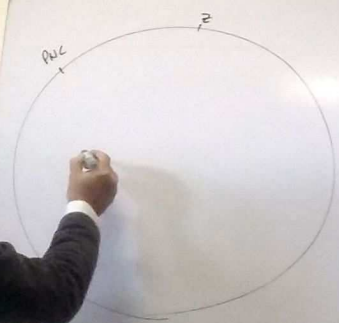
ÁREA PLANETA: πR^2

FASE = $\frac{\text{ÁREA ILUMINADA}}{\text{ÁREA PLANETA}} = \frac{1}{2} (1 + \cos \phi) = \text{"FASE"}$

BRILLO = $\frac{1}{2} (1 + \cos \phi) \cdot \frac{1}{r^2} \cdot \frac{1}{R^2} \cdot \text{CTE}$

\uparrow $\frac{1}{r^2}$ GEOCÉNTRICA
 \uparrow $\frac{1}{R^2}$ HELIOC.





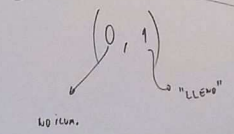
AREA TOTAL ILUMINADA: $(A) + (B) = \frac{\pi R^2}{2} (1 + \cos \phi)$

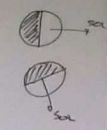
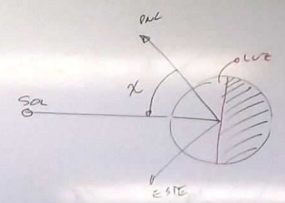
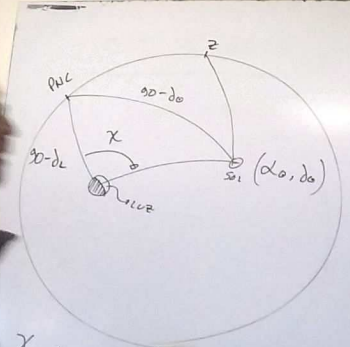
AREA PLANETA: πR^2

FASE = $\frac{\text{AREA ILUMINADA}}{\text{AREA PLANETA}} = \frac{1}{2} (1 + \cos \phi) = \text{"FASE"}$

BRILLO = $\frac{1}{2} (1 + \cos \phi) \cdot \frac{1}{p^2} \cdot \frac{1}{r^2} \cdot \text{CTE}$

\uparrow p^2 \uparrow r^2
 GEOCENTRICA \uparrow HELIOC.



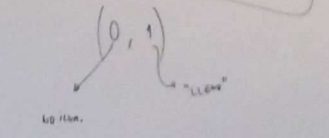


AREA TOTAL: $\textcircled{A} + \textcircled{B} = \frac{\pi R^2}{2} (1 + \cos \varphi)$
 ILUMINADA

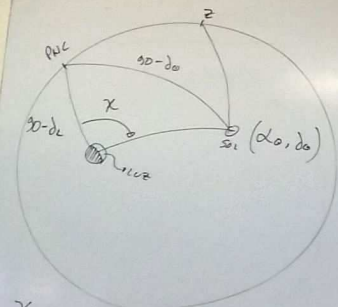
AREA: πR^2
 PLANETA

FASE = $\frac{\text{AREA ILUMINADA}}{\text{AREA PLANETA}} = \frac{1}{2} (1 + \cos \varphi) = \text{"FASE"}$

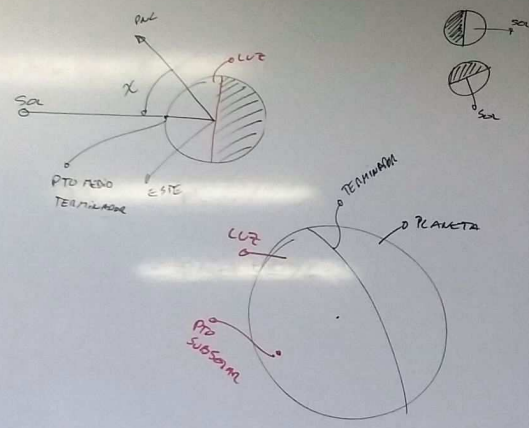
BRILLO = $\frac{1}{2} (1 + \cos \varphi) \cdot \frac{1}{r^2} \cdot \frac{1}{r^2} \cdot \text{CTE}$
 Geocéntrica Helioc.



χ ANGULO DE ...



χ ANGULO DE POSICIÓN

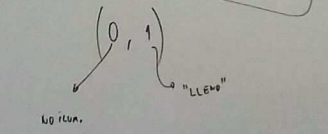


AREA TOTAL ILUMINADA: $(A) + (B) = \frac{\pi R^2}{2} (1 + \cos \phi)$

AREA PLANETA: πR^2

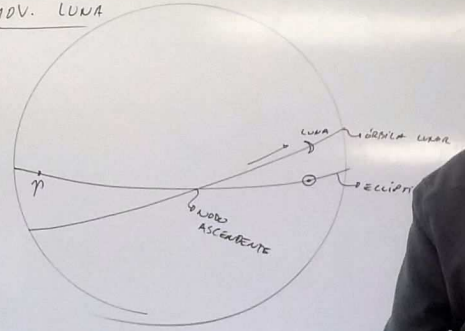
FASE = $\frac{\text{AREA ILUMINADA}}{\text{AREA PLANETA}} = \frac{1}{2} (1 + \cos \phi) = \text{"FASE"}$

BRILLO = $\frac{1}{2} (1 + \cos \phi) \cdot \frac{1}{p^2} \cdot \frac{1}{r^2} \cdot \text{CTE}$
 (Labels: p^2 - GEOCENTRICA, r^2 - HELIOC.)



OCULTACIONES Y ECLIPSES

MOV. LUNA

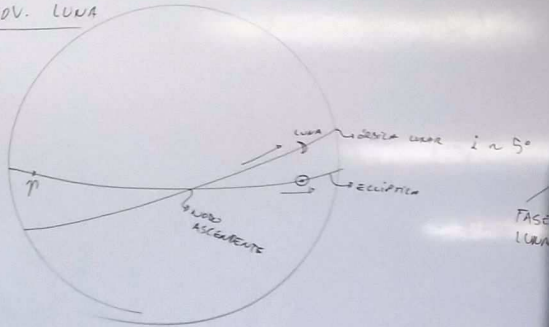


(MES)
PERIODO SIDEREO
(τ)



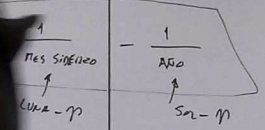
OCULTACIONES Y ECLIPSES

MOV. LUNA



$P_{LEO} = 27 \text{ Días } 3217$

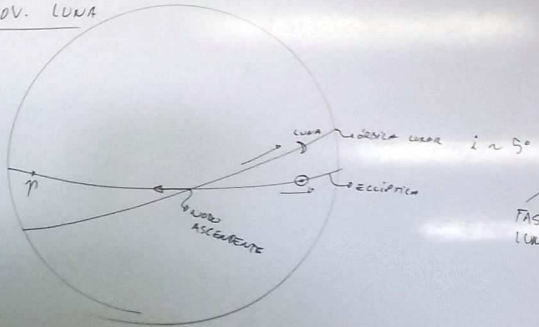
$$\frac{1}{P_{LUNA}} = \frac{1}{27,3217} - \frac{1}{P_{SOLAR}}$$



$- W_{SM}$

OCULTACIONES Y ECLIPSES

MOV. LUNA



(MES)
PERIODO SIDEREO = $27^{\text{días}}.3217$
(η)

$$\frac{1}{\text{MES SIDEREO}} = \frac{1}{\text{MES SIDEREO}} - \frac{1}{\text{AÑO}}$$

\uparrow LUNA-SOL \uparrow LUNA- η \uparrow SOL- η

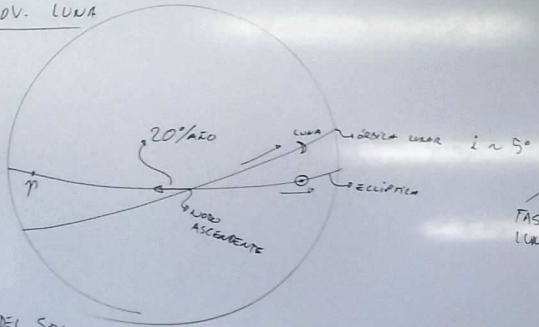
FASES LUNARES
 $\omega_{L-S} = \omega_{L-\eta} - \omega_{S-\eta}$

$$\frac{1}{\text{M. SIDEREO}} = \frac{1}{27.3217} - \frac{1}{365.25} \Rightarrow \text{MES SIDEREO} = 29.53^{\text{días}}$$

LUNA NUEVA : $\lambda_L = \lambda_{\odot}$

OCULTACIONES Y ECLIPSES

MOV. LUNA



(MES)
PERIODO SIDEREO = $27^{\text{días}}, 3217$
(π)

$$\frac{1}{\text{MES SIDEREO}} = \frac{1}{\text{MES SIDEREO}} - \frac{1}{\text{AÑO}}$$

\uparrow LUNA-SOL \uparrow LUNA- π \uparrow SOL- π

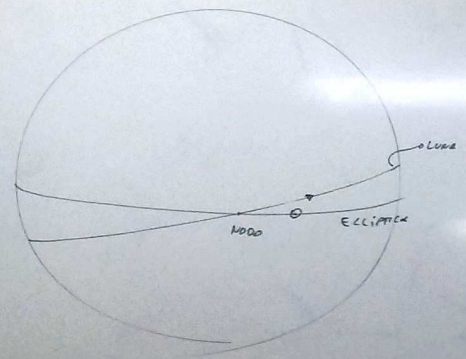
FASES LUNARES
 $\omega_{L-S} = \omega_{L-\pi} - \omega_{S-\pi}$

$$\frac{1}{\text{M. SIDEREO}} = \frac{1}{27,3217} - \frac{1}{365,25} \Rightarrow \text{MES SIDEREO} = 29,53^{\text{días}}$$

LUNA NUEVA : $\lambda_L = \lambda_{\odot}$

CRUCE DEL SOL
POR EL NODO ASCENDENTE
LUNAR $\Rightarrow 346,6$ años

☉ CRUCE NODOS : 173,3 años



PARCIAL:

¿ VIERNES 23
¿ LUNES 26 ?

- PRECESIÓN, NUTACIÓN
- MOV. PROPIO
- MOV. Y CONFIG. PLANET.

$(\text{Sol}) \text{ CRUZA } (\text{Nodo}) \text{ CADA: } 346.62 \text{ días} \times 19 = 6585.8 \text{ días}$

$(\text{LUNA}) \text{ RESPECTO AL } (\text{Sol}): 29.53 \text{ días} \times 223 = 6585.3 \text{ días}$

$\Rightarrow 18 \text{ años} \times 11 \text{ días}$

SAROS

26 FEB 2017

9 MAR 2035

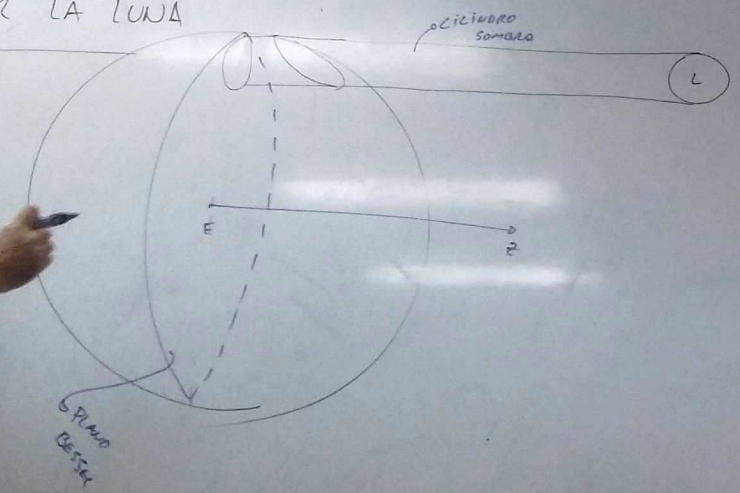
PARCIAL:

¿ VIERNES 23
LUNES 26 ?

PRECESIÓN, NUTACIÓN
MOV. PROPIO
MOV. Y CONFIG. PLANET.

OCULTACIONES DE ESTRELLAS POR LA LUNA

> BESSEL



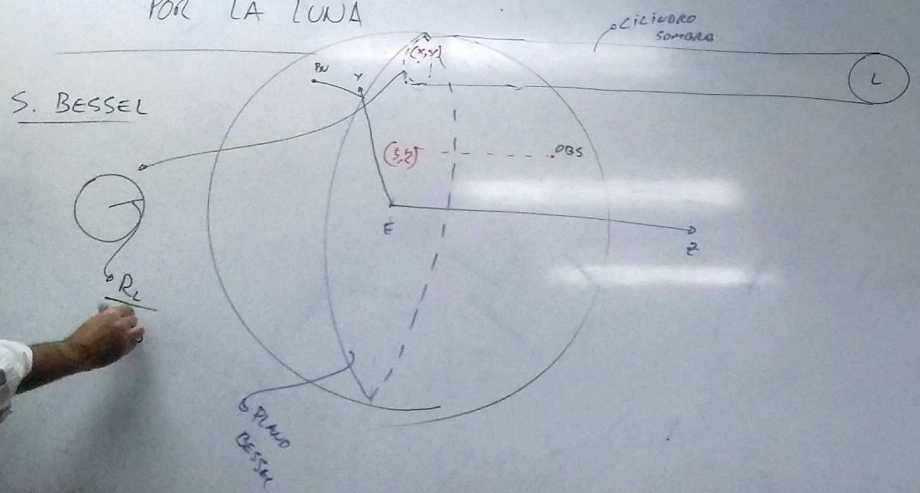
PARCIAL:

¿ VIERNES 23
LUNES 26 ?

- PRECESIÓN, NUTACIÓN
- MOV. PPI0
- MOV. Y CONFIG. PLANET.



OCULTACIONES DE ESTRELLAS POR LA LUNA



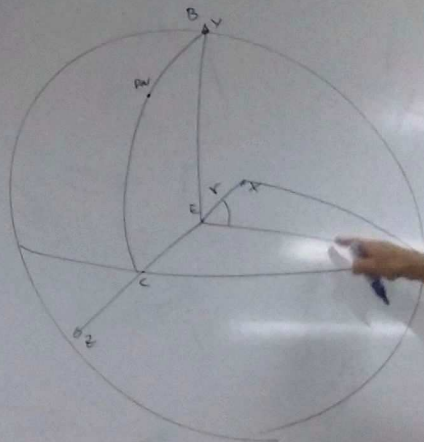
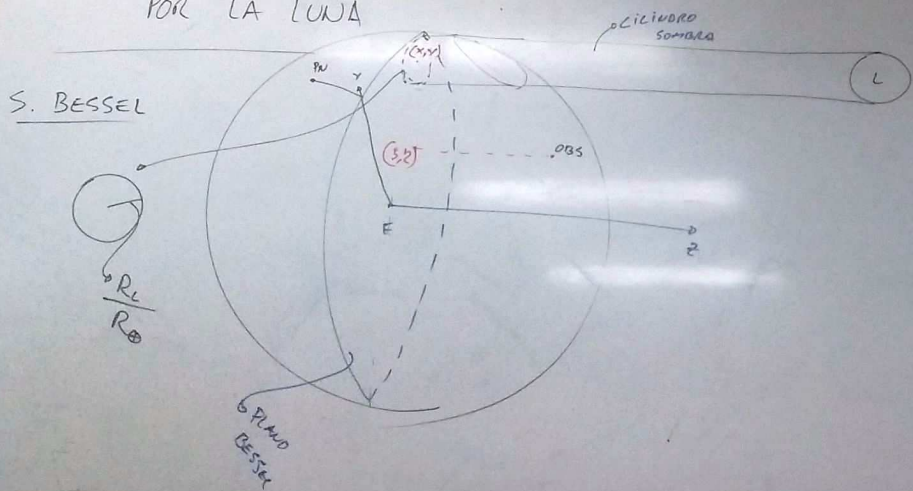
PARCIAL:

¿ VIERNES 23
LUNES 26 ?

- PRECESIÓN, ROTACIÓN
- MOV. PROPIO
- MOV. Y CONFIG. PLANET.



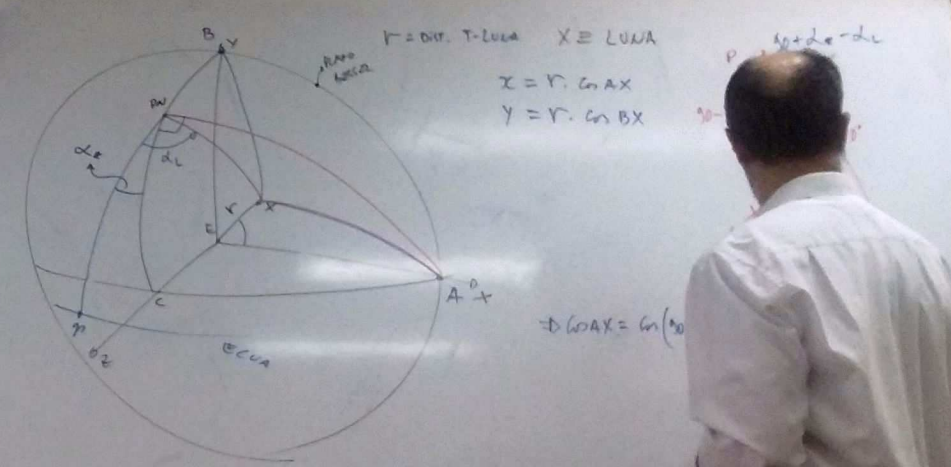
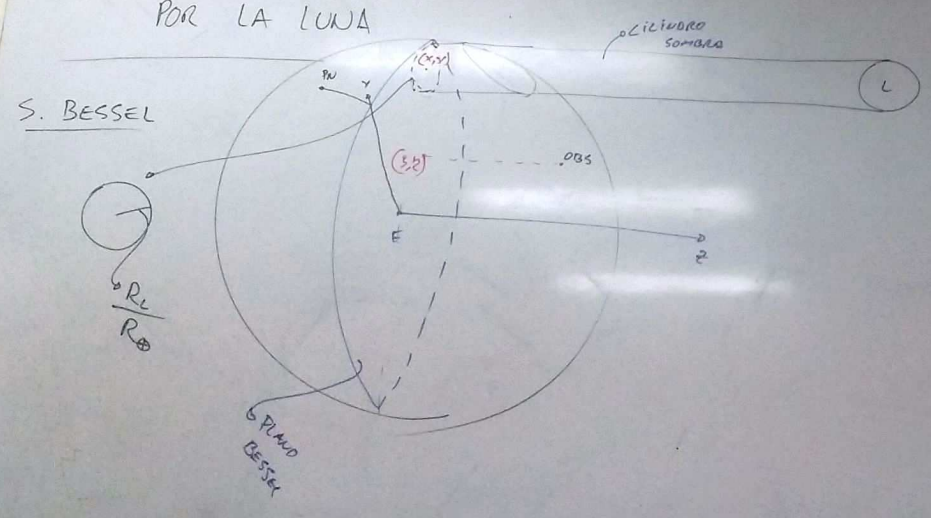
OCULTACIONES DE ESTRELLAS POR LA LUNA



$x = r$
 $x \equiv LUNA$
PARCIAL:
¿ VIENE 23 ?
¿ LUNES 26 ?
RESIÓN, ROTACIÓN
DID
FIG. PLAWET.



OCULTACIONES DE ESTRELLAS POR LA LUNA



$$X = r \cdot \cos \alpha X$$

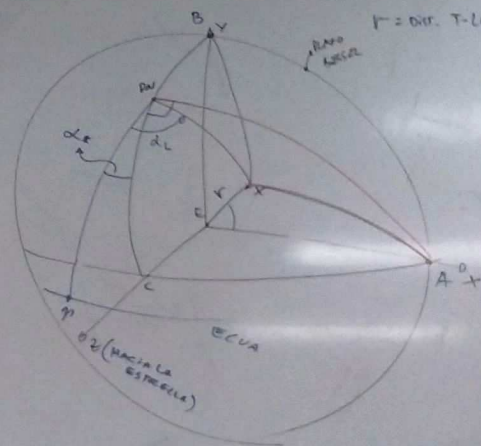
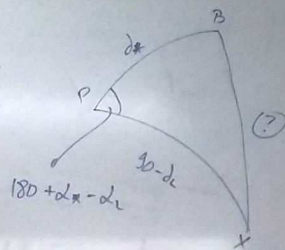
$$Y = r \cdot \cos \beta X$$

$$\rightarrow \cos \alpha X = \cos(\dots)$$



OCULTACIONES DE ESTRELLAS POR LA LUNA

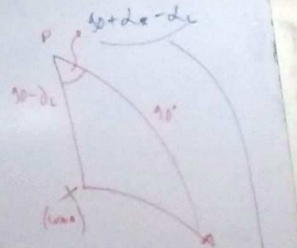
$$= \cos d_* \cdot \cos(90 - d_c) +$$



$r = \text{DIR. T-LUNA}$ $X \equiv \text{LUNA}$

$$x = r \cdot \cos AX$$

$$y = r \cdot \cos BX$$



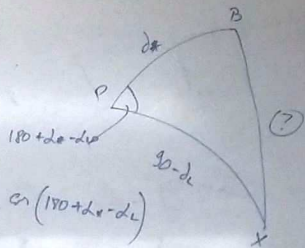
$$\Rightarrow \cos AX = \cos(90 - d_c) \cdot \cos(90 + d_* - d_c) + \sin(90 - d_c) \cdot \sin(90 + d_* - d_c) \cdot \cos 90^\circ$$

$$\Rightarrow \cos AX = 0 + \cos d_c \cdot \sin(d_* - d_c)$$

$$\Rightarrow x = r \cdot \cos d_c \sin(d_* - d_c)$$

$$\sin \pi_L = \frac{R_0}{r}$$

OCULTACIONES DE ESTRELLAS POR LA LUNA

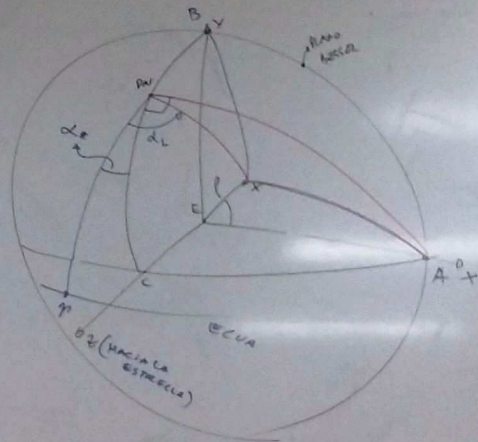


$$\cos BX = \cos d^* \cdot \cos(90-d_L) + \sin d^* \sin(90-d_L) \cdot \cos(180+d_L-d^*)$$

$$= \cos d^* \sin d_L - \sin d^* \cos d_L \cdot \cos(d_L-d^*)$$

$$Y = r \cdot \cos BX$$

$$\frac{R_{\oplus}}{\sin \theta_L}$$

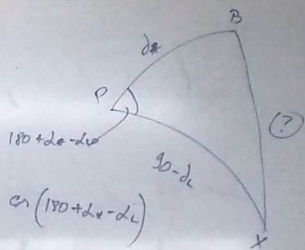


$$\sin \theta_L = \frac{R_{\oplus}}{r}$$

R_{\oplus} = dist. geocentrica da obs.



OCULTACIONES DE ESTRELLAS POR LA LUNA

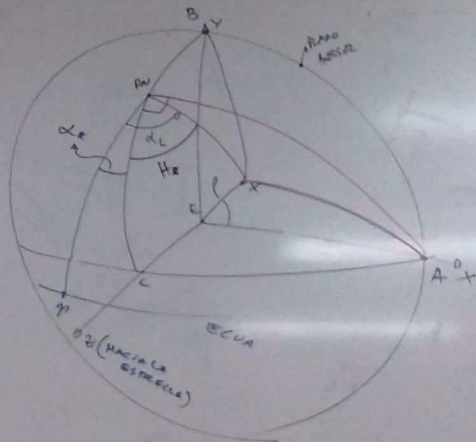


SIST. BESSÉL
 $R_{\oplus} = 1$

$$\cos d_* \cdot \cos(90-d_L) + \sin d_* \sin(90-d_L) \cdot \cos(180+d_L-d_L)$$

$$= \cos d_* \sin d_L - \sin d_* \cos d_L \cdot \cos(d_L-d_*)$$

BX



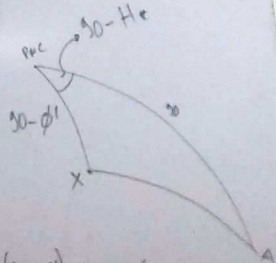
$$\sin \pi_2 = \frac{R_{\oplus}}{r}$$

ρ = DIST. GEODÉSICA en cos.

X = OBSERVADOR

$$\beta = \rho \cdot \cos AX$$

$$\gamma = \rho \cdot \cos BX$$



$$\cos AX = 0 + \sin(90-\phi') \cdot 1 \cdot \cos(90-H_x)$$

$$\cos AX = \cos \phi' \cdot \sin H_x$$

$$\beta = \rho \cdot \cos \phi' \cdot \sin H_x$$

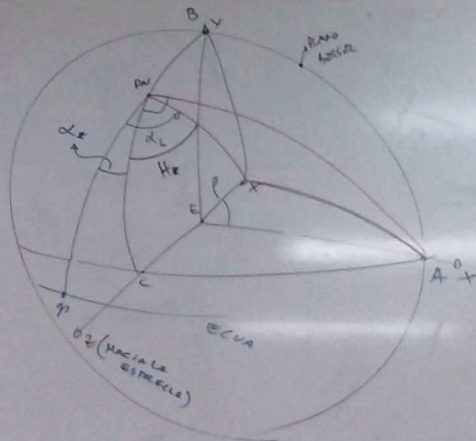
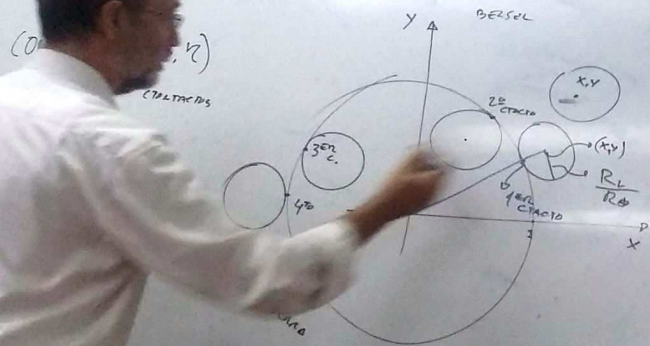
EN RADIOS TERREST.

OCULTACIONES DE ESTRELLAS

POR LA LUNA

SIST. BESSÉL

$$R_{\oplus} = 1$$

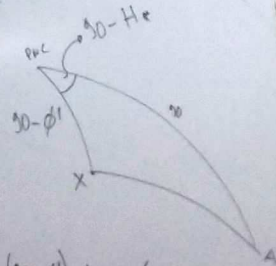


P = DIST. GEOCÉNTRICA DE OBS.

X = OBSERVADOR

$$z = P \cdot \cos AX$$

$$y = P \cdot \cos BX$$



$$\cos AX = 0 + \sin(90-\phi) \cdot 1 \cdot \cos(90-H_e)$$

$$\cos AX = \cos \phi' \cdot \sin H_e$$

$$z = P \cdot \cos \phi' \cdot \sin H_e$$

EN RADIOS TERREST.

$$\sin \pi_L = \frac{R_{\oplus}}{r}$$

OCULTACIONES DE ESTRELLAS POR LA LUNA

SIST. BESSOL
 $R_{\oplus} = 1$

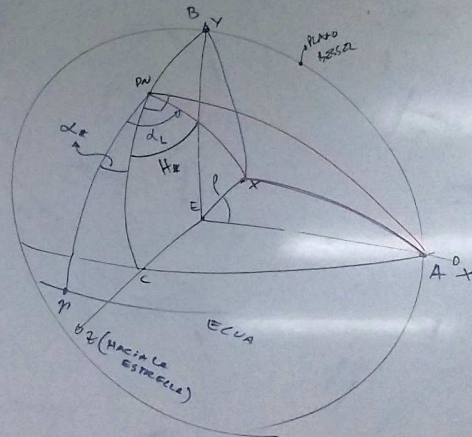
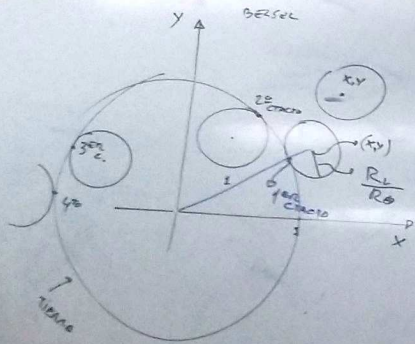
CONDICIONES

1^{er} y 4^{to} cuadrantes

$$x^2 + y^2 = \left(1 + \frac{R_L}{R_{\oplus}}\right)^2$$

h

$$x^2 + y^2 = (1 - h)^2$$



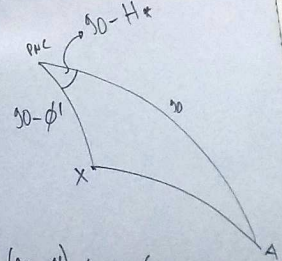
$$\sin \pi_L = \frac{R_{\oplus}}{r}$$

$\rho =$ DIST. GEODÉSICA OJA OBS.

X \equiv OBSERVADOR

$$\xi = \rho \cdot \cos AX$$

$$\eta = \rho \cdot \cos BX$$



$$\cos AX = 0 + \sin(90 - \phi') \cdot 1 \cdot \cos(90 - H_{\oplus})$$

$$\cos AX = \cos \phi' \cdot \sin H_{\oplus}$$

$$\xi = \rho \cdot \cos \phi' \cdot \sin H_{\oplus}$$

EN RADIOS TERREST.

OCULTACIONES DE ESTRELLAS POR LA LUNA

SIST. BESSEL
 $R_{\oplus} = 1$

LÍNEA DE CENTRALIDAD

$$\xi(t) = x(t)$$

$$\eta(t) = y(t)$$

ϕ', λ

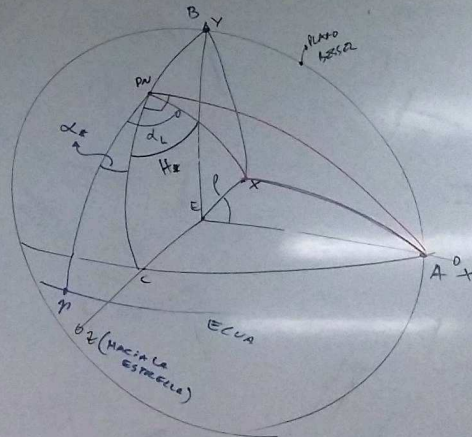
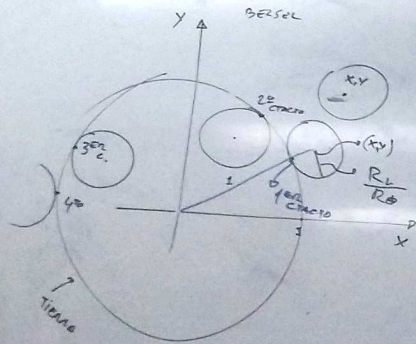
CONDICIONES

1^{ra} y 4^{ta} CUALTIERS

$$x^2 + y^2 = \left(1 + \frac{R_L}{R_{\oplus}}\right)^2$$

$$x^2 + y^2 = (1 - h)^2$$

$z \approx 3^{\text{ra}}$



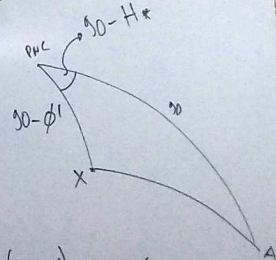
$$\sin \pi_L = \frac{R_{\oplus}}{r}$$

$p = \text{DIST. GEODÉSICA DEL OBS.}$

$X \equiv \text{OBSERVADOR}$

$$\xi = p \cdot \cos \alpha X$$

$$\eta = p \cdot \cos \beta X$$



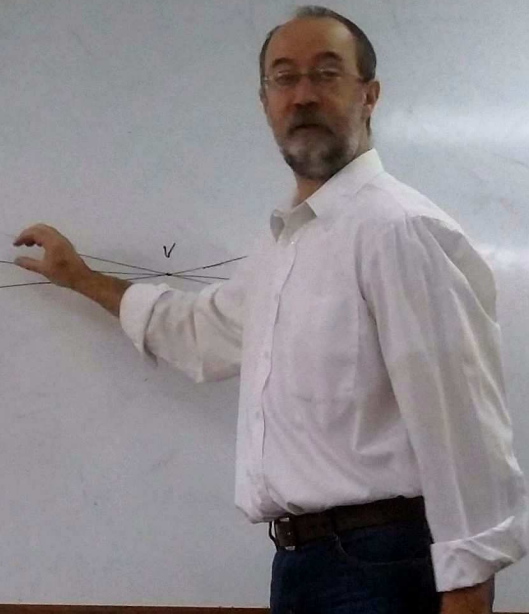
$$\cos \alpha X = 0 + \sin(\theta_0 - \phi) \cdot 1 \cdot \cos(\theta_0 - H_{\oplus})$$

$$\cos \alpha X = \cos \phi' \cdot \sin H_{\oplus}$$

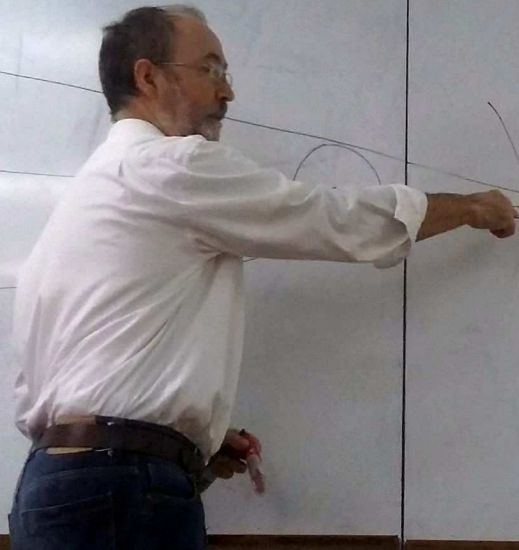
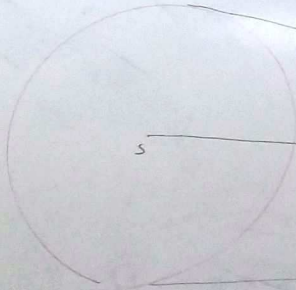
$$\xi = p \cdot \cos \phi' \cdot \sin H_{\oplus}$$

EN RADIOS TERREST.

ECLIPSES



ECLIPSES

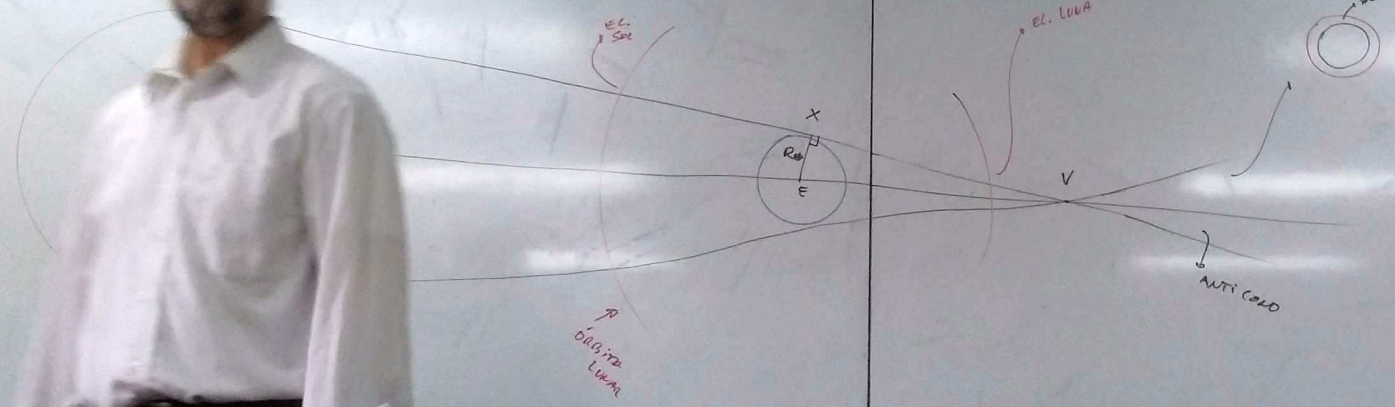


V

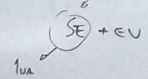
ANTI SOL



ECLIPSES



$$\left. \begin{matrix} \Delta VXE \\ \Delta VAS \end{matrix} \right\} \frac{EV}{SV} = \frac{R_{\oplus}}{R_{\odot}}$$



$$\frac{1u + EV}{EV} = \frac{R_{\oplus}}{R_{\odot}}$$

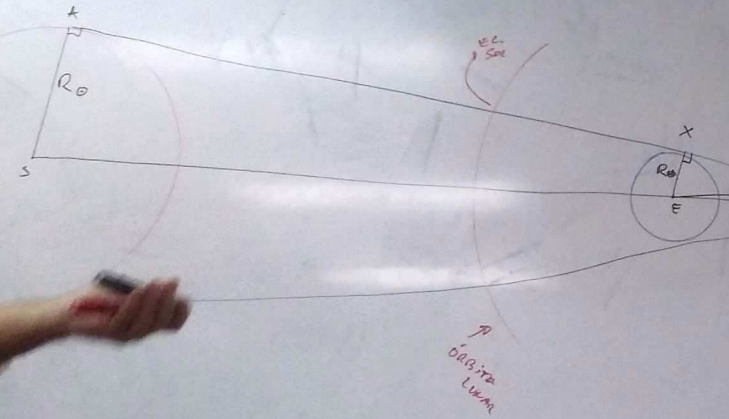
$$\frac{1u}{EV} + 1 = \frac{R_{\oplus}}{R_{\odot}}$$

$$\frac{1u}{EV} = \frac{R_{\oplus} - R_{\odot}}{R_{\odot}}$$

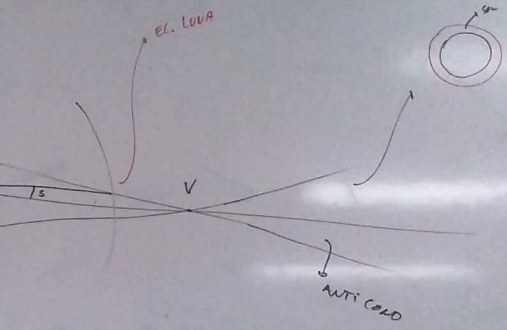
$$\Rightarrow EV = \frac{R_{\oplus}}{\frac{R_{\oplus} - R_{\odot}}{R_{\odot}}} \cdot 1u$$

$\frac{R_{\oplus} - R_{\odot}}{R_{\odot}}$
 \downarrow
 $696,000 \quad 6,371,000$

ECLIPSES



S: SEMIDIAMETRO DEL CORDO DE SOMBRRA DE LA TIERRA A LA DISTANCIA DE LA LUNA



$$\left. \begin{matrix} \Delta VXE \\ \Delta VAS \end{matrix} \right\} \frac{EV}{SV} = \frac{R_{\oplus}}{R_{\odot}}$$

$\begin{matrix} \text{SE} + EV \\ \uparrow \\ 1ua \end{matrix}$

$$\frac{1ua + EV}{EV} = \frac{R_{\oplus}}{R_{\odot}}$$

$$\frac{1ua}{EV} + 1 = \frac{R_{\oplus}}{R_{\odot}}$$

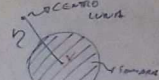
$$\frac{1ua}{EV} = \frac{R_{\oplus} - R_{\odot}}{R_{\odot}}$$

$$\Rightarrow EV = \frac{R_{\oplus}}{\frac{R_{\oplus} - R_{\odot}}{R_{\odot}}} \cdot 1ua$$

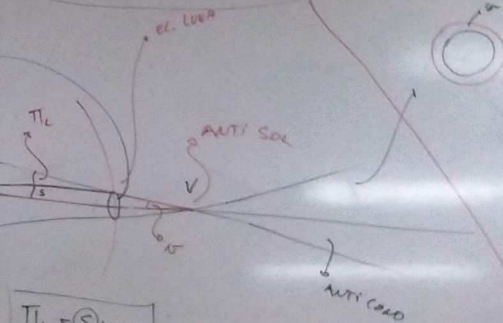
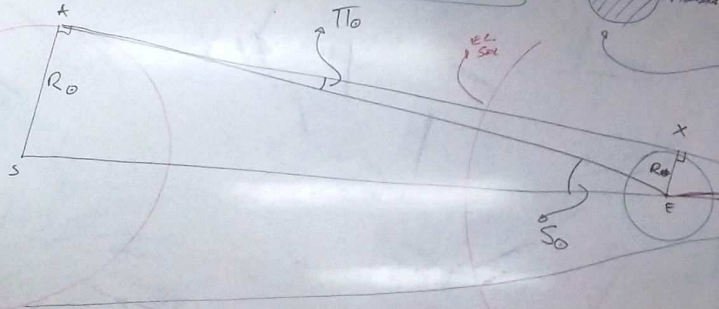
$\begin{matrix} R_{\oplus} - R_{\odot} \\ \downarrow \\ 656.000 \quad 16.100 \end{matrix}$

ECLIPSES

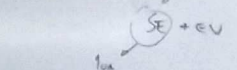
$2 L + S_L$
ECLIPSE LUNA



S: SEMIDIAMETRO DEL CIRCULO DE SOMBRAS DE LA TIERRA A LA DISTANCIA DE LA LUNA



$$\frac{\Delta VE}{\Delta VS} \left\{ \frac{EV}{SV} = \frac{R_0}{R_0} \right.$$



$$\frac{1u + EV}{EV} = \frac{R_0}{R_0}$$

$$\frac{1u}{EV} + 1 = \frac{R_0}{R_0}$$

$$\frac{1u}{EV} = \frac{R_0 - R_0}{R_0}$$

$$T_L = S + N$$

$$\Delta VEA$$

$$S_0 = T_0 + N$$

SEMIDIAMETRO SOL

$$T_L - S_0 = S - T_0$$

$$S = T_L - S_0 + T_0$$

$$\Rightarrow EV = \frac{R_0}{R_0 - R_m} \cdot 1u$$

696,000 1740

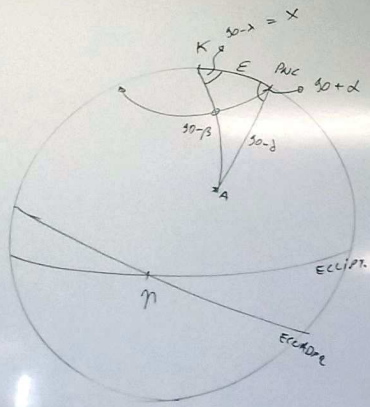
ORBITA LUNA

EE LUNA

ANTI SOL

ANTI CORD

2



$$\frac{\text{Núm } X}{\text{Núm } 90 - \beta} = \frac{\text{Núm } (90 + \alpha)}{\text{Núm } (90 - \beta)}$$

$$\rightarrow X = 62.14$$

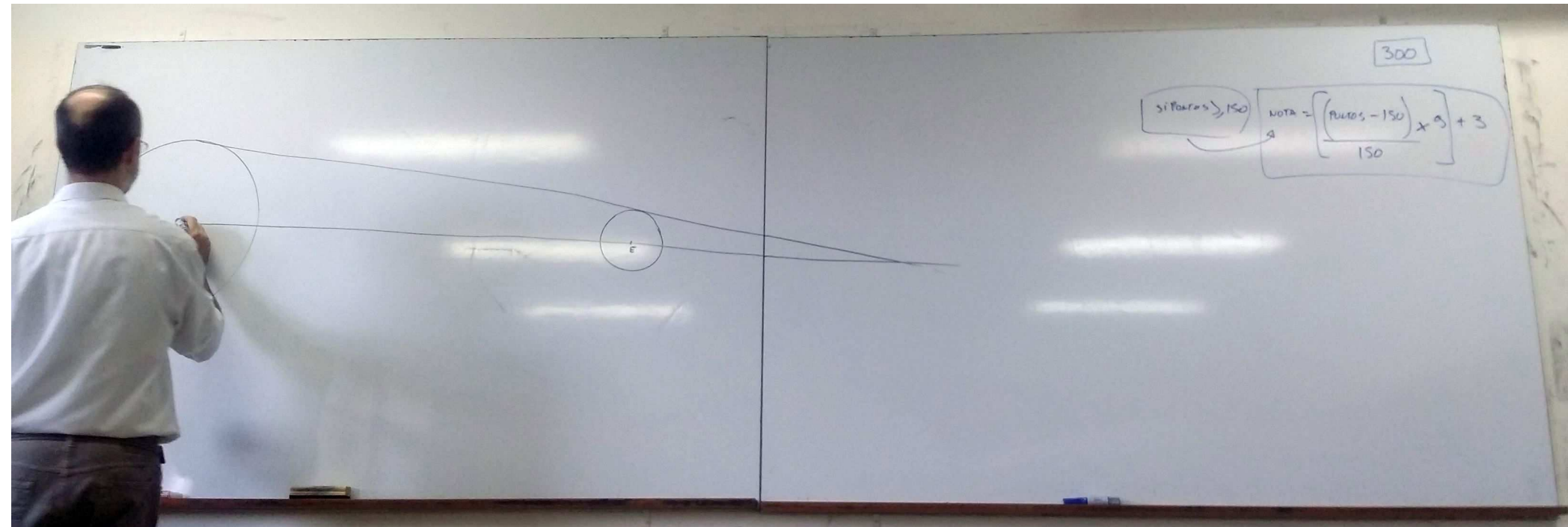
$$360 \rightarrow 26.000$$

$$62.14 \rightarrow \text{○}$$

300

si puntos > 150

$$\text{NOTA} = \frac{(\text{Puntos} - 150)}{150} \cdot 9 + 3$$

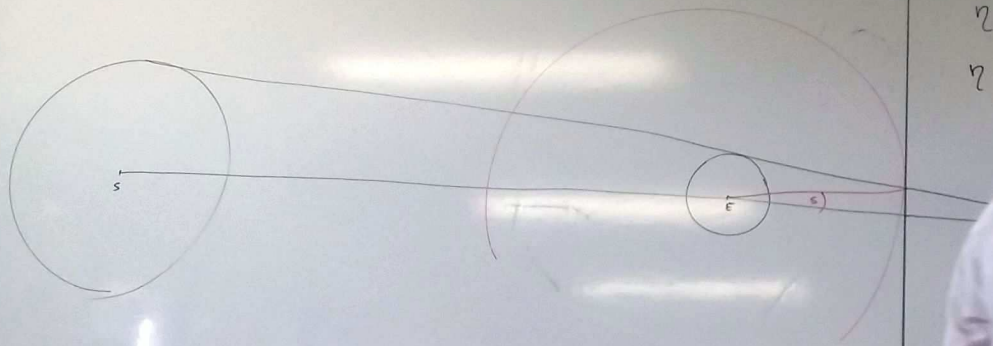


300

si Puntos > 150

$$NOTA = \left[\frac{PUNTO - 150}{150} \times 9 \right] + 3$$

E



$$S = \pi_c + \pi_o - S_o$$

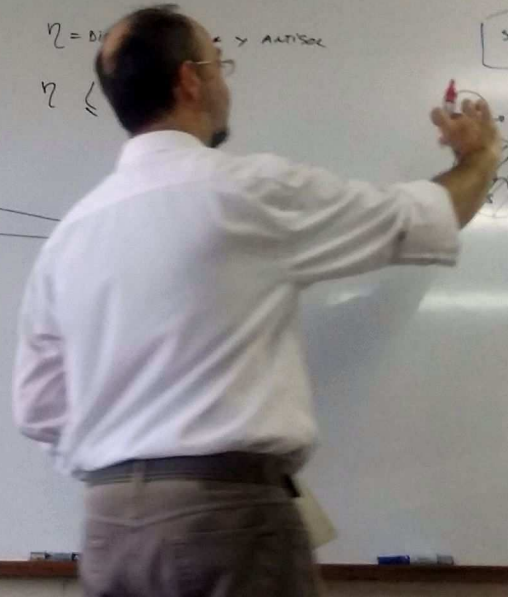
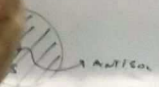
$$\eta = \text{Dist} \times \gamma \text{ ANTISOL}$$

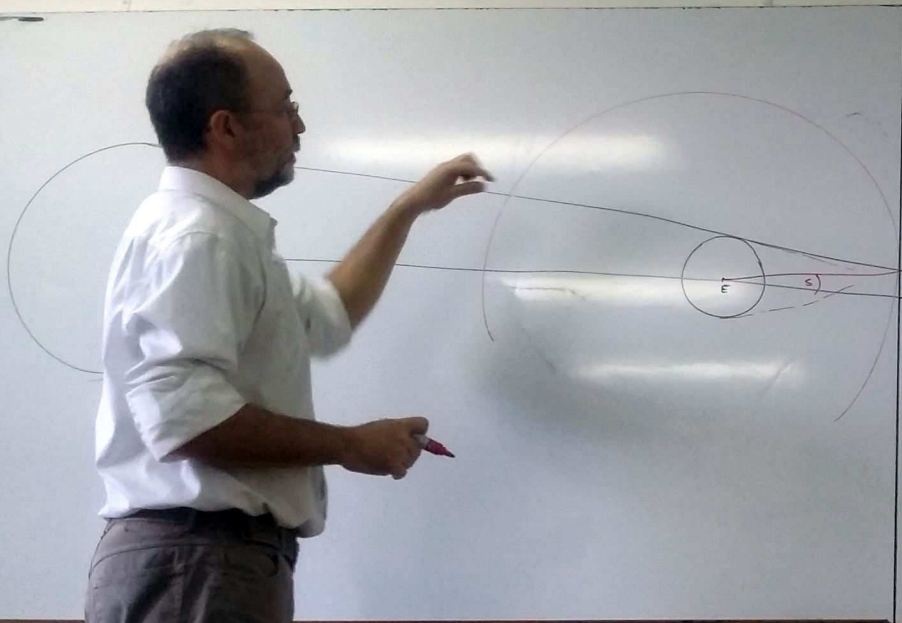
$$\eta \leq$$

300

si Puntos > 150

$$\text{NOTA} = \left[\frac{\text{Puntos} - 150}{150} \times 9 \right] + 3$$





$$s = \pi_L + \pi_0 - s_0$$

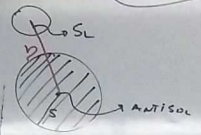
η = DIST. CENTRO LUNA y ANTISOL

$$\eta \leq s + S_L$$

CONDICION DE ECLIPSE LUNA: $\eta \leq (\pi_L + \pi_0 - s_0) + S_L$

$$\eta = s_0 - \pi_0$$

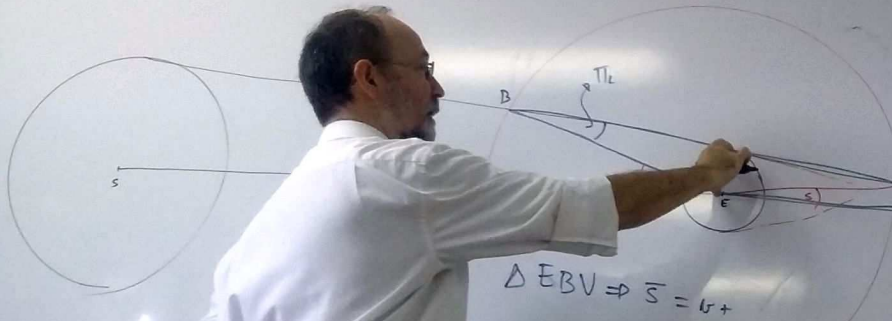
ATMÓSF.



300

SI PUNTOS > 150

$$NOTA = \left[\frac{PUNTOS - 150}{150} \times 9 \right] + 3$$



$$\Delta EBV \Rightarrow \bar{s} = r +$$

$$S = \pi_L + \pi_0 - S_0$$

η = DIST. CENTRO LUNA Y ANTISOL

$$\eta \leq s + S_L$$

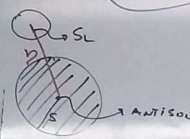
CONDICION DE ECLIPSE LUNA: $\eta \leq (\pi_L + \pi_0 - S_0) (1.02 + S_L)$

$$r = S_0 - \pi_0$$

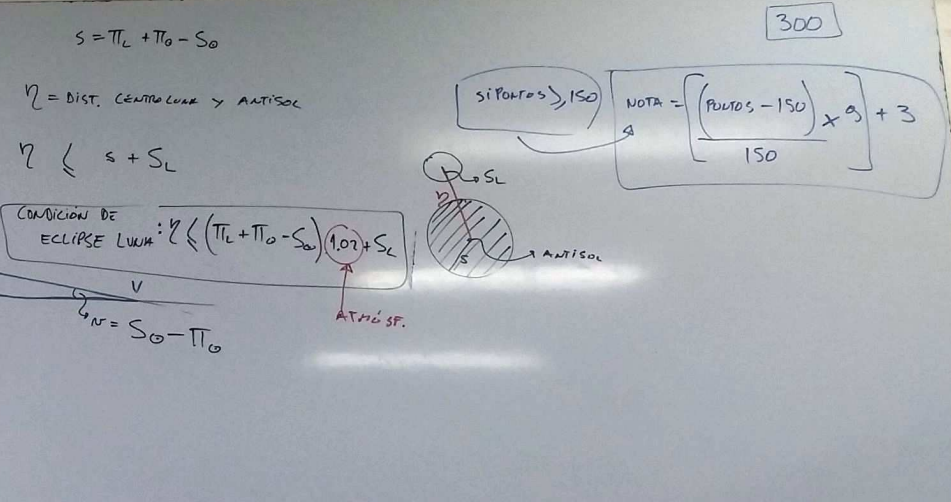
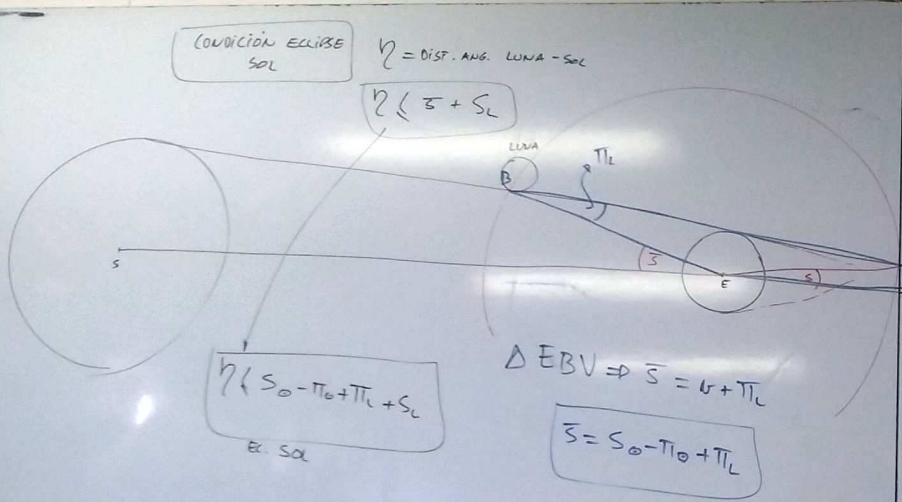
300

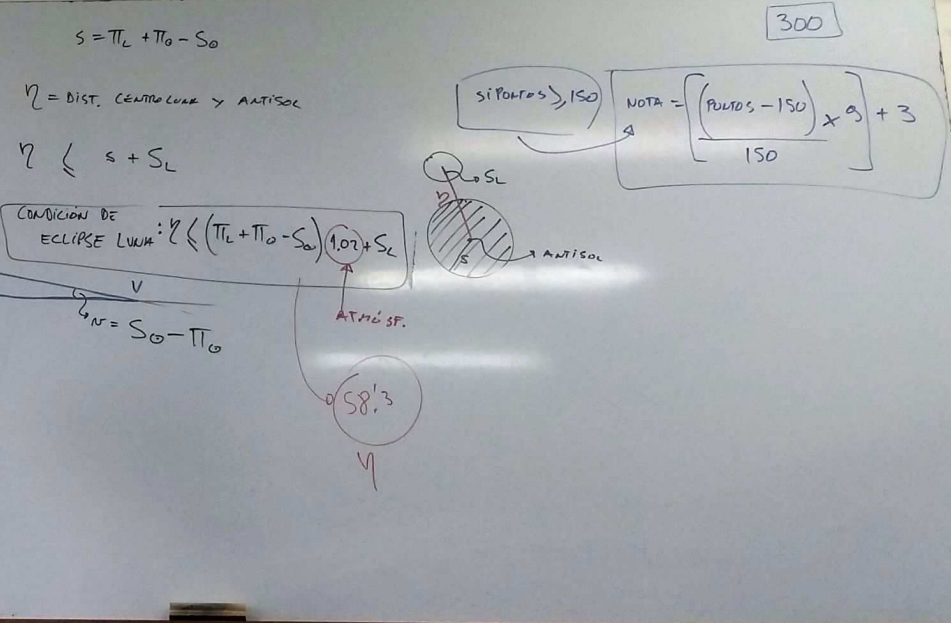
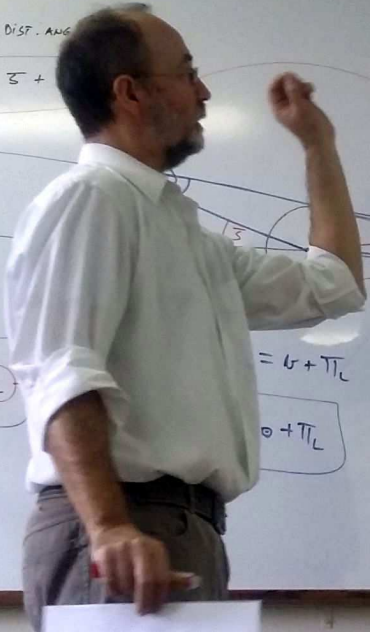
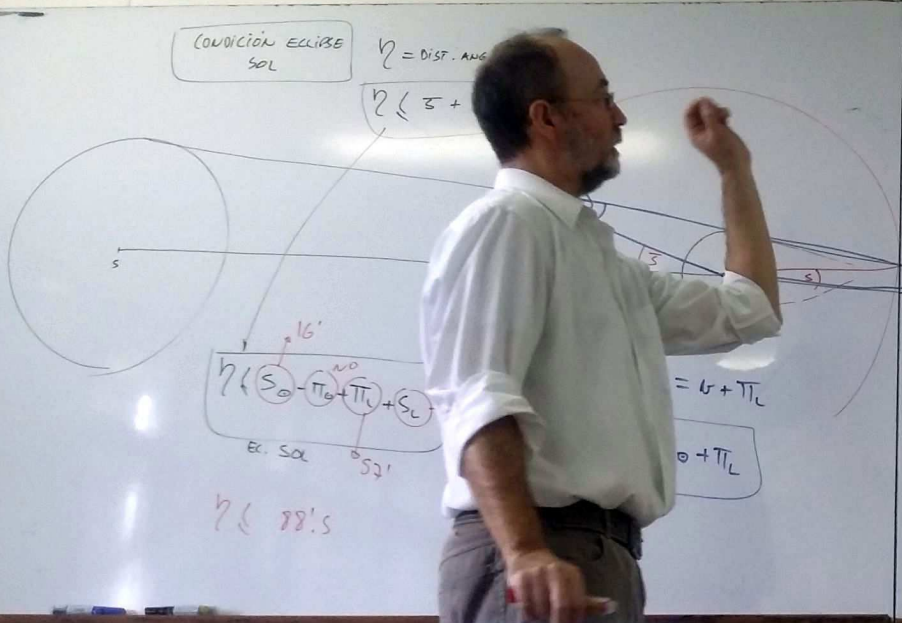
SI PUNTOS > 150

$$NOTA = \left[\frac{PUNTOS - 150}{150} \times 9 \right] + 3$$



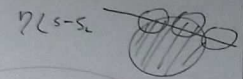
ATMÓS.



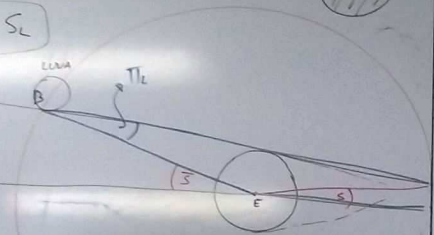


CONDICIÓN ECLIPSE SOL

$\eta = \text{DIST. ANG. LUNA - SOL}$



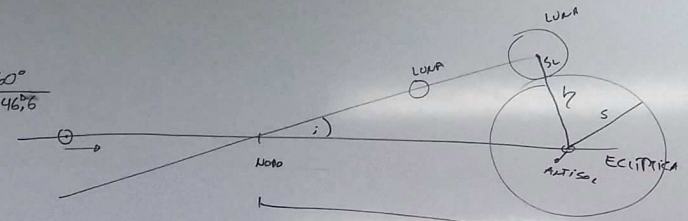
$\eta \leq S + S_L$



$\Delta EBV \Rightarrow \bar{S} = \eta + \pi_L$

$\bar{S} = S_0 - \pi_{10} + \pi_L$

$\frac{360^\circ}{346.6}$



$\Delta \lambda = \eta / \sin i$

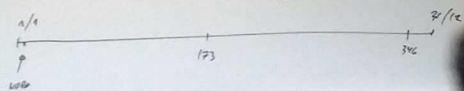
$\mu_{\eta i} = \frac{\eta}{\Delta \lambda}$

$2\Delta \lambda = \frac{2\eta \rightarrow \text{Sp's}}{\mu_{\eta i}} \cong 22^\circ$

$360 \rightarrow 346.6$
 $22^\circ \rightarrow$

21 DIAS

PERIODO SIMBIO LUNA
29.5



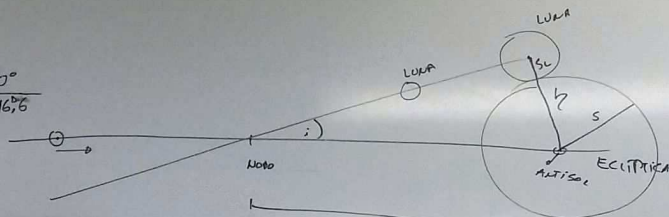
MAX N° EC. LUNA → 3

MIN " " → 0

→ 5

$$\eta_{lim} = 88' S$$

$$\frac{360^\circ}{346,76}$$



$$\Delta\lambda = \eta / \sin i$$

$$\eta_{mi} = \frac{\eta}{\Delta\lambda}$$

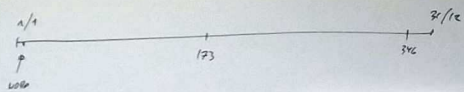
$$2\Delta\lambda = \frac{2 \overset{SP'S}{\eta}}{\eta_{mi}} \cong 22^\circ$$

$$360 \rightarrow 346,76$$

$$22 \rightarrow$$

21 DIAS

PERÍODO SÍMBOLO LUNA
29,5



MAX N° EC. LUNA → 3
 MIN. N° " → 0

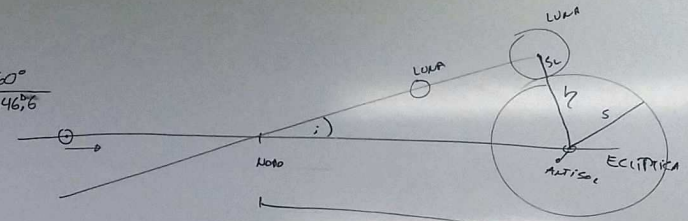
EC. SOL → 3
 $\eta_{lim} = 88' S$

MIN. N° EC SOL → 2
 MAX N° EC SOL → 5

$$2\Delta\lambda = \frac{2\eta_{lim}}{\tan i} \Rightarrow \Delta\lambda \approx 31.5^\circ$$

Sol

$$\frac{360^\circ}{346.76}$$



$$\Delta\lambda = \eta / \tan i$$

$$\tan i = \frac{r}{\Delta\lambda}$$

$$360 \rightarrow 346.4$$

$$22 \rightarrow$$

$$2\Delta\lambda = \frac{2\eta^{Sp's}}{\tan i} \approx 22^\circ$$

Δt 21 DIAS
 LUNA

PERÍODO SÍMBIO LUNA
 29^d