Introduction to Ring Dynamics Lecture #2

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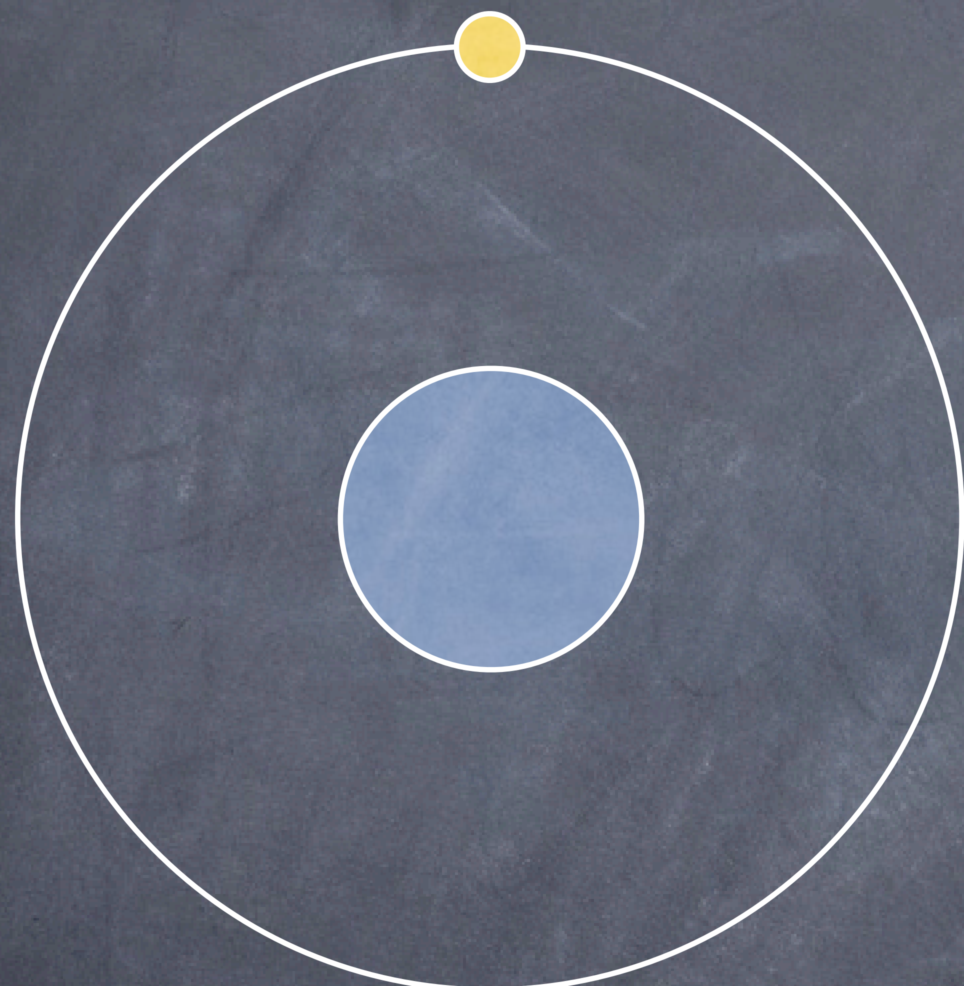
SETI Institute

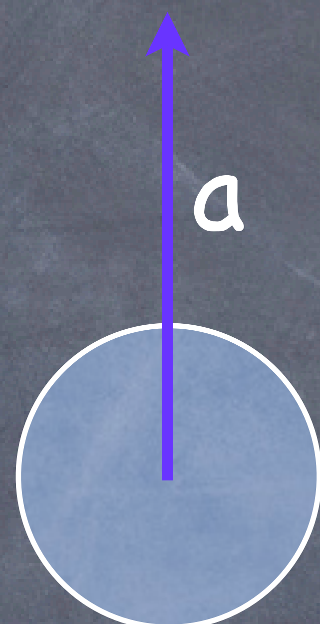
COSPAR WORKSHOP
Wednesday, July 23, 2007

Circular, Equatorial Motion



Top View,
Inertial Frame

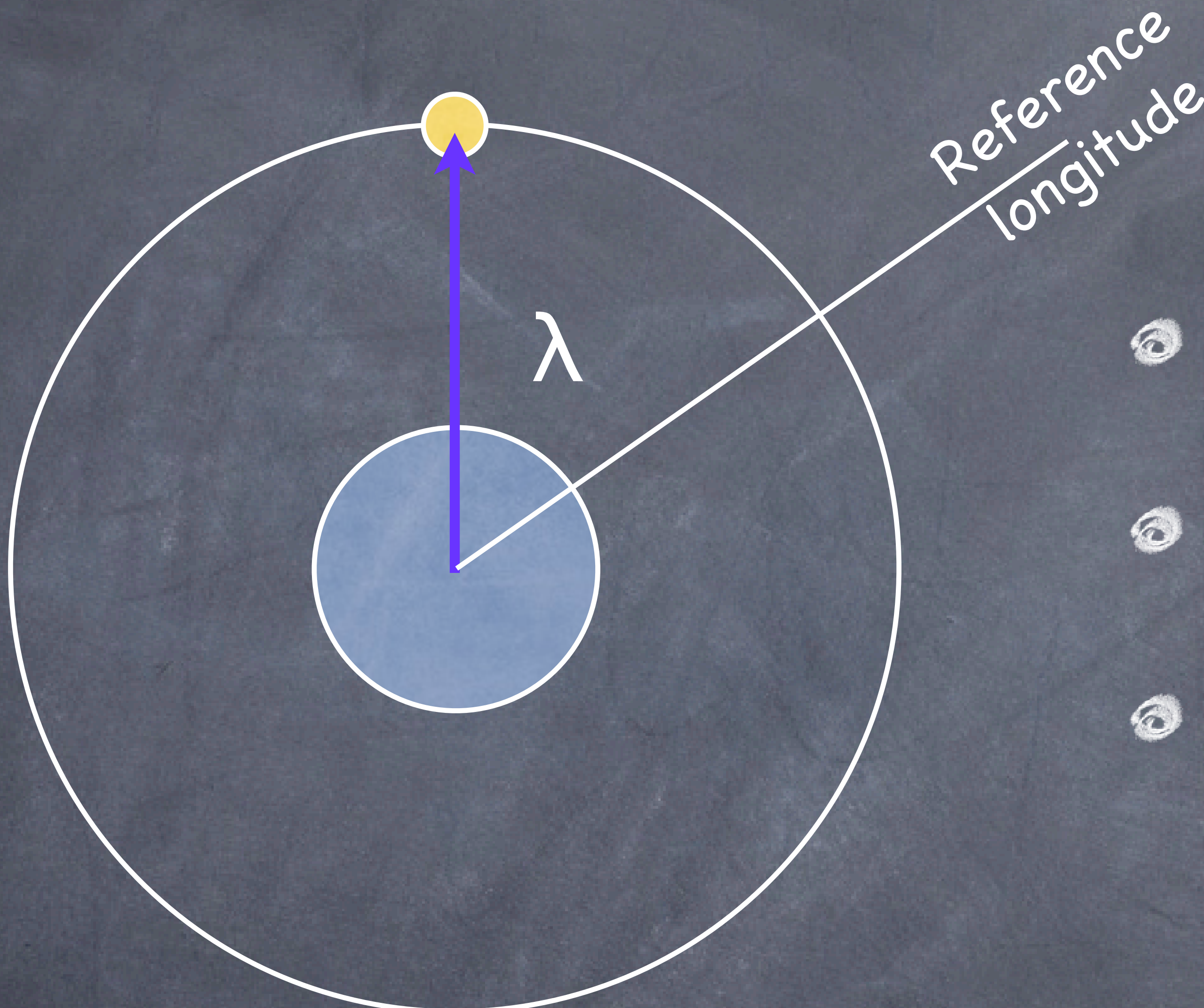
Circular, Equatorial Motion



- Semimajor axis = a
- Mean motion = n
- Orbital period = P
  - P = 2π/n

Top View,
Inertial Frame

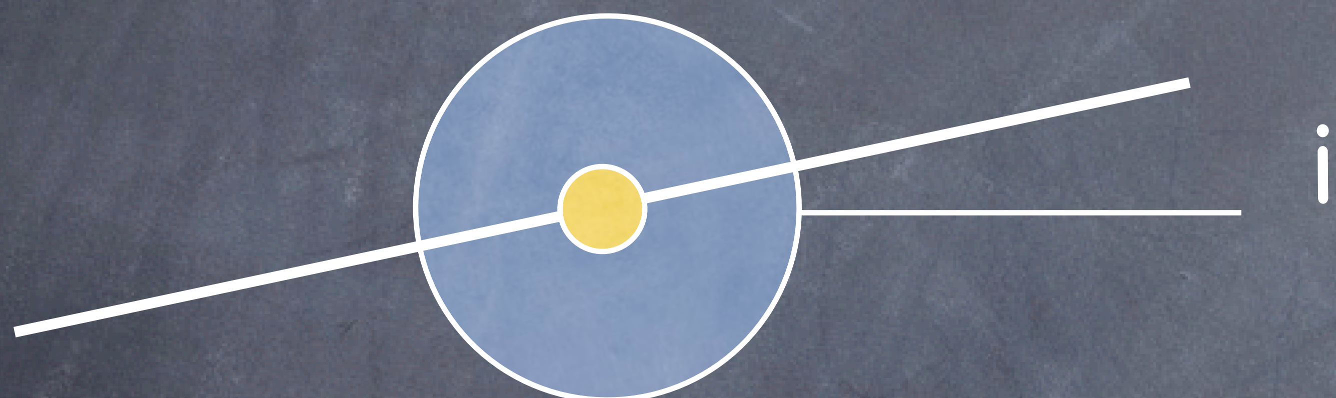
Circular, Equatorial Motion



- Semimajor axis = a
- Mean motion = n
- Orbital period = P
  - P = 2π/n
- Mean longitude at epoch = λ

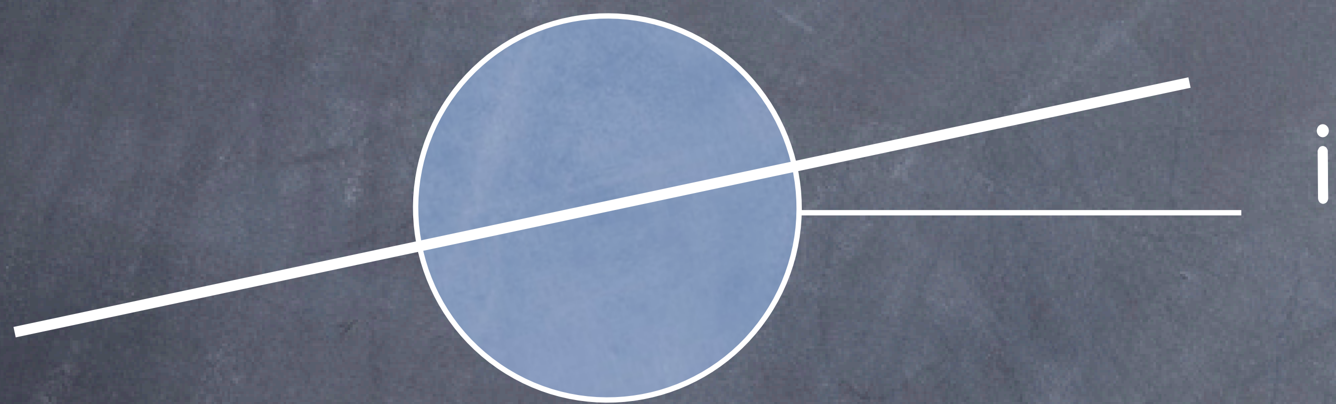
Top View,
Inertial Frame

Inclined Motion



Side View,
Inertial Frame

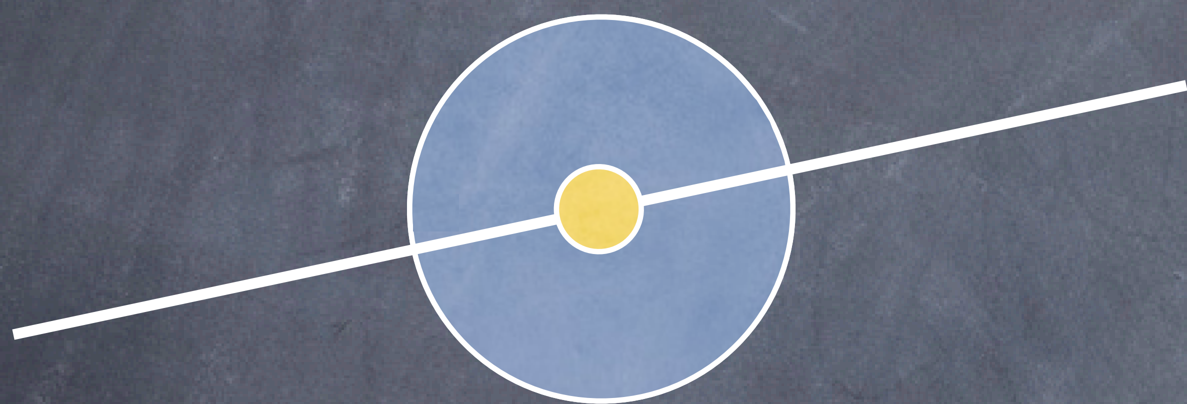
Inclined Motion



- Inclination = i
- Vertical frequency = ν

Side View,
Inertial Frame

Inclined Motion

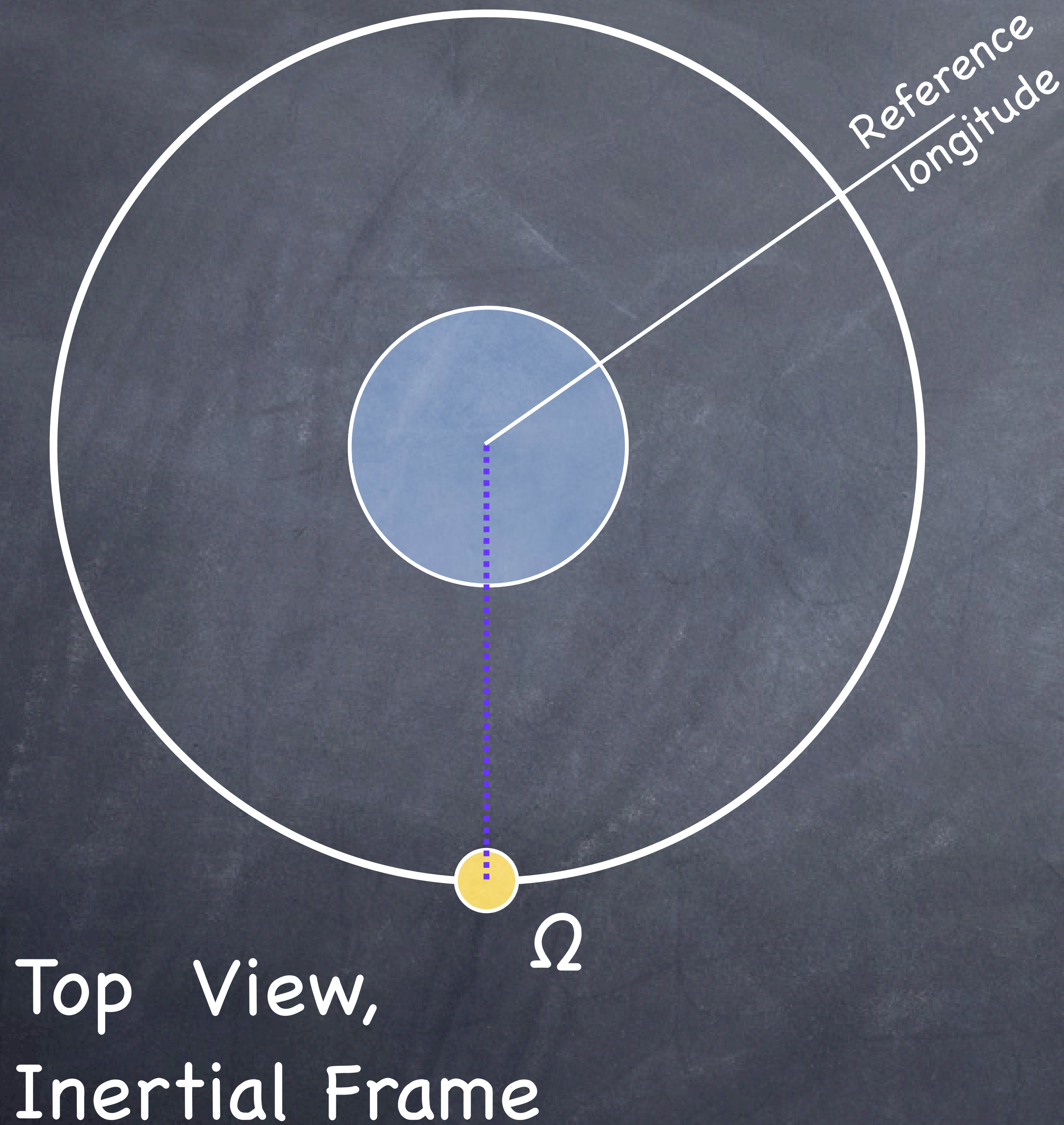


Side View,
Inertial Frame

- Inclination = i
- Vertical frequency = ν
- Longitude of ascending node = Ω

(crossing from below to above the equator)

Inclined Motion



- Inclination = i
- Vertical frequency = ν
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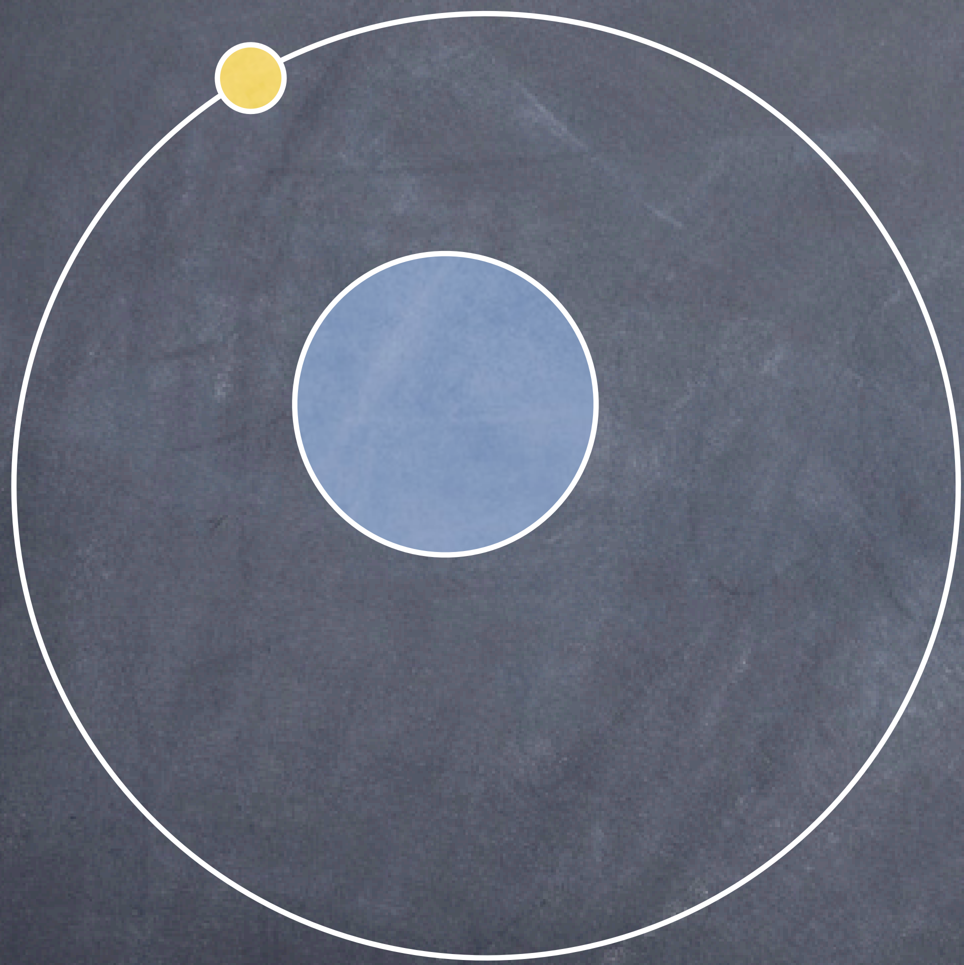
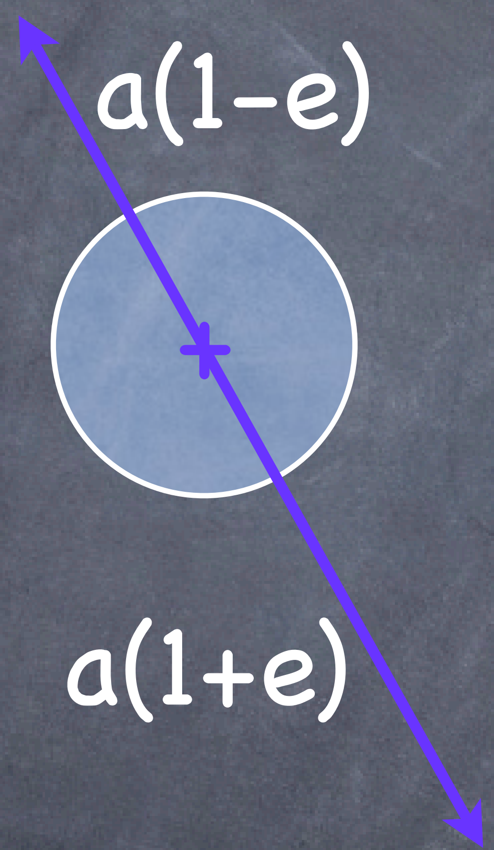
  (crossing from below to above the equator)

Eccentric Motion



Top View,
Inertial Frame

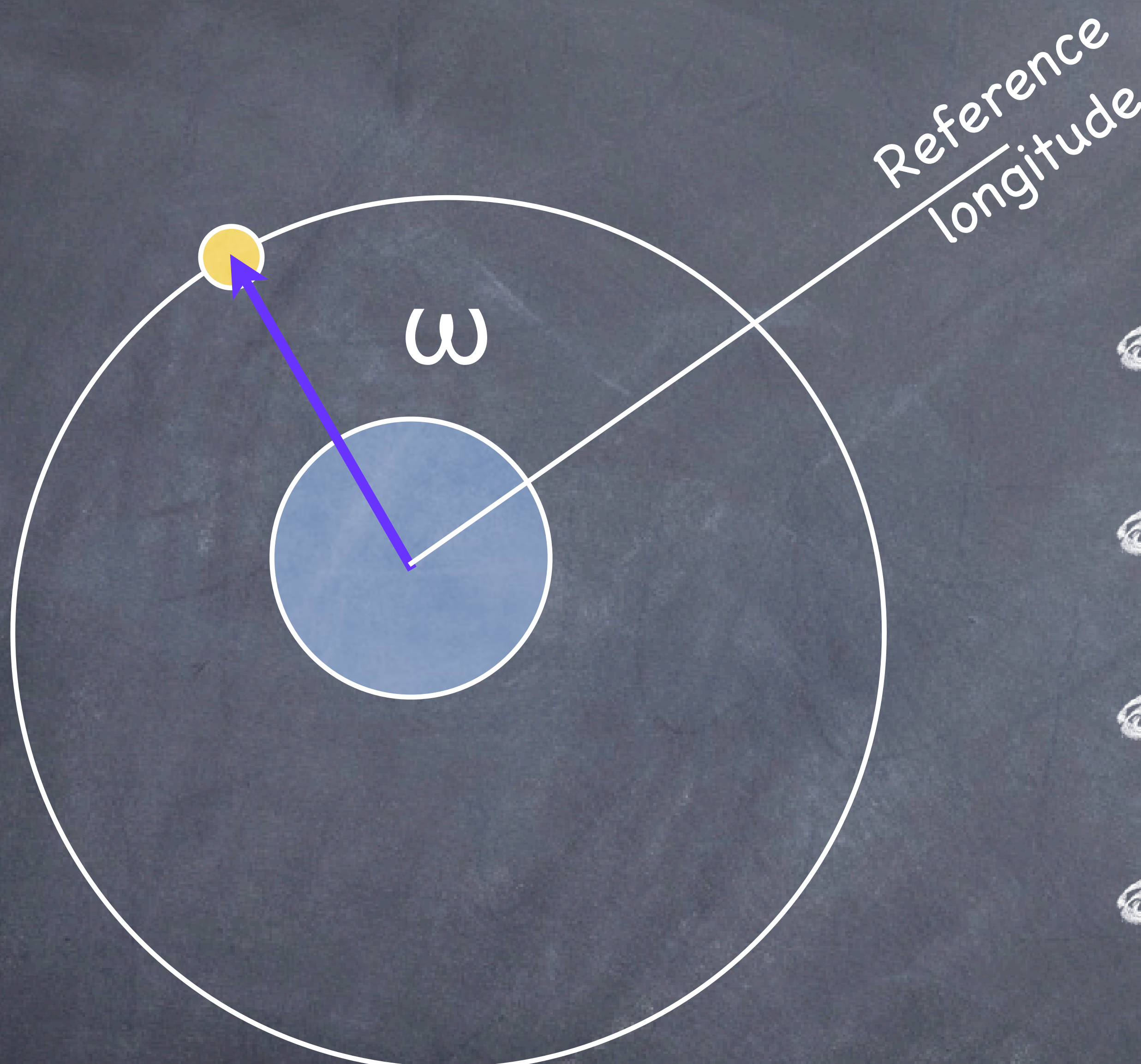
Eccentric Motion



- Eccentricity = e
- Pericenter at a(1-e)
- Apocenter at a(1+e)
- Radial ("epicyclic") frequency = κ

Top View,
Inertial Frame

Eccentric Motion

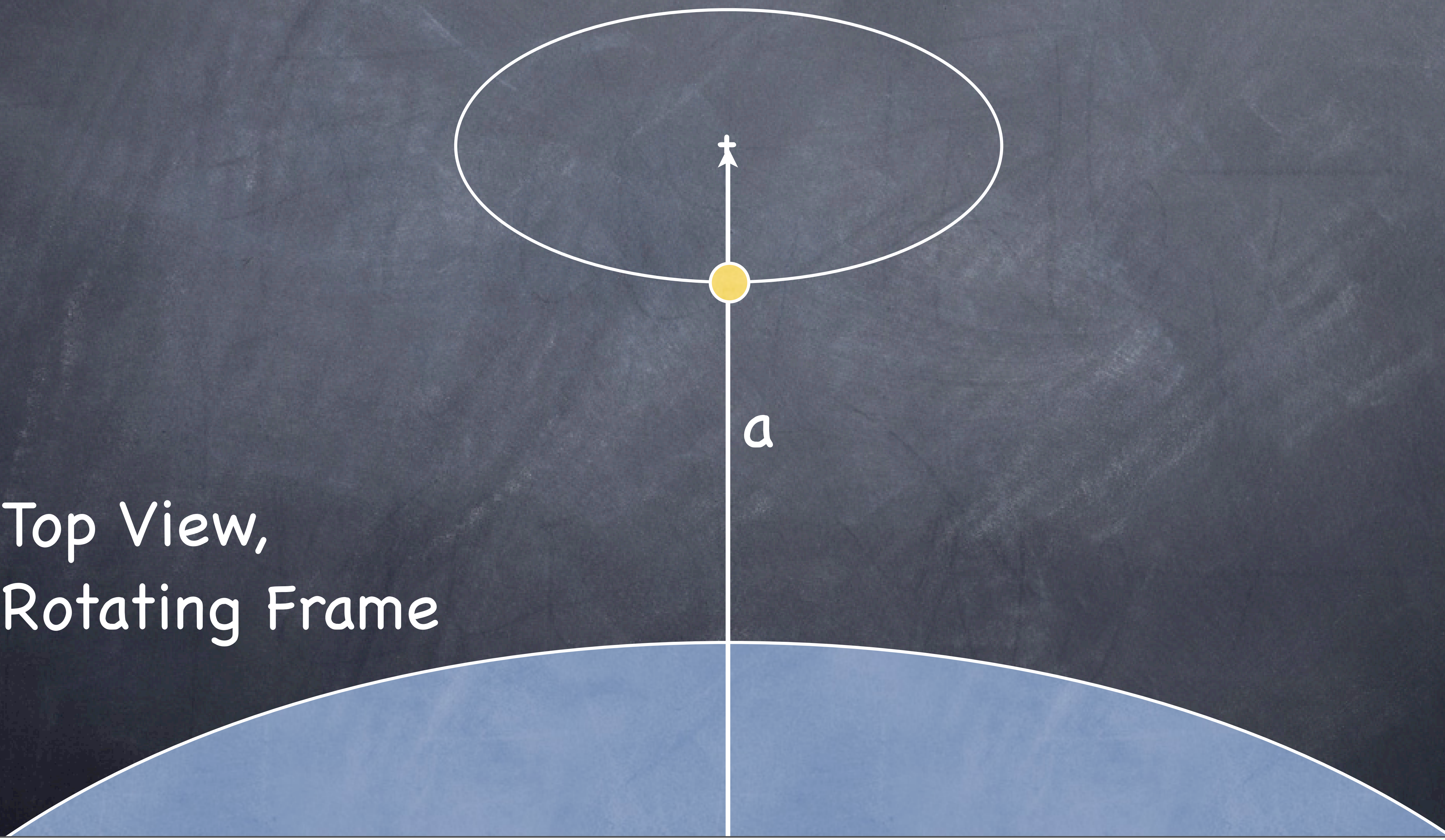


- Eccentricity = e
- Pericenter at $a(1-e)$
- Apocenter at $a(1+e)$
- Radial ("epicyclic") frequency = κ
- Longitude of pericenter = ω

Top View,
Inertial Frame

Epicyclic Motion:

Eccentric Motion viewed in a Rotating Frame



Top View,
Rotating Frame

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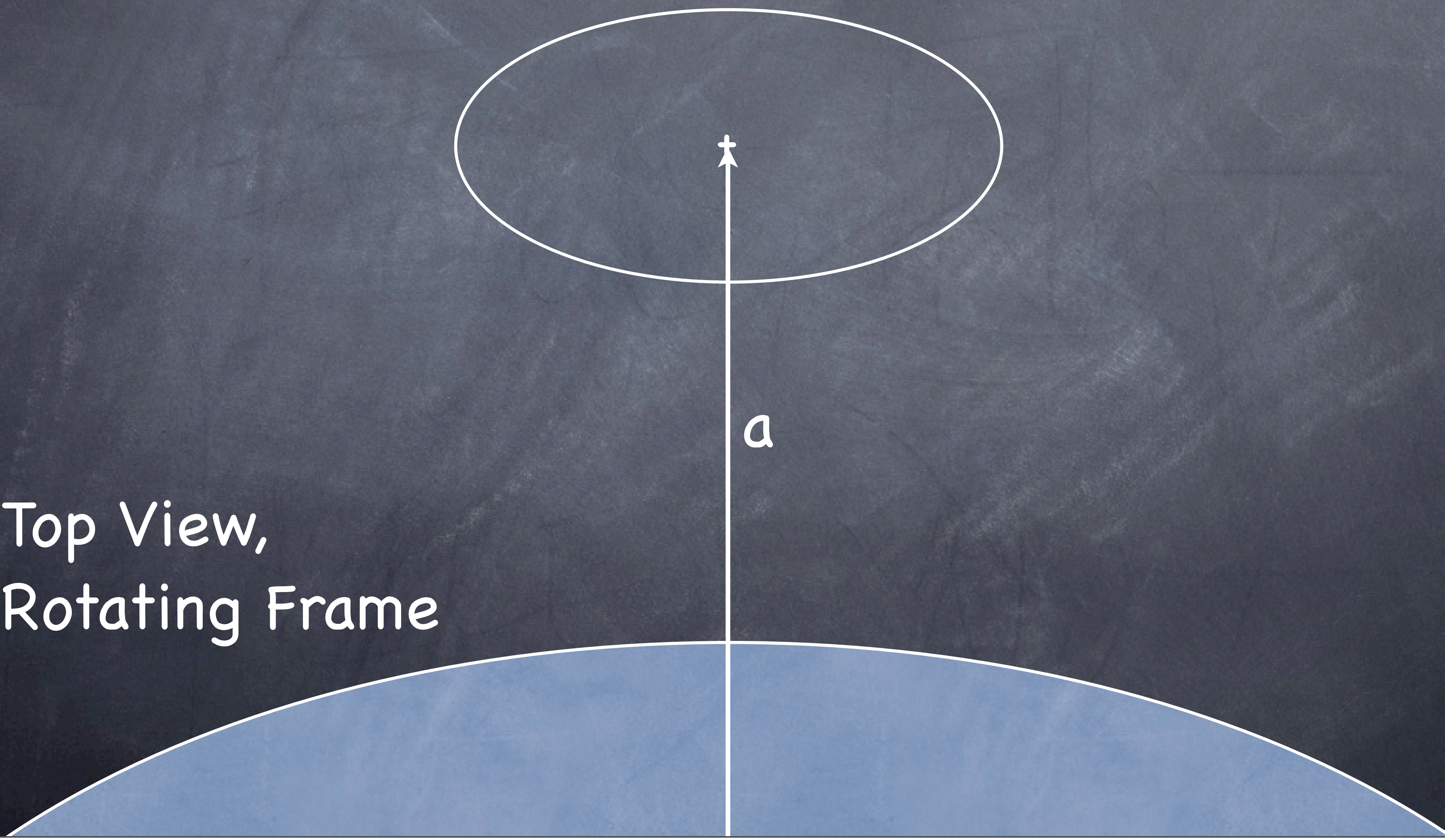

Top View,
Rotating Frame

Epicyclic Motion:

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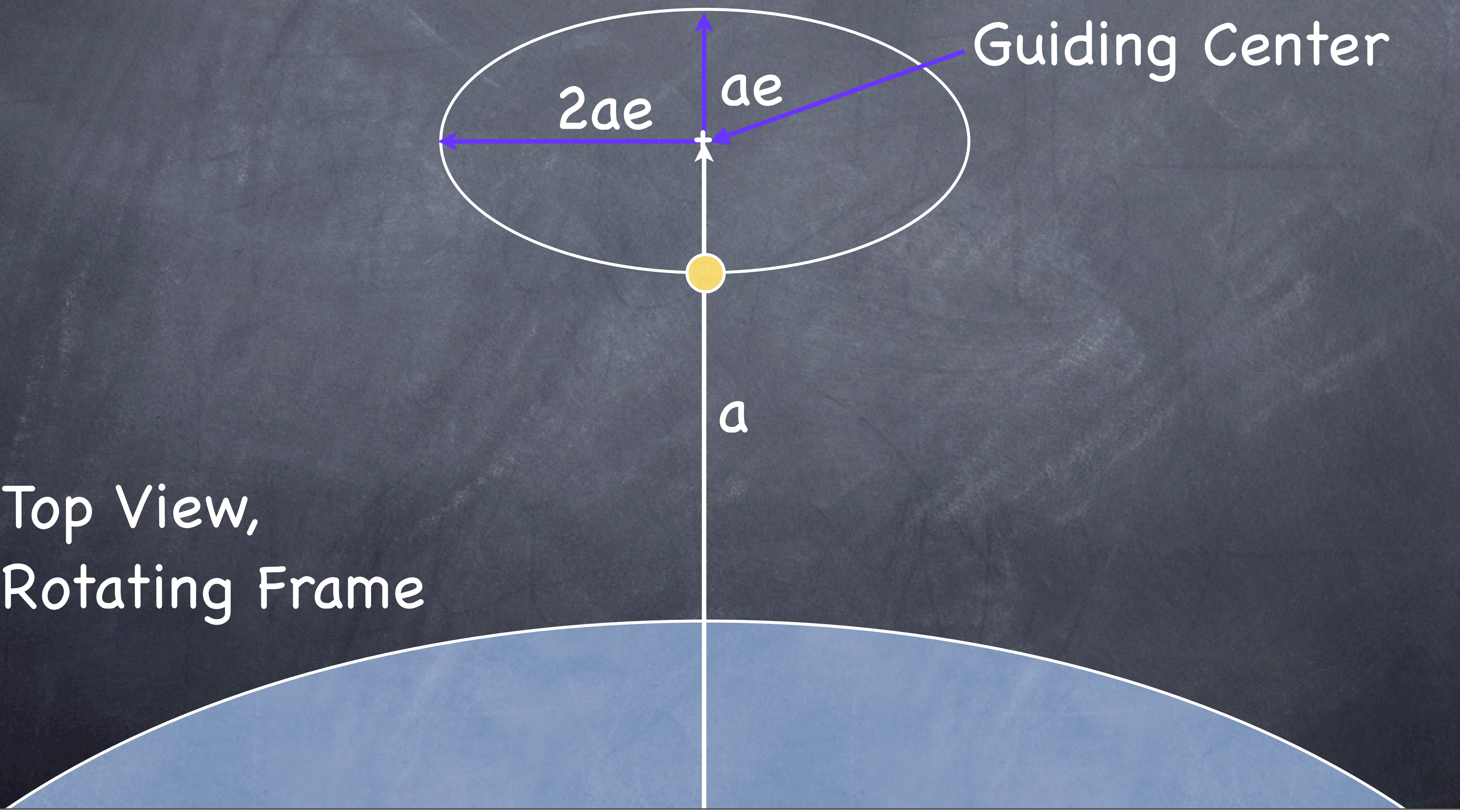
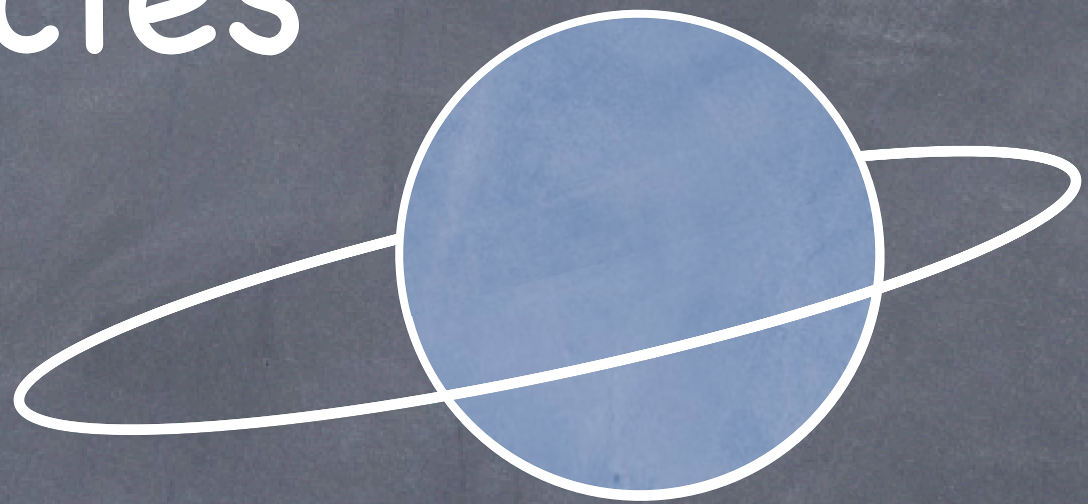


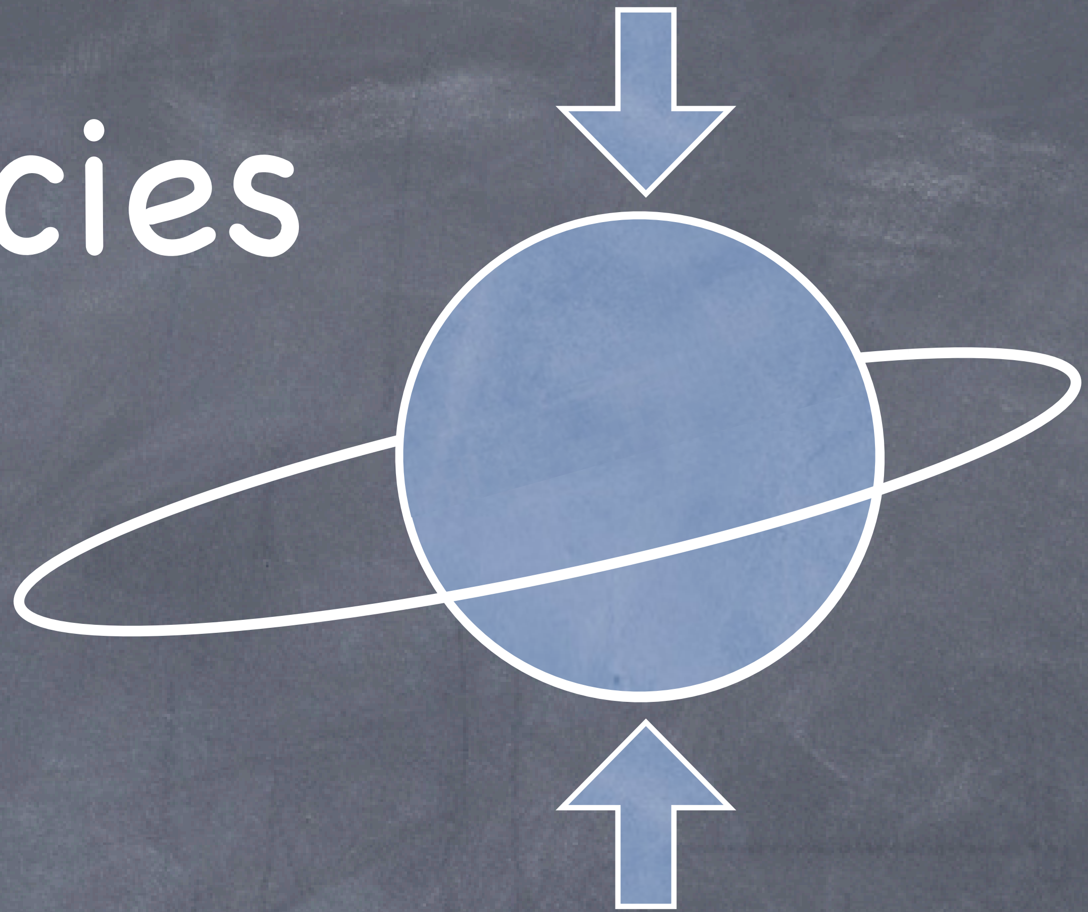
Top View,
Rotating Frame

Three Frequencies



- Mean motion n
  - $n^2 = GM_p/a^3$
- Epicyclic frequency κ
  - $\kappa^2 = GM_p/a^3$
- Vertical frequency ν
  - $\nu^2 = GM_p/a^3$

Three Frequencies



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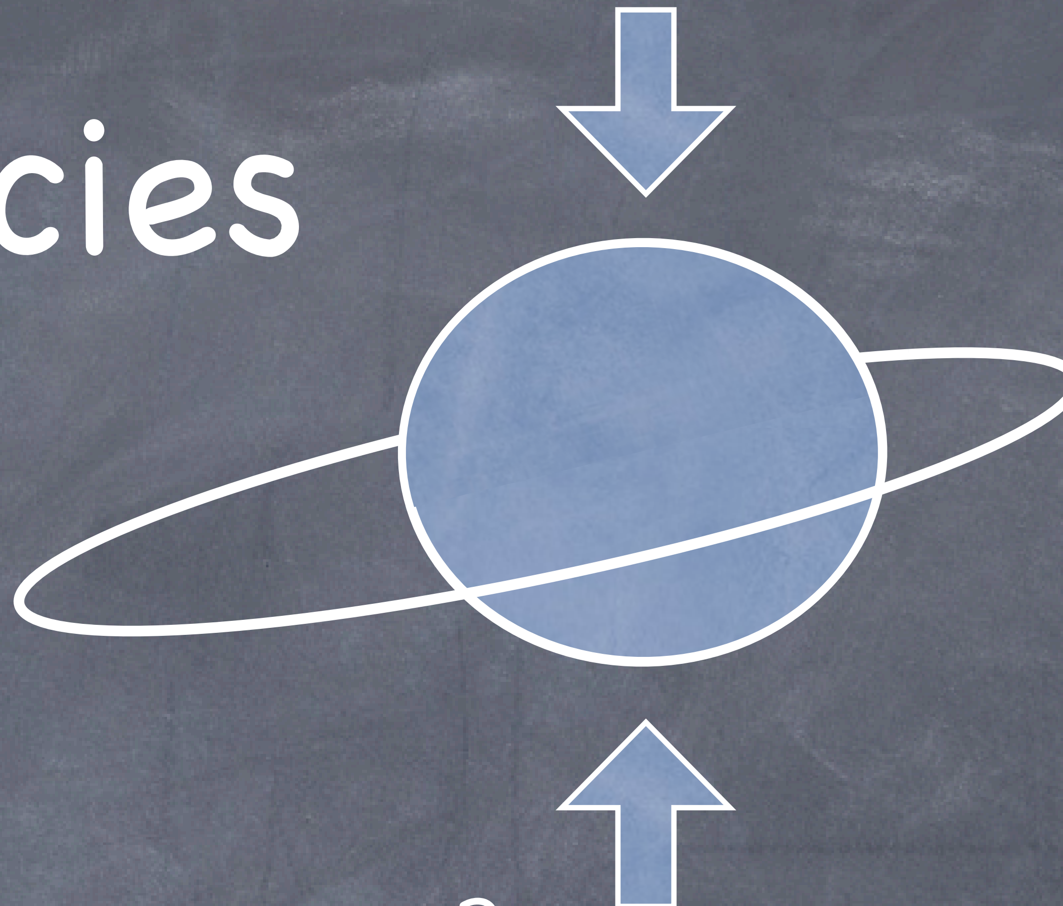
Three Frequencies



- Mean motion n
  - $n^2 = GM_P/a^3 \, [1 + \frac{3}{2} J_2(\frac{R_P}{a})^2 - \frac{15}{8} J_4(\frac{R_P}{a})^4 \ldots]$
- Epicyclic frequency κ
  - $\kappa^2 = GM_P/a^3 \, [1 - \frac{3}{2} J_2(\frac{R_P}{a})^2 + \frac{45}{8} J_4(\frac{R_P}{a})^4 \ldots]$
- Vertical frequency ν
  - $\nu^2 = GM_P/a^3 \, [1 + \frac{9}{2} J_2(\frac{R_P}{a})^2 - \frac{75}{8} J_4(\frac{R_P}{a})^4 \ldots]$

Three Different Frequencies

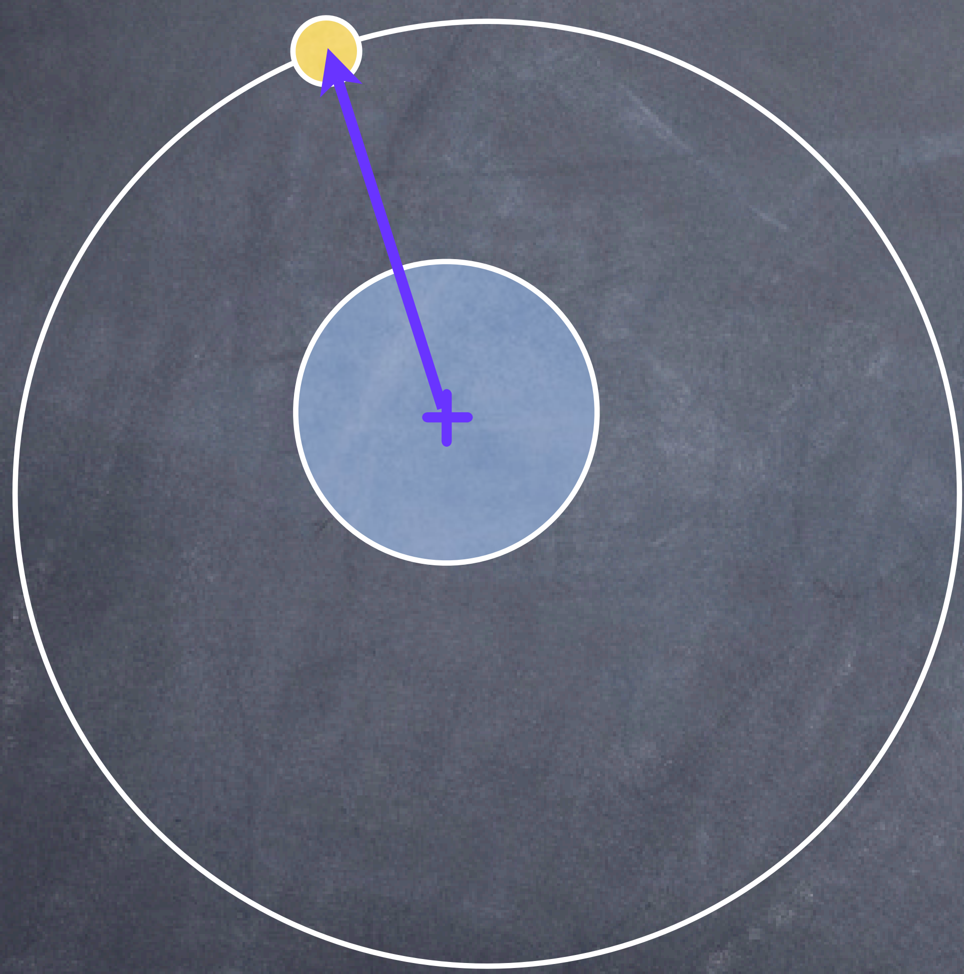
$$n^2 = GM_p/a^3 \left[1 + \frac{3}{2} J_2\left(\frac{R_p}{a}\right)^2 - \frac{15}{8} J_4\left(\frac{R_p}{a}\right)^4 \ldots \right]$$

$$\kappa^2 = GM_p/a^3 \left[1 - \frac{3}{2} J_2\left(\frac{R_p}{a}\right)^2 + \frac{45}{8} J_4\left(\frac{R_p}{a}\right)^4 \ldots \right]$$

$$\nu^2 = GM_p/a^3 \left[1 + \frac{9}{2} J_2\left(\frac{R_p}{a}\right)^2 - \frac{75}{8} J_4\left(\frac{R_p}{a}\right)^4 \ldots \right]$$

- J2, J4, ... are the "gravitational moments".
- J2 can be ~ 1%.
- Terms matter less as semimajor axis increases.
- $\kappa < n < \nu$.

$\kappa < n$: Pericenter Precession

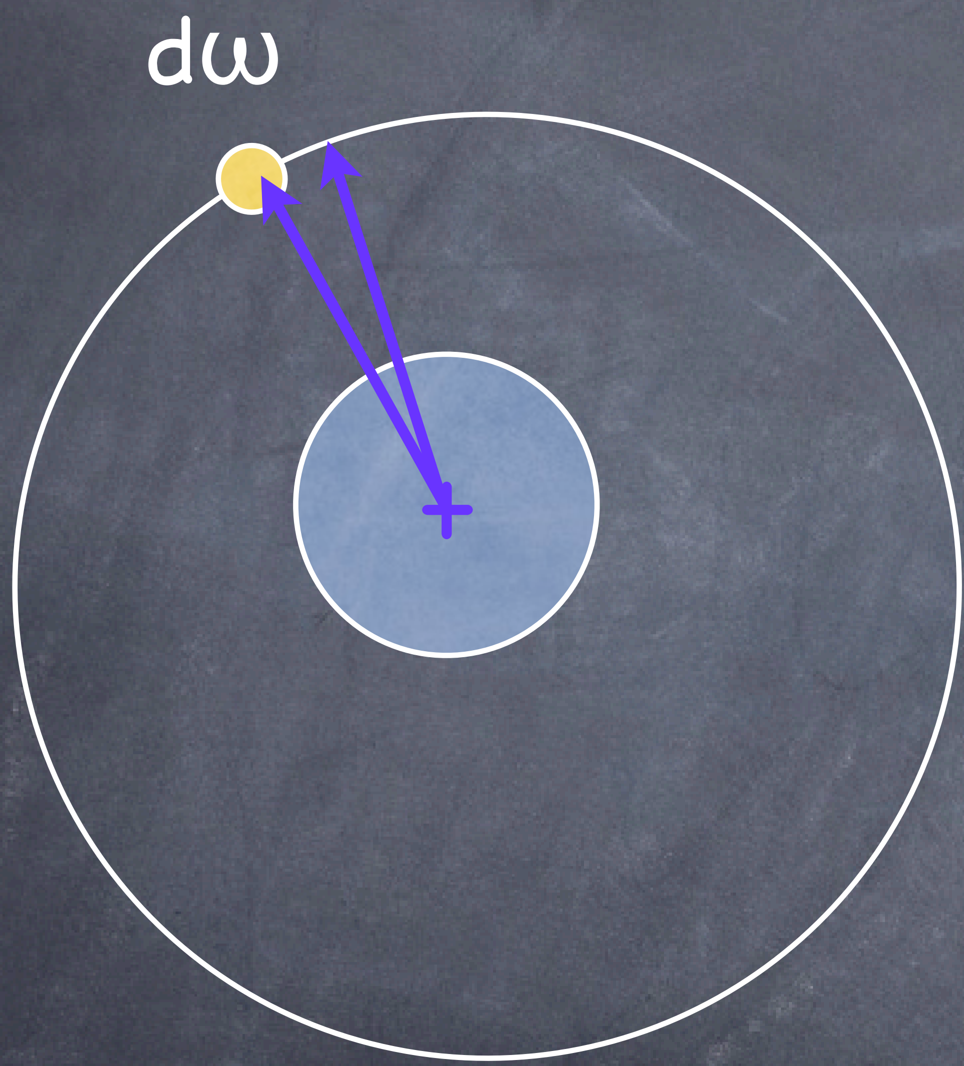


- Epicyclic period $T = 2\pi/\kappa$.
- Moon advances nT ($> 2\pi$).
- Pericenter ω advances

$$nT - 2\pi$$

- Precession rate:
- $\dot{\omega} = n - 2\pi/T = n - \kappa$.

$\kappa < n$: Pericenter Precession



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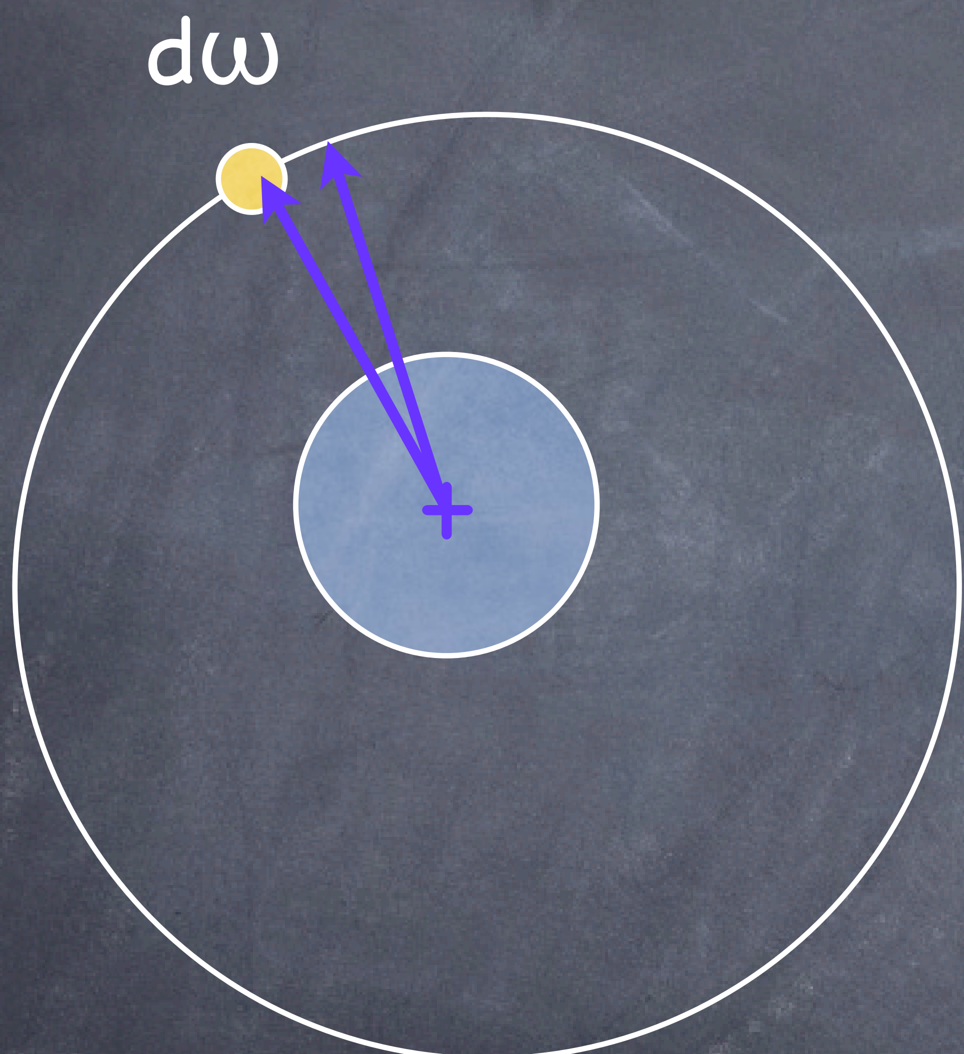

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- Moon advances nT ($> 2\pi$).
- Pericenter ω advances

$$nT - 2\pi$$

- Precession rate:
- $\dot{\omega} = n - 2\pi/T = n - \kappa$.

- Similarly, $n < \nu$ leads to nodal <u>regression</u> at a rate:

$$\dot{\Omega} = n - \nu$$

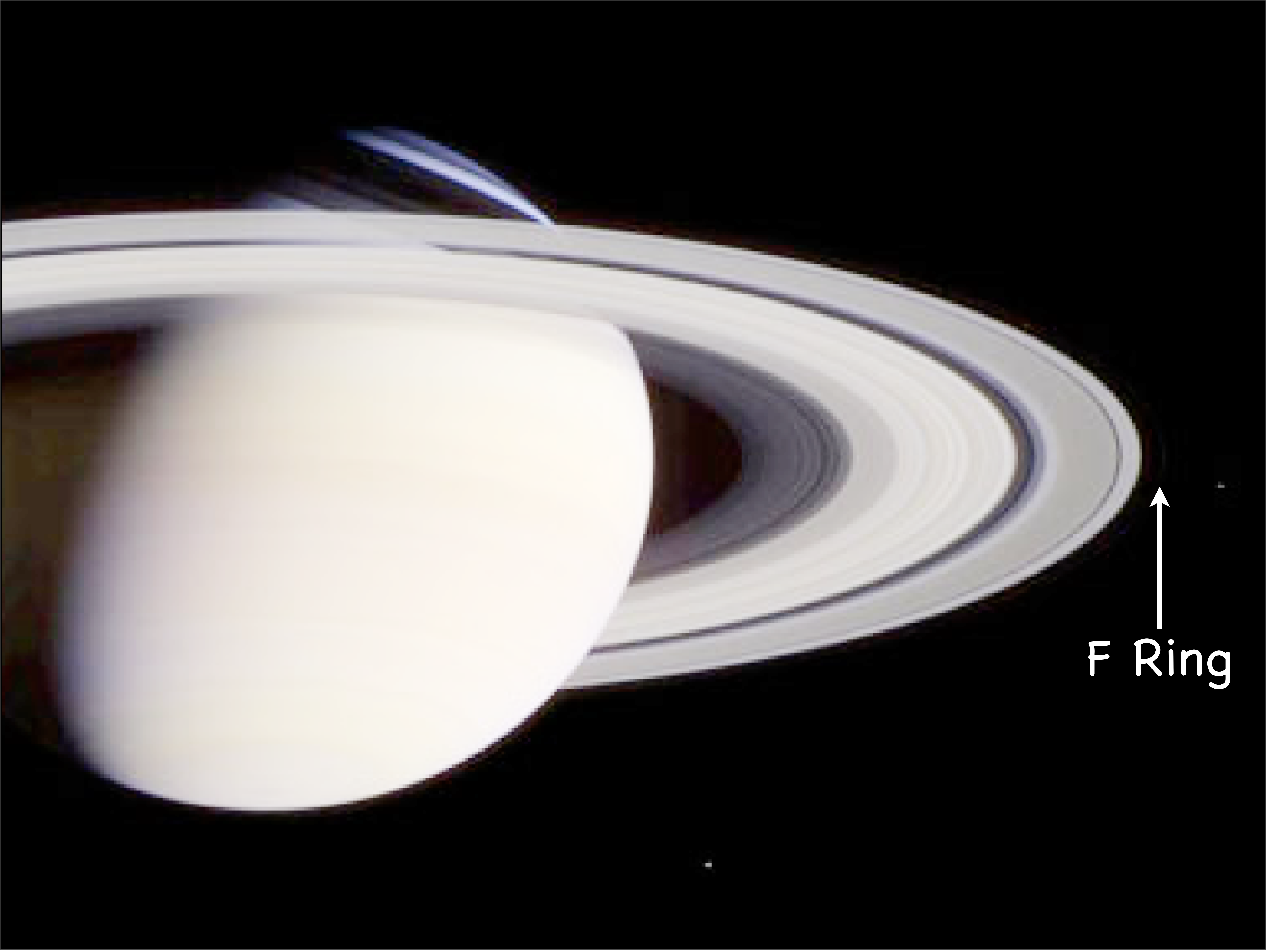
Kepler Shear

- All frequencies are functions of semimajor axis a.
- "Nearby" features do not stay nearby for long.
- Lifetime of a clump of length $\Delta\theta$ and width Δa:

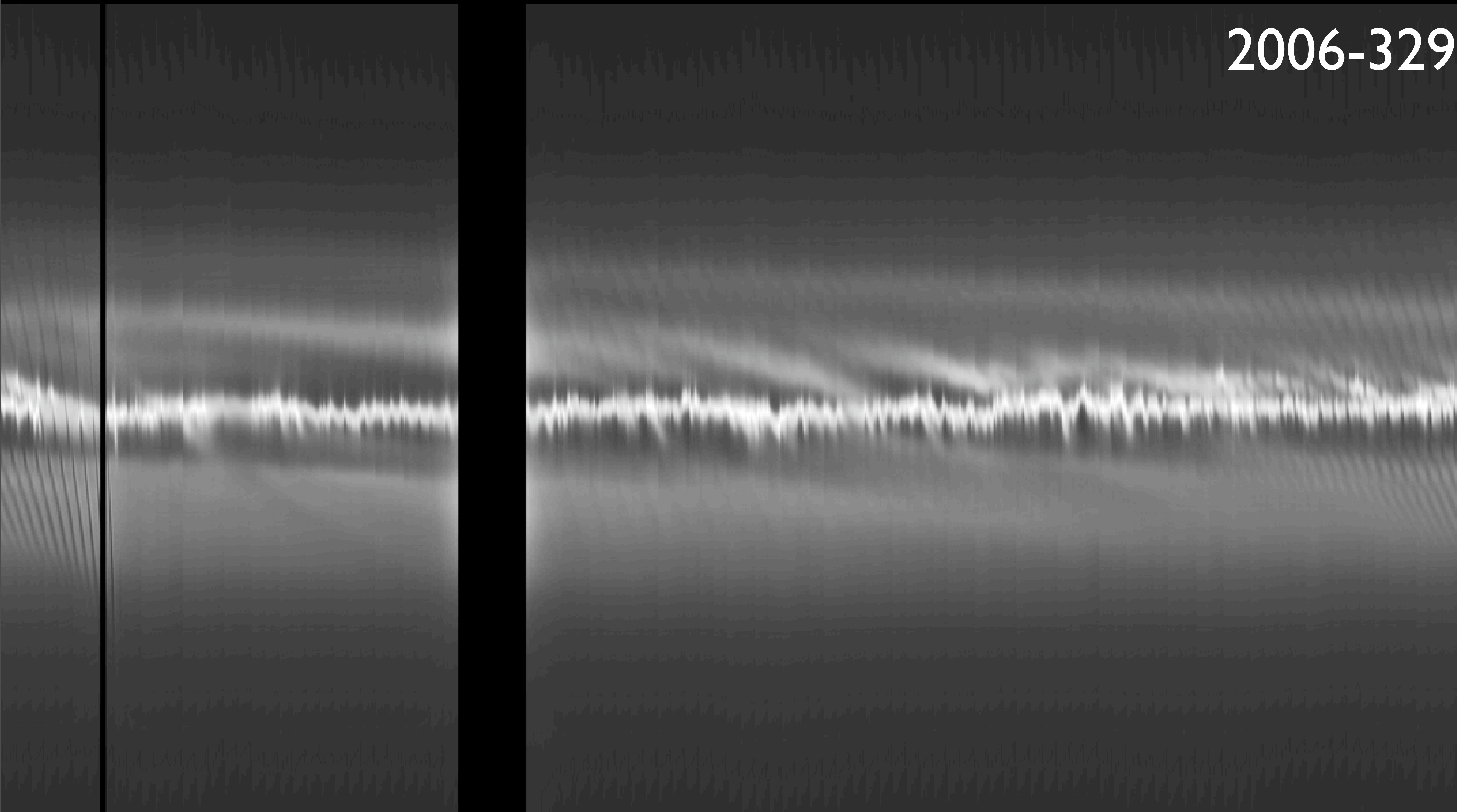
$$\Delta\theta/\Delta n = 2/3\ P\ [\Delta\theta/2\pi]\ [a/\Delta a]$$

⇒ A Saturn feature 1 km × 1° in size at 100,000 km is only ~ 1 year old.

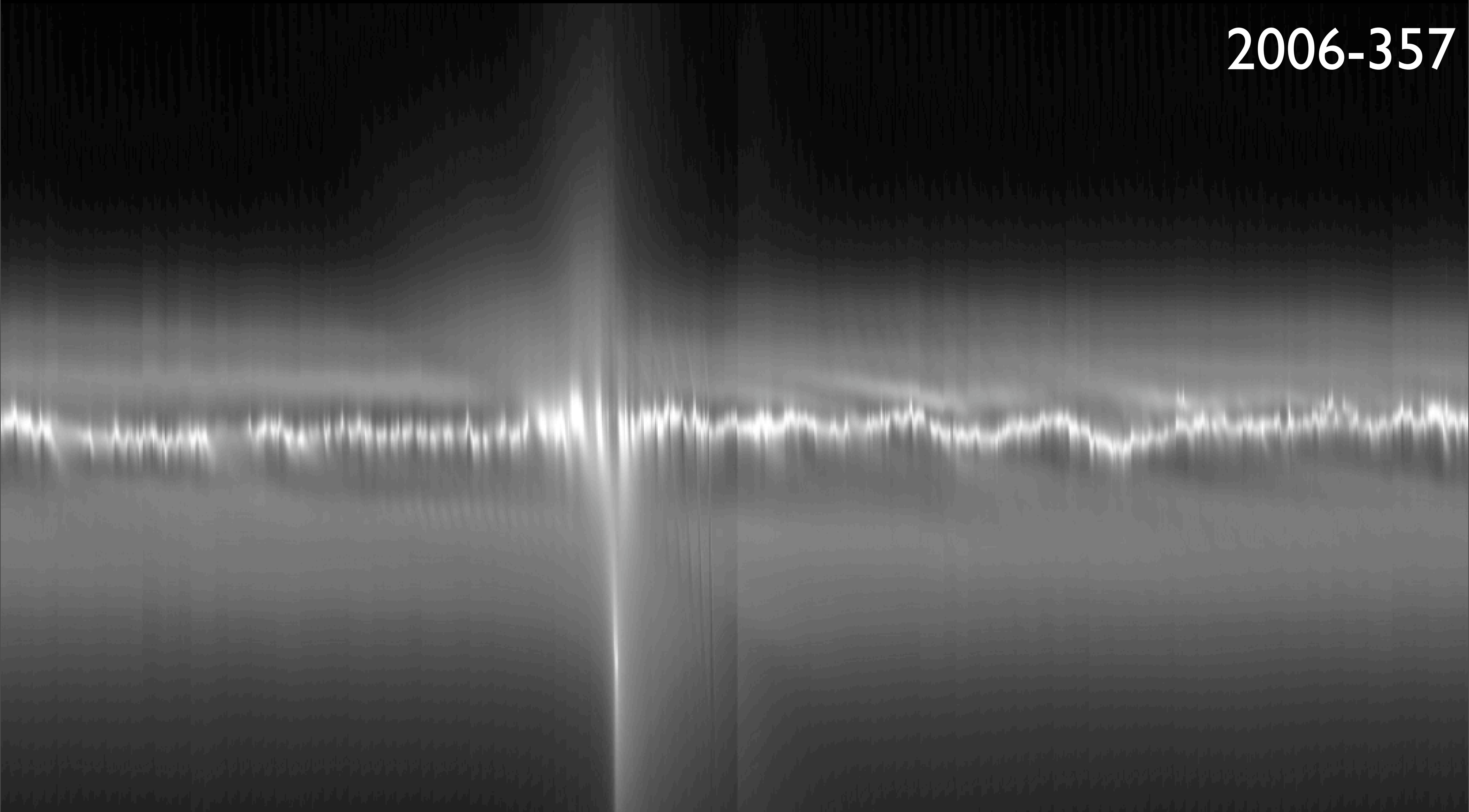
⇒ Clumps in planetary rings must be either young or confined.


F Ring

Transient Structures in Saturn's F Ring



Transient Structures in Saturn's F Ring



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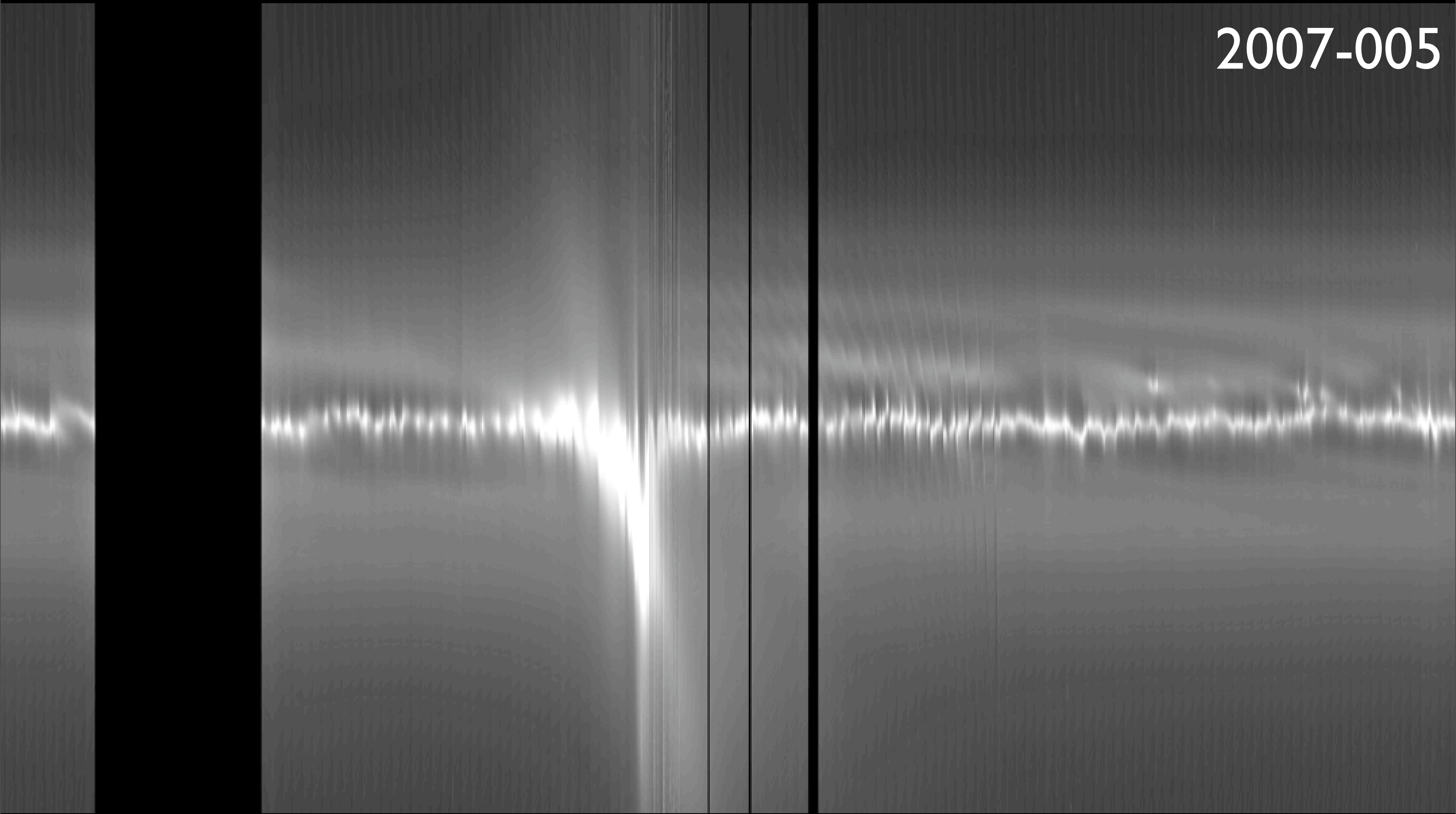

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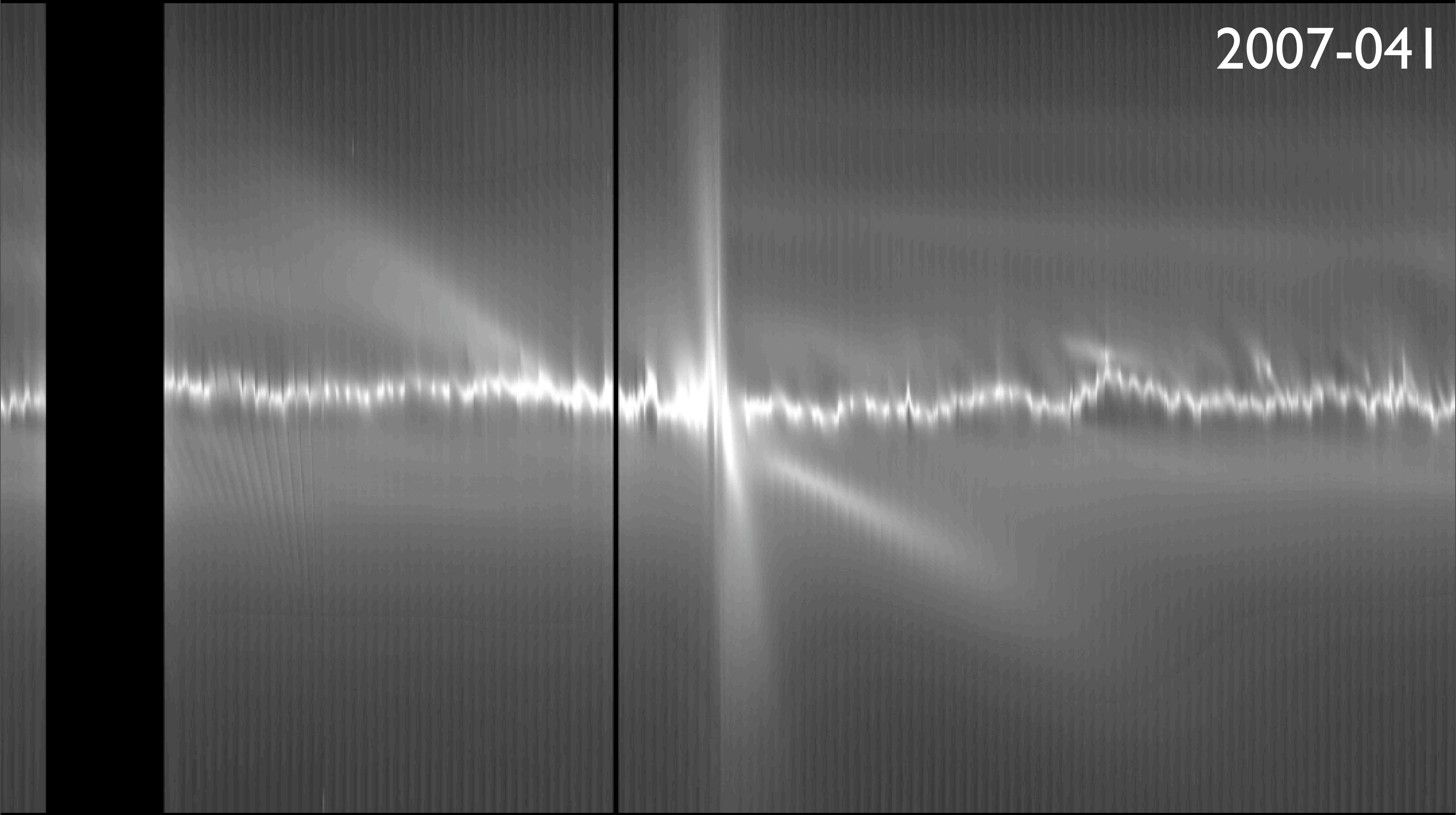

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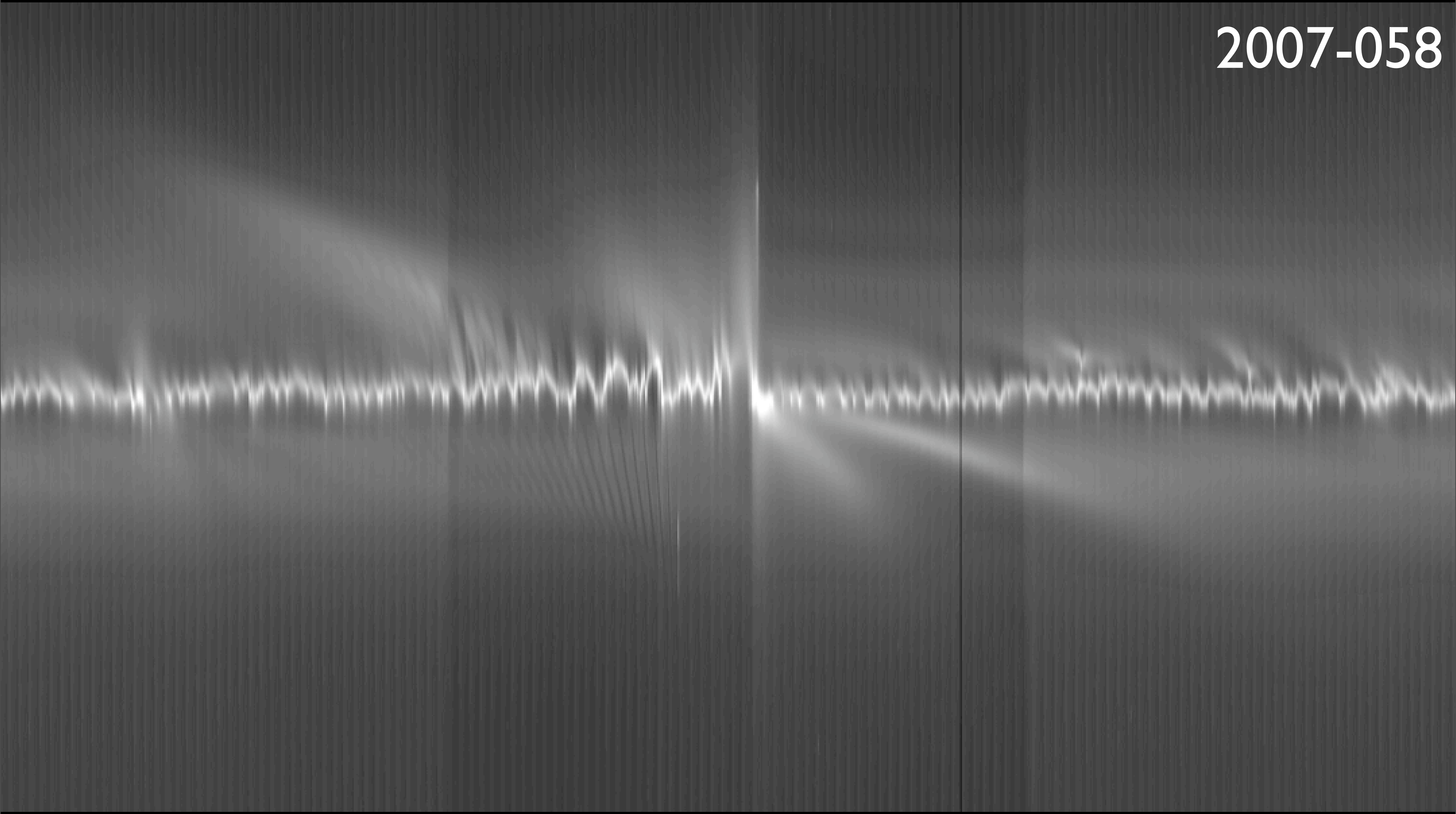
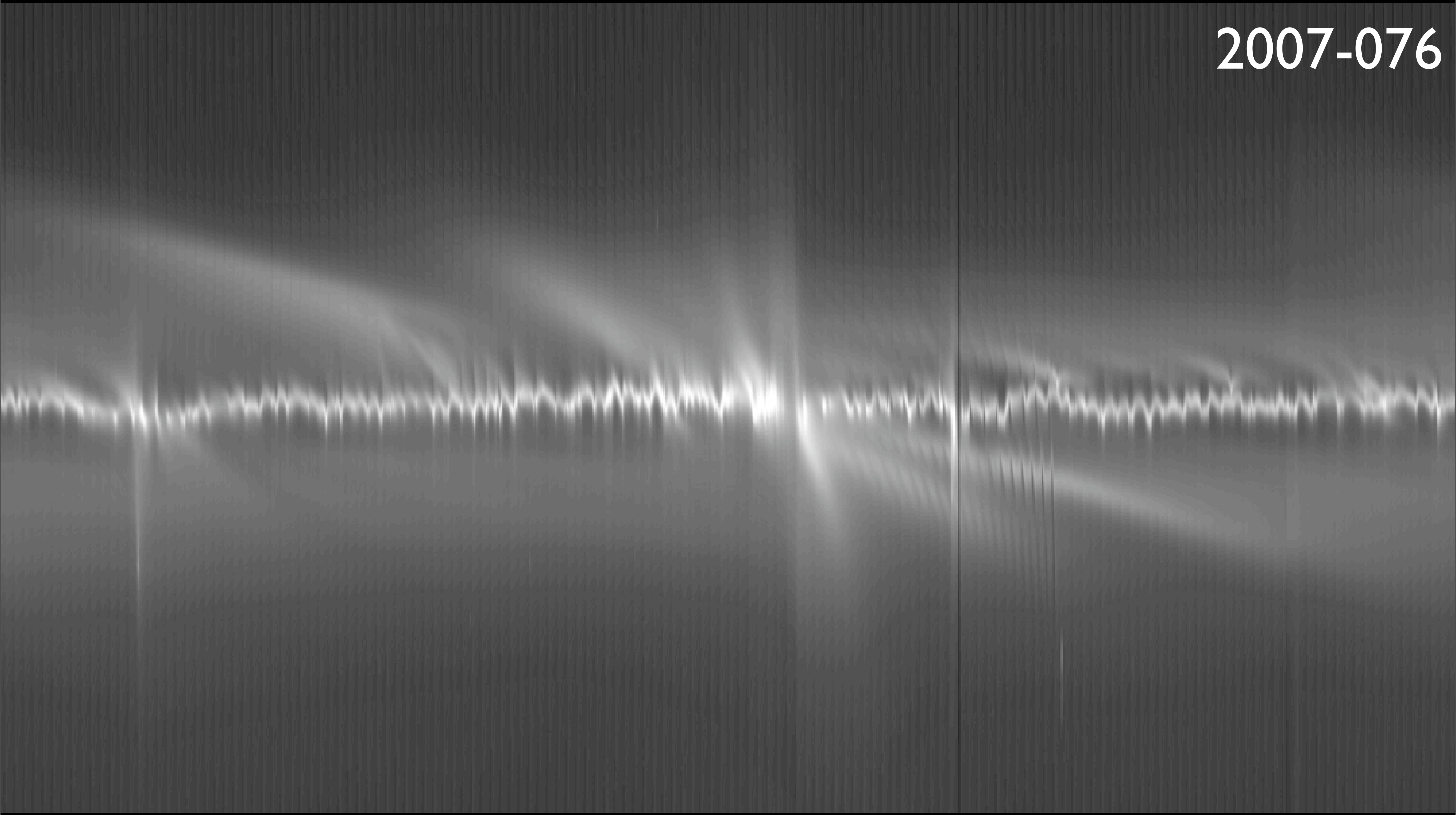


Transient Structures in Saturn's F Ring



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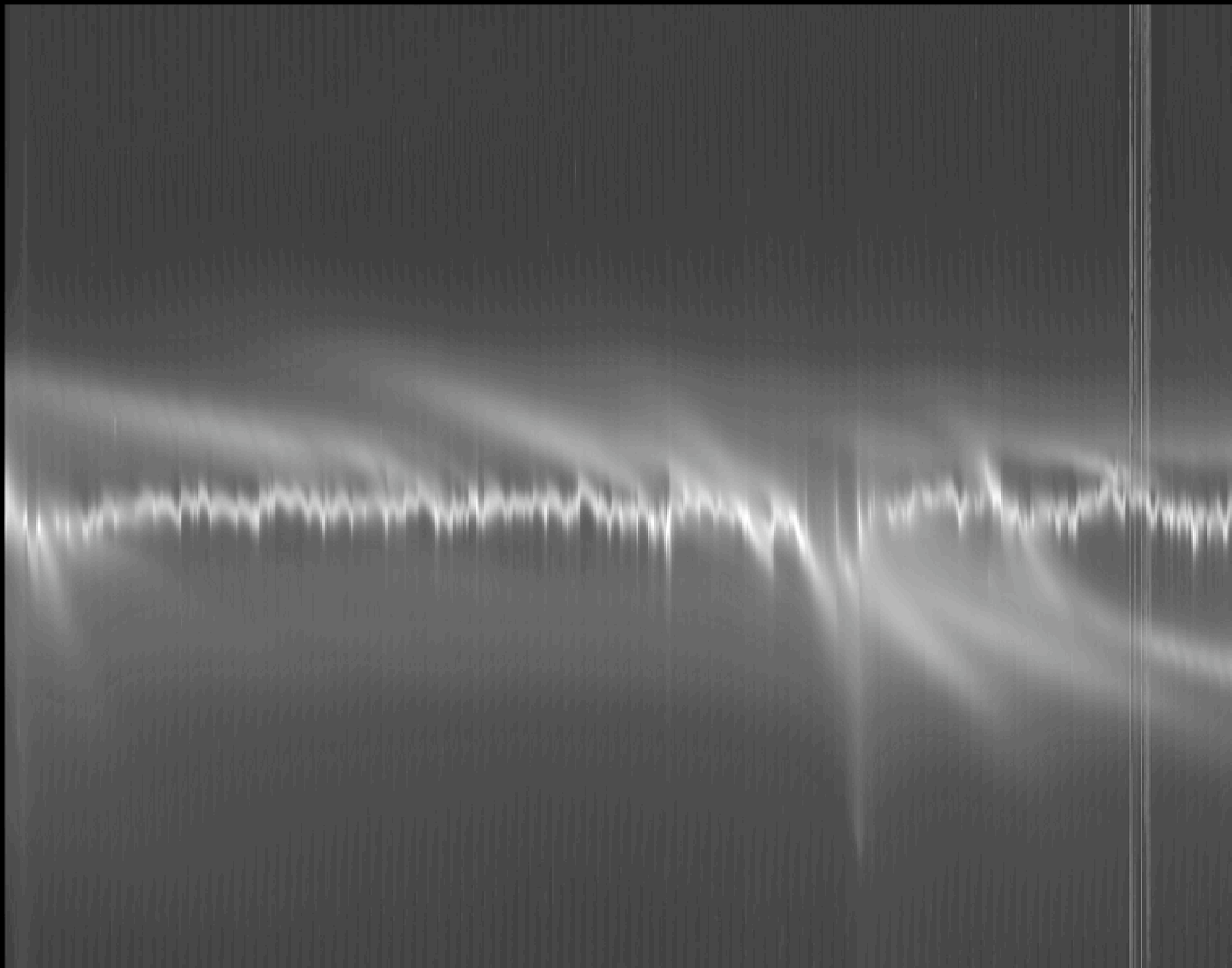

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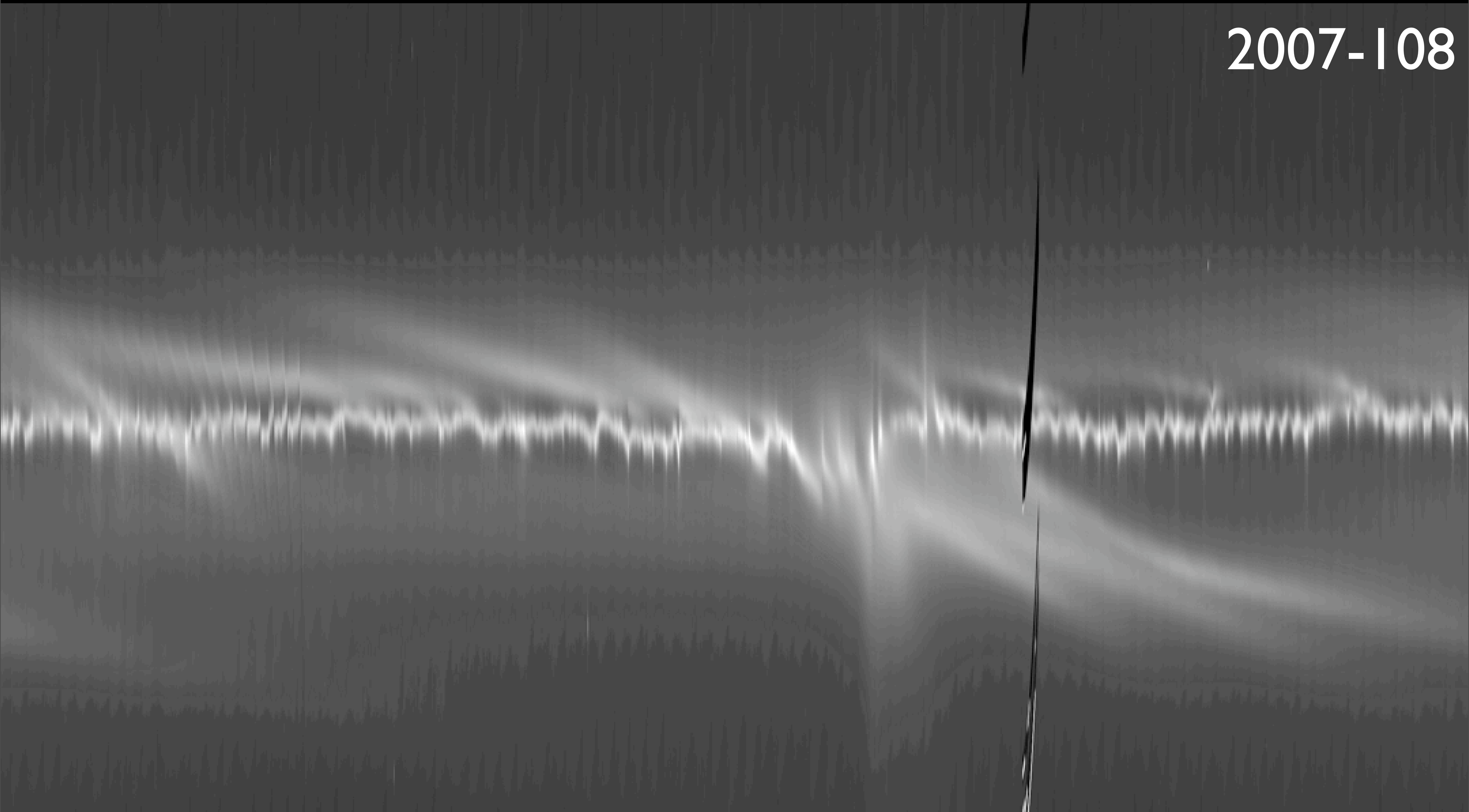
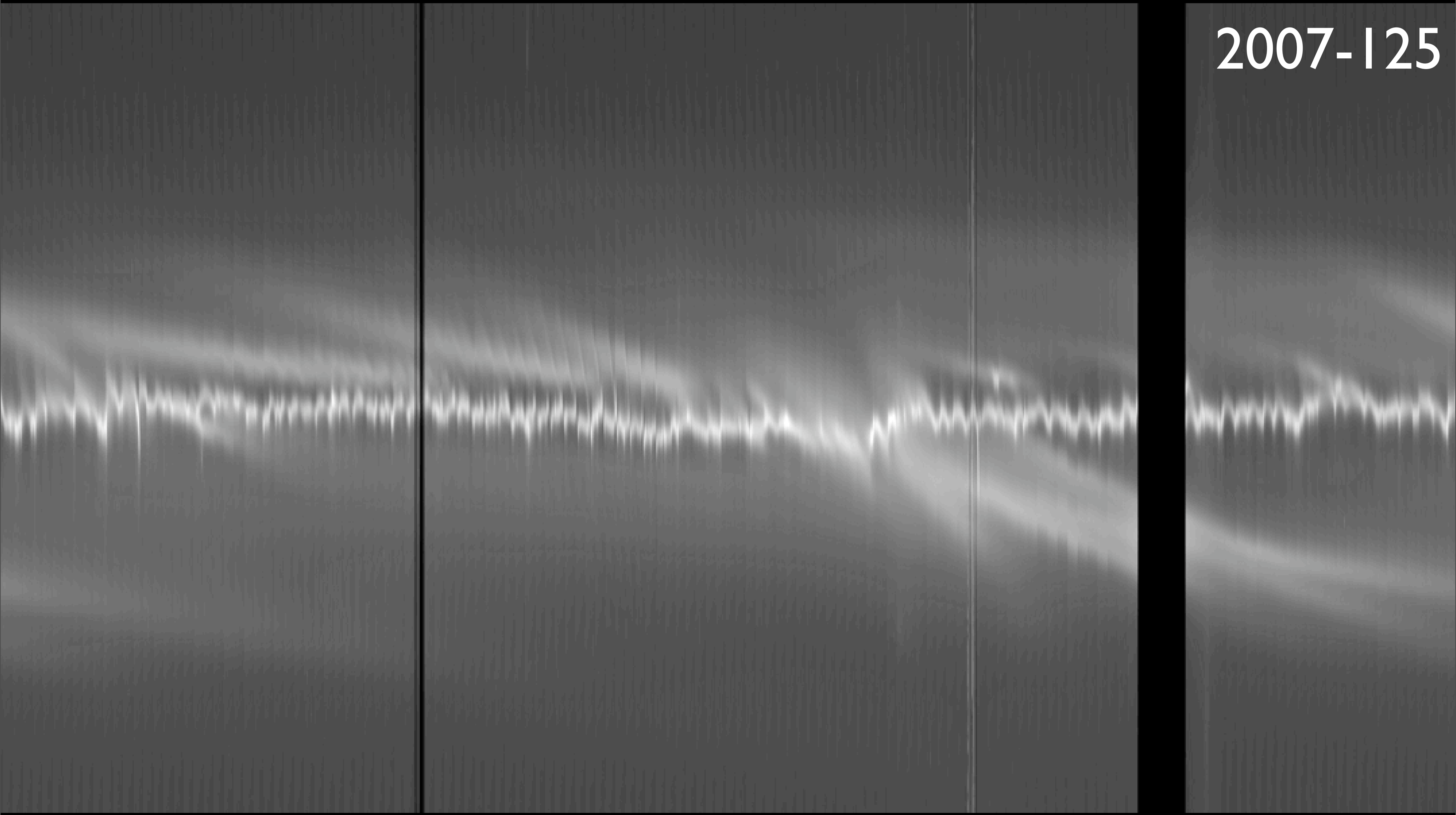
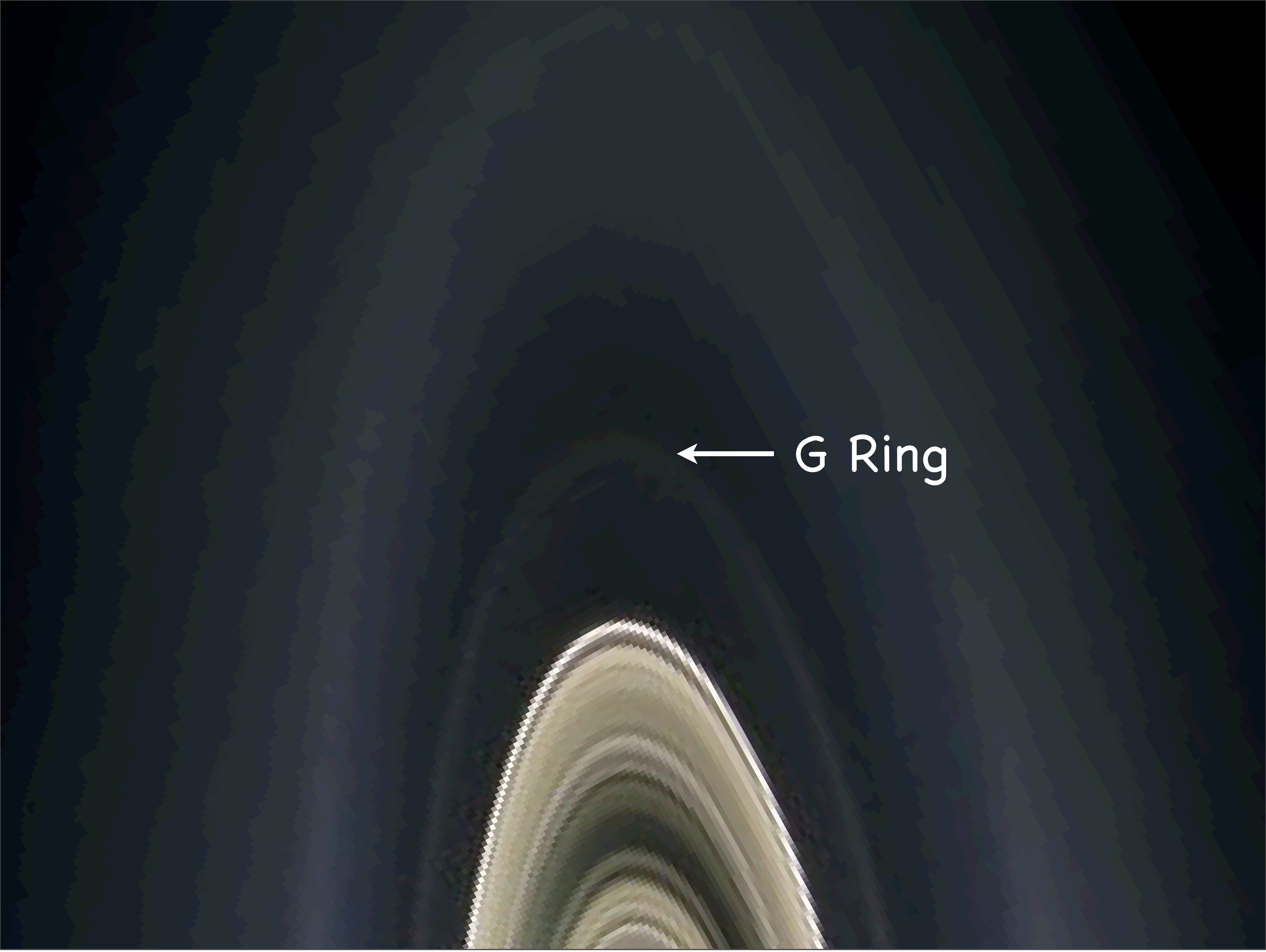
Transient Structures in Saturn's F Ring



Transient Structures in Saturn's F Ring

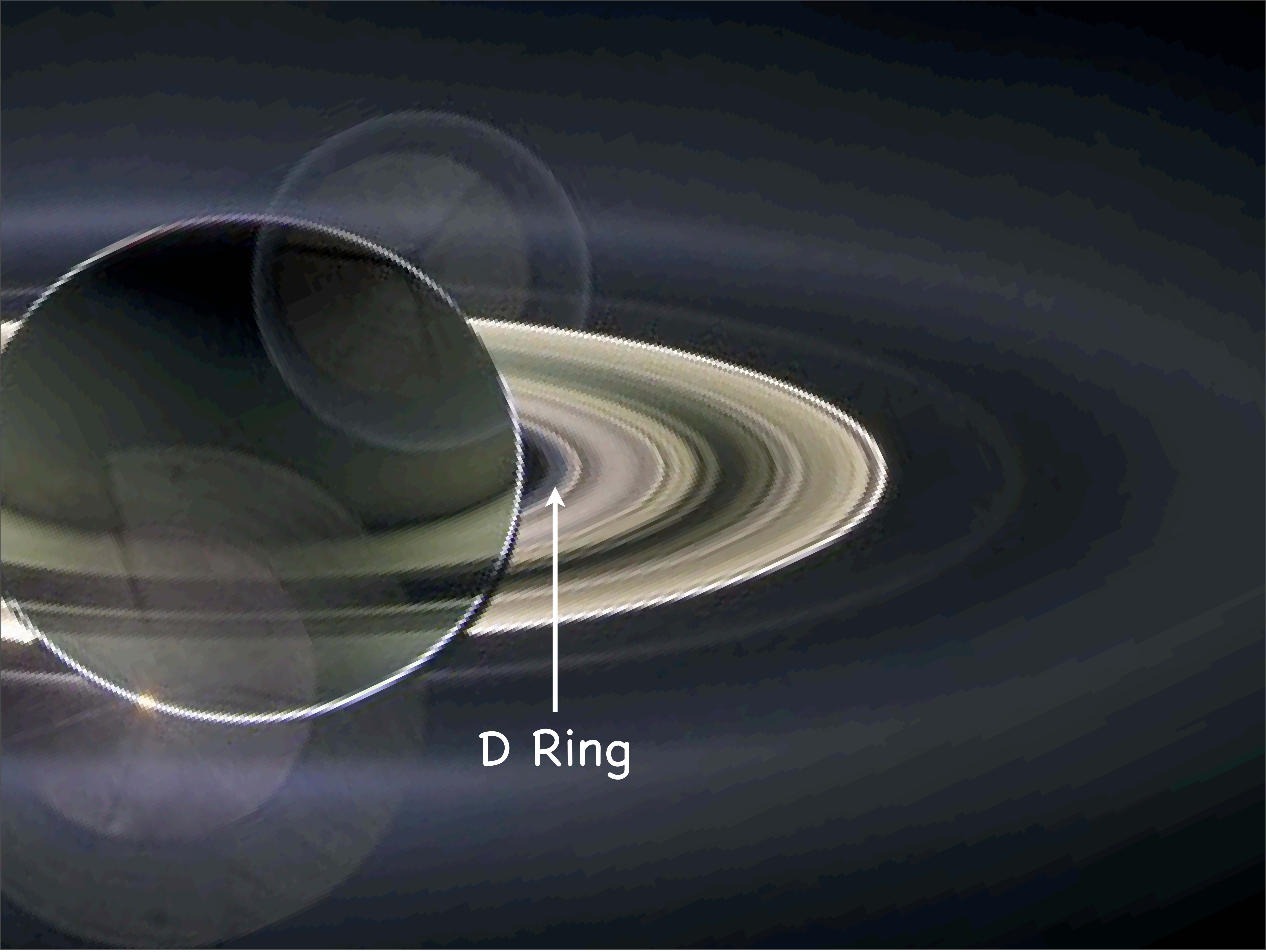


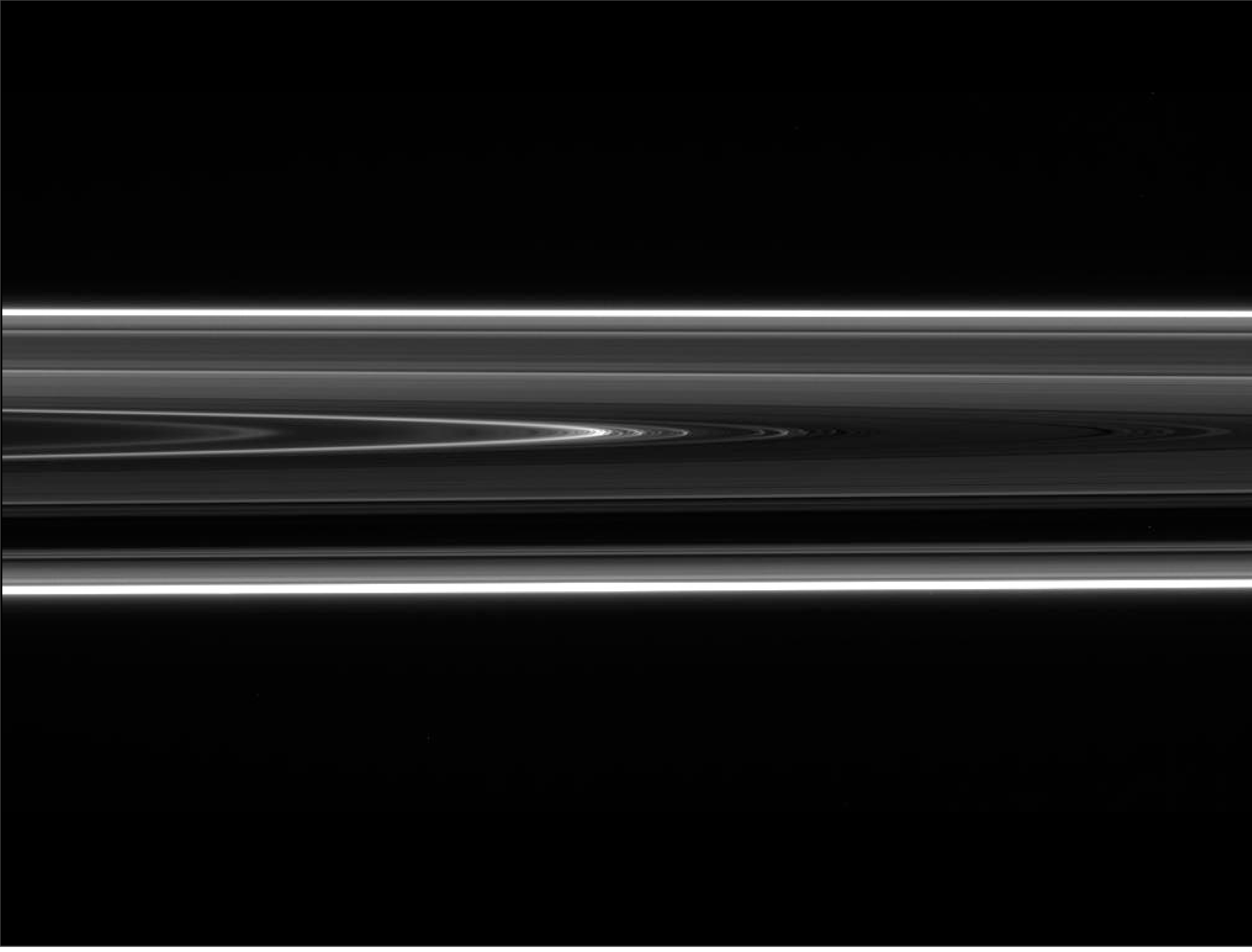

G Ring

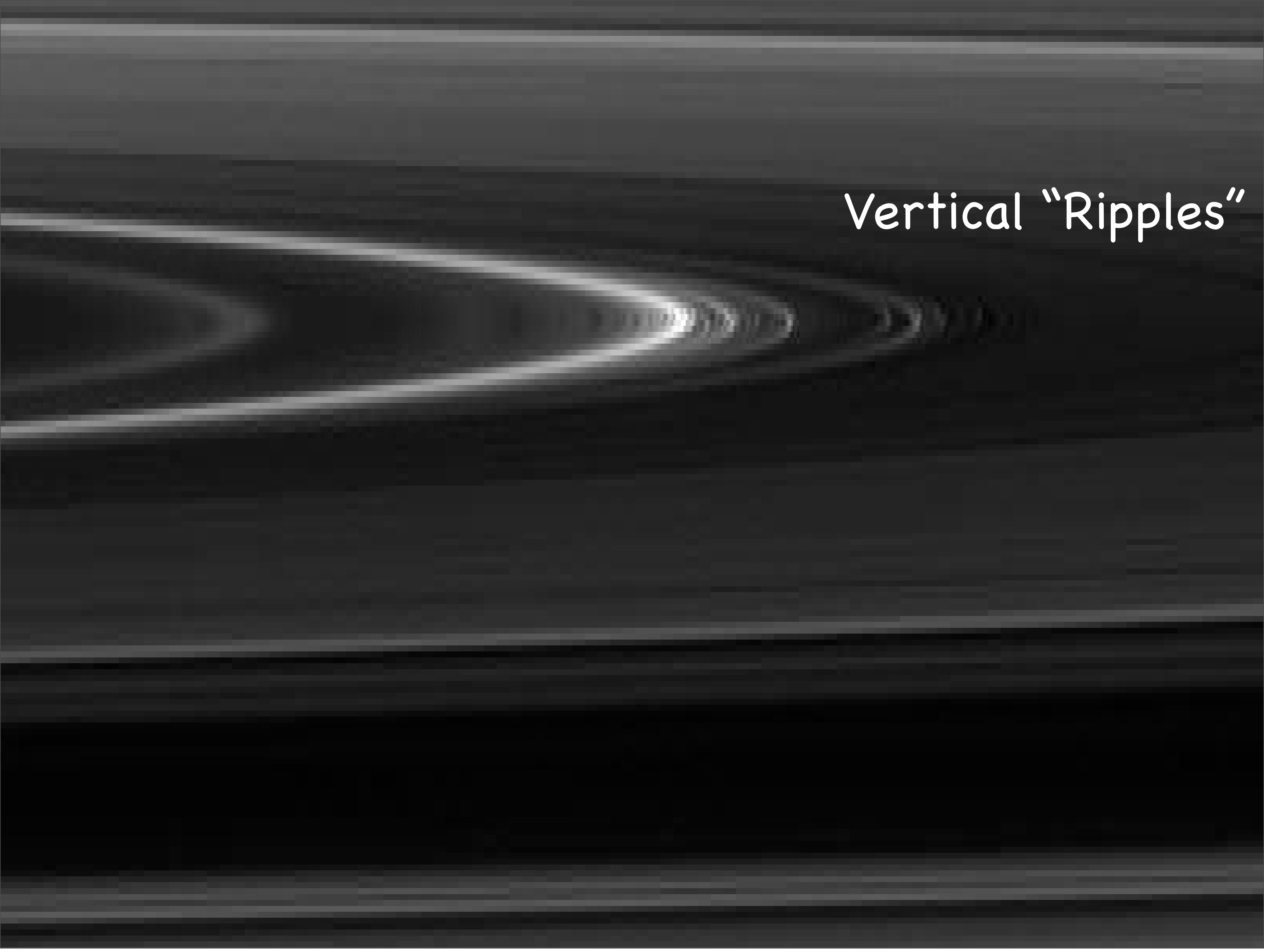
A Confined Arc in Saturn's G Ring

Other Types of Shear

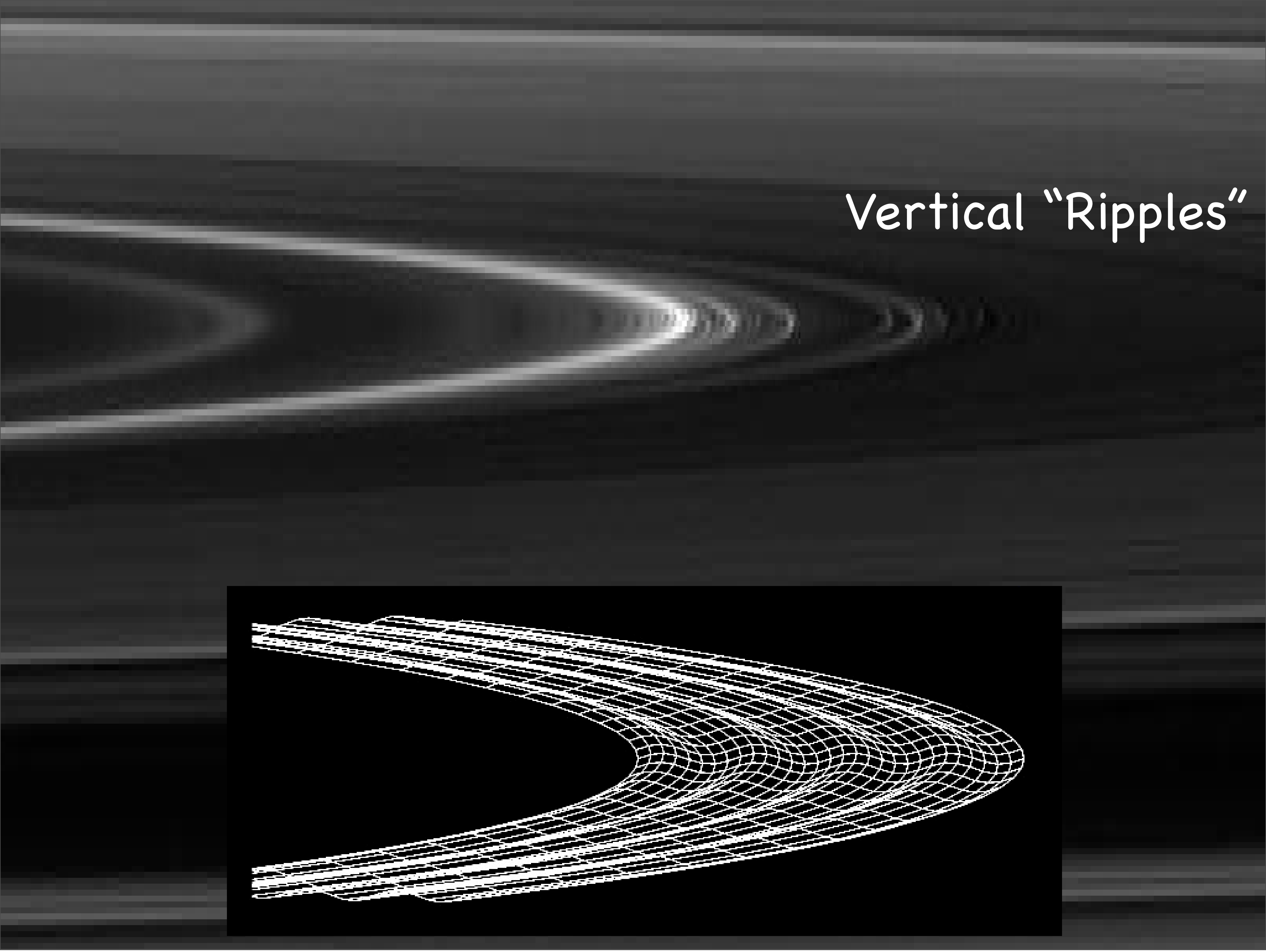
- n, κ and ν are all similar in magnitude.
  - Typical periods ~ 10 hours in rings.
- Precession rate $\dot{\omega}$ and regression rate $\dot{\Omega}$ are much slower.
  - Typical periods are ~ 100 days.
- Shearing rates for pericenters and nodes are correspondingly much slower.

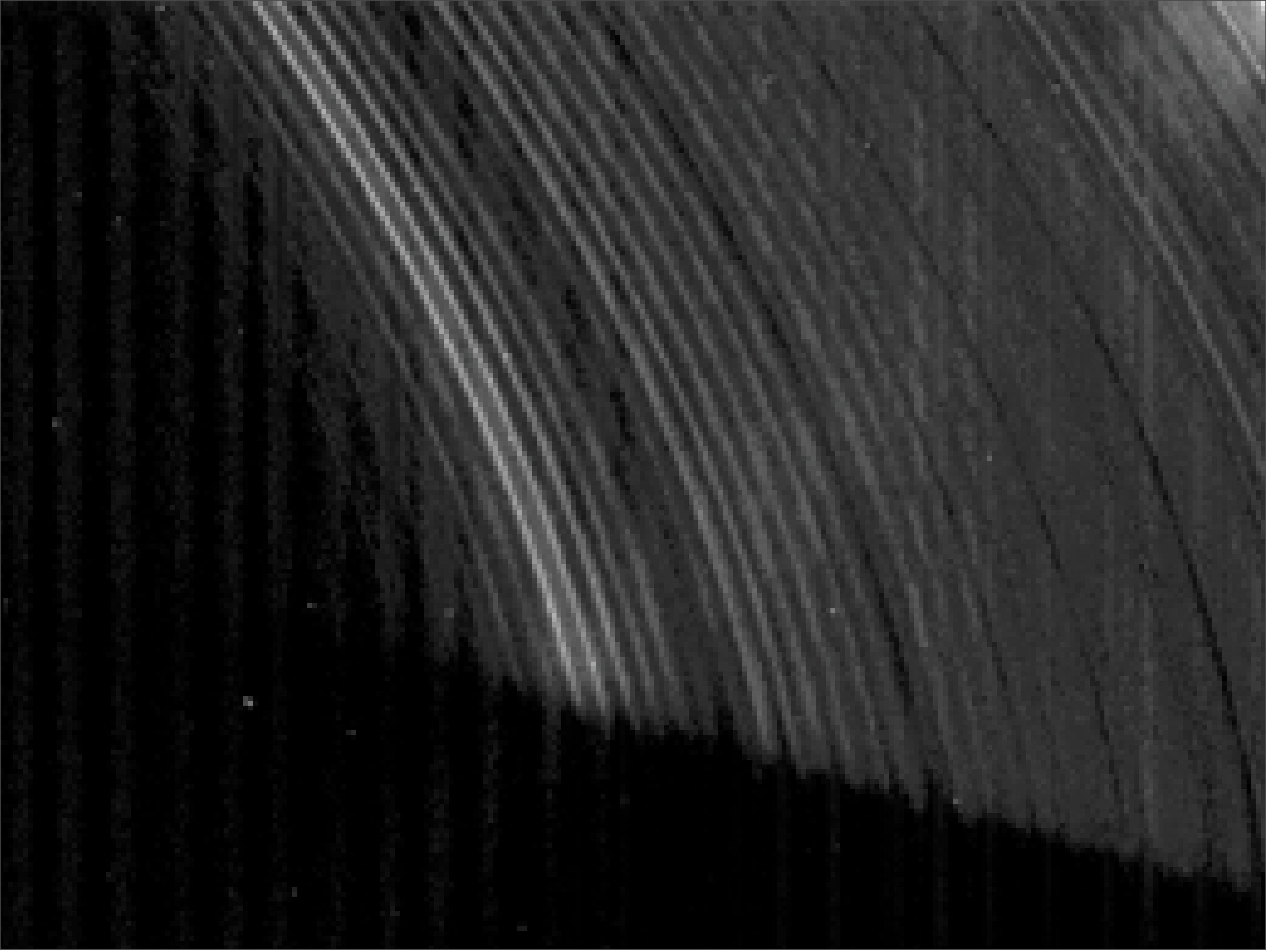

D Ring



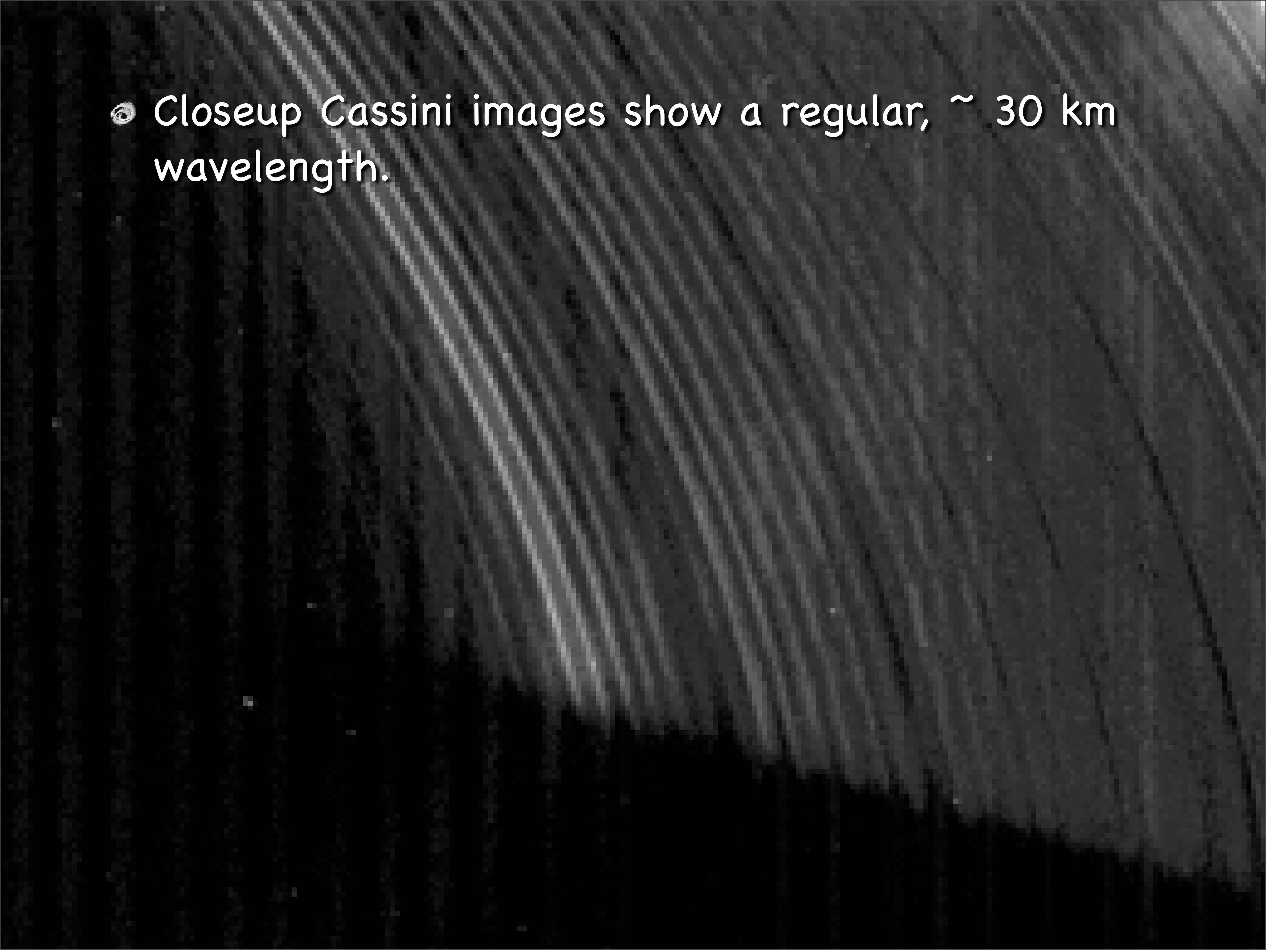

Vertical “Ripples”

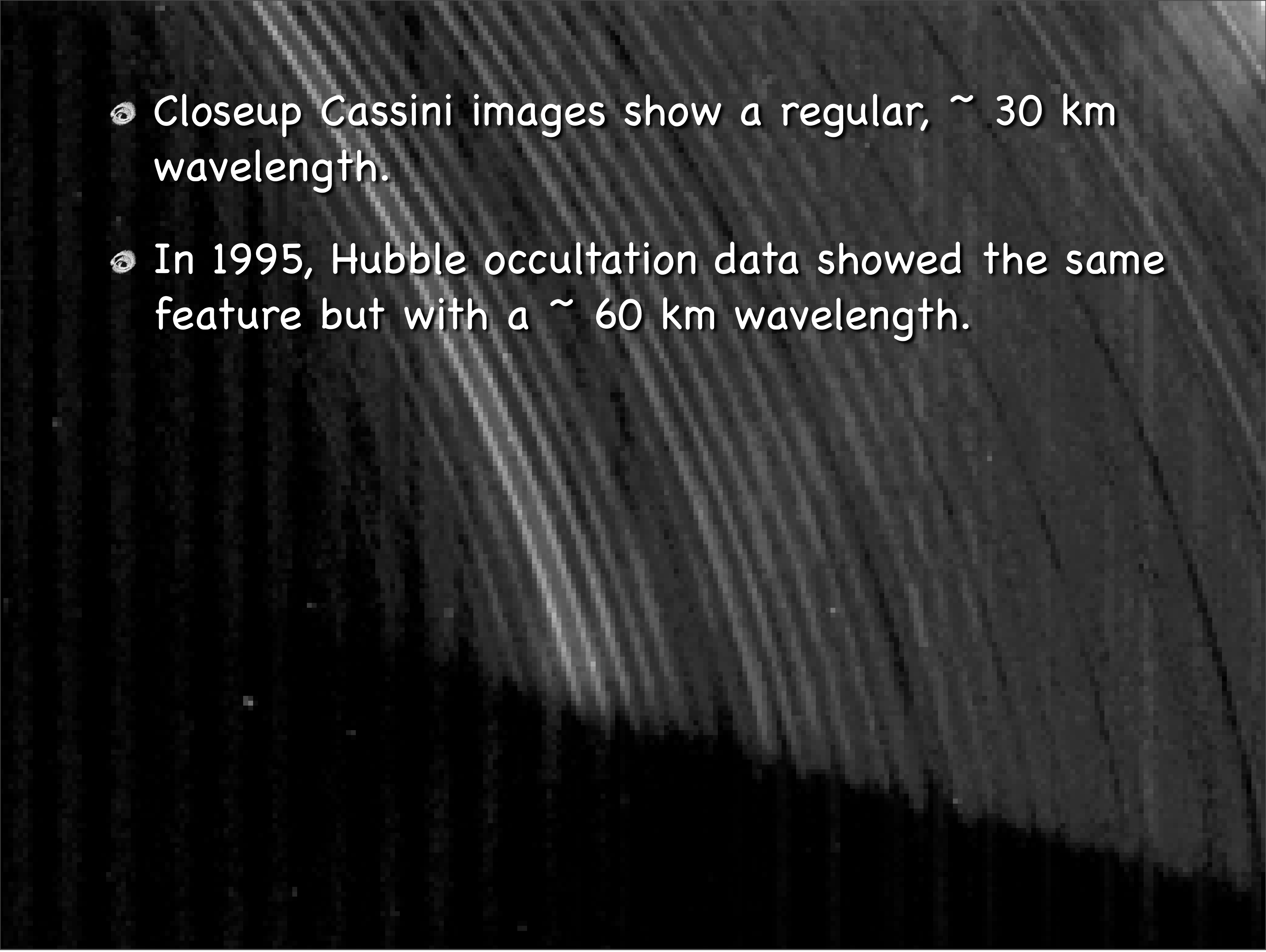
Vertical “Ripples”





- Closeup Cassini images show a regular, ~ 30 km wavelength.





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- In 1995, Hubble occultation data showed the same feature but with a ~ 60 km wavelength.

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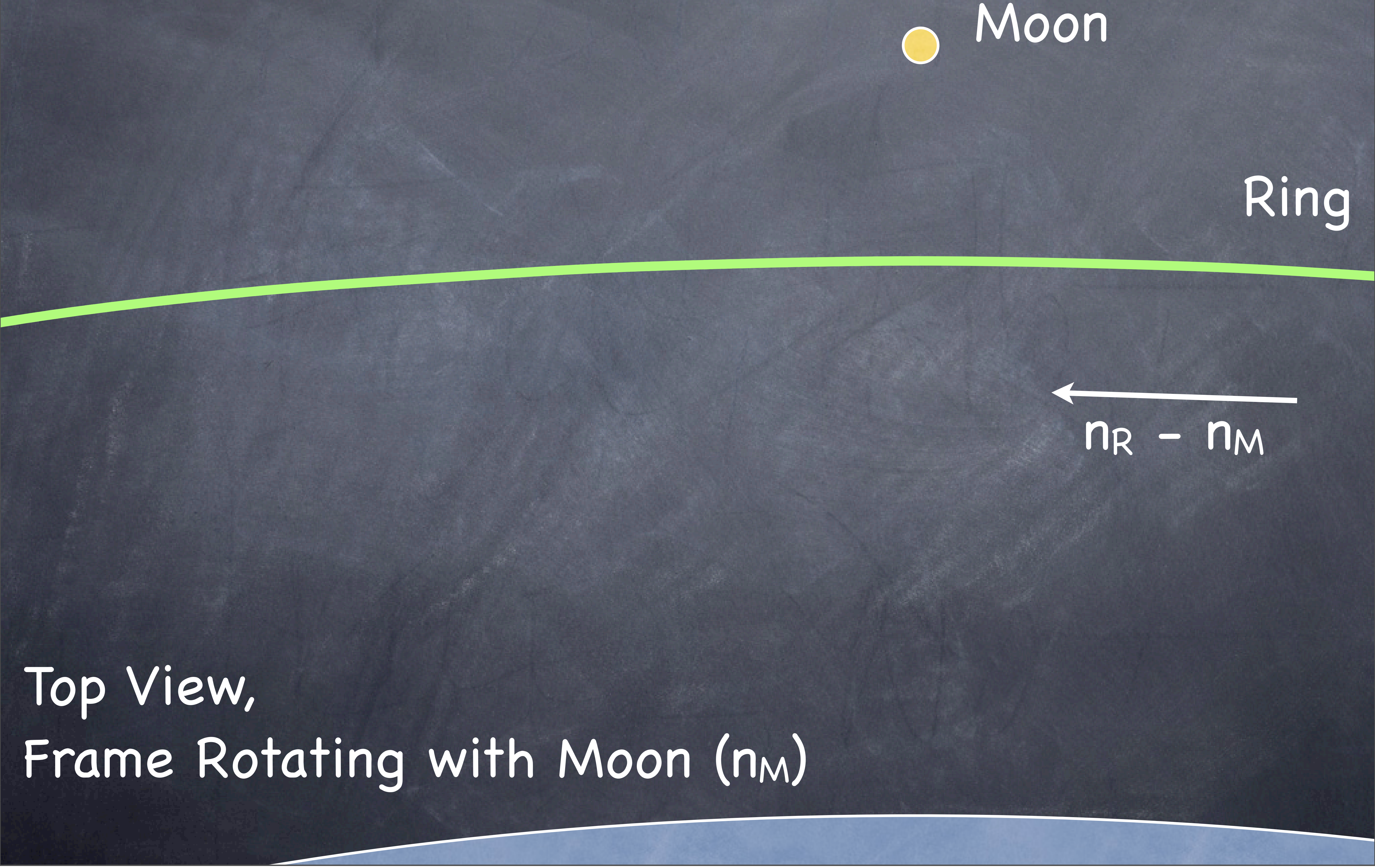
- In Cassini images, it continues to wind tighter at a rate exactly consistent with $d\dot{\Omega}/dr$.

- Closeup Cassini images show a regular, ~ 30 km wavelength.
- In 1995, Hubble occultation data showed the same feature but with a ~ 60 km wavelength.
- In Cassini images, it continues to wind tighter at a rate exactly consistent with $d\Omega/dr$.
- Playing the process backwards, something warped the ring in early 1984.

Ring-Moon Interactions

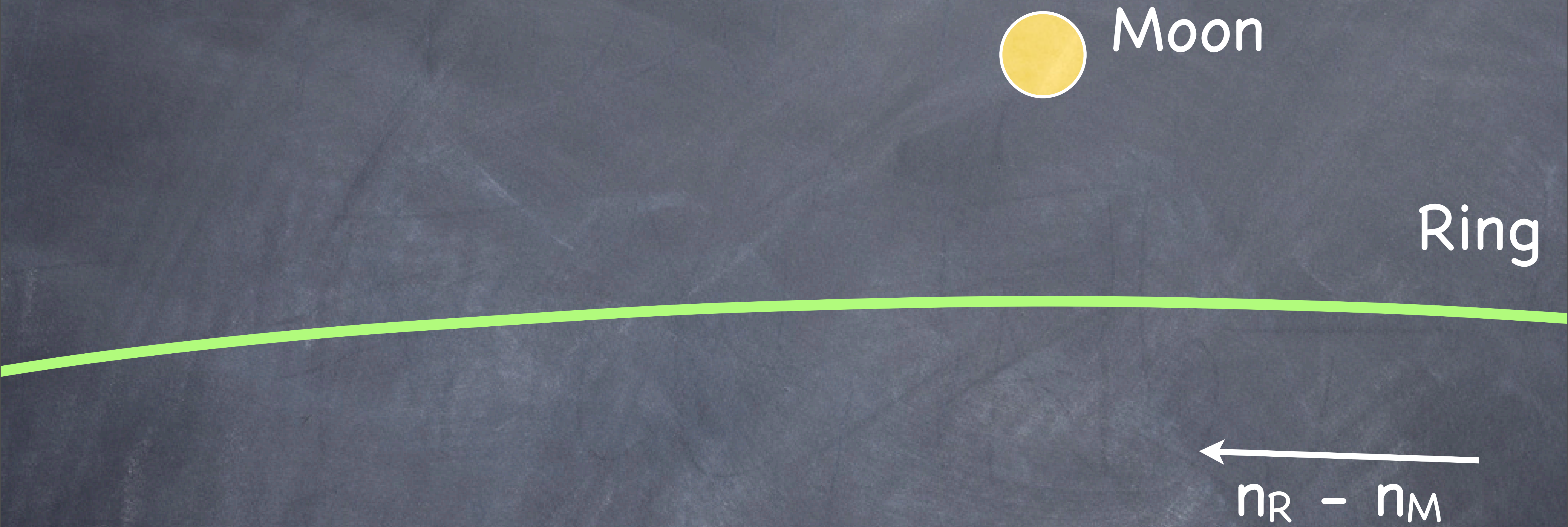
Top View,
Frame Rotating with Moon (n_M)

Ring-Moon Interactions



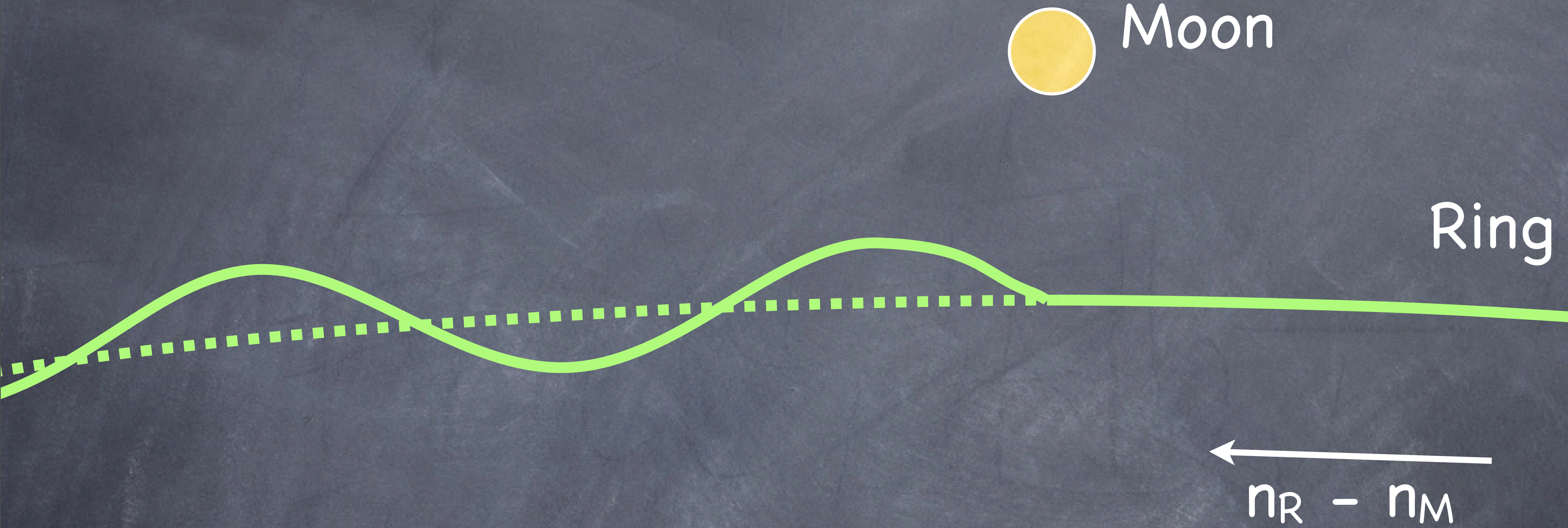
Top View,
Frame Rotating with Moon (n_M)

Ring-Moon Interactions



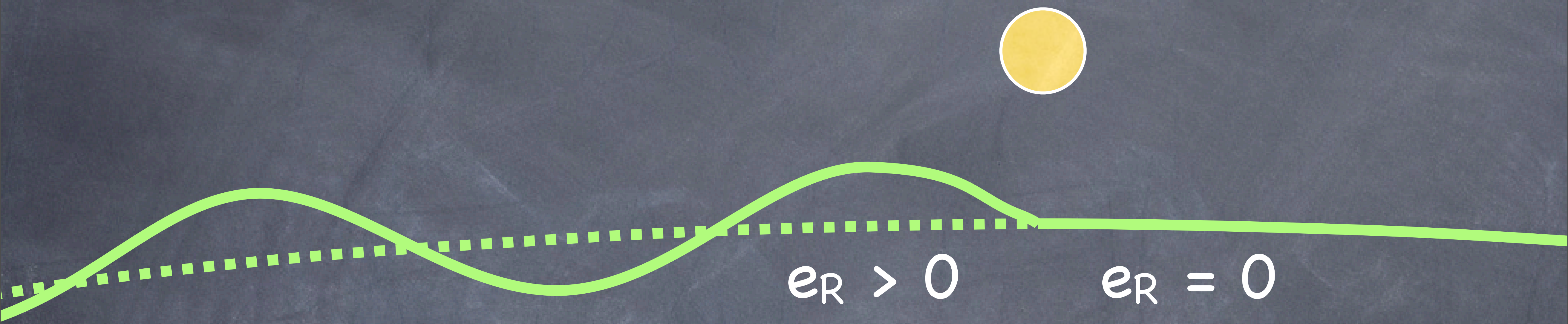
Top View,
Frame Rotating with Moon (n_M)

Ring-Moon Interactions



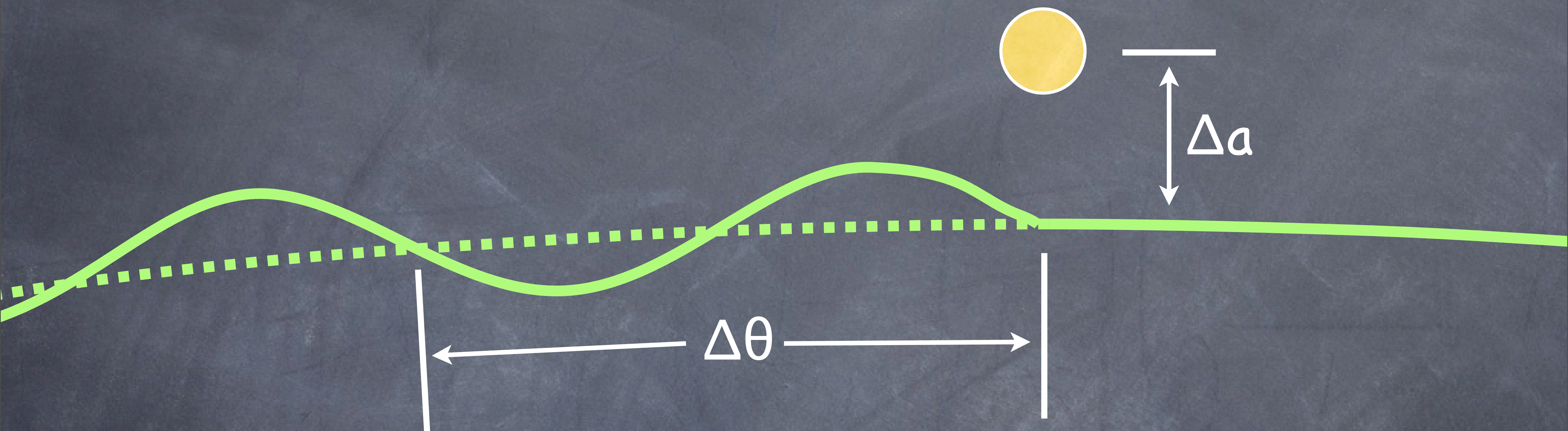
Top View,
Frame Rotating with Moon (n_M)

Gravitational Deflection



- Ring particle is deflected by moon's gravity
- Epicycles form:
  - $e_R \approx 2.24 \dfrac{M_M}{M_P} \dfrac{a}{|a_M - a_R|}$ (Julian & Toomre, 1966)
  - Formula valid for a small moon and a nearby ring

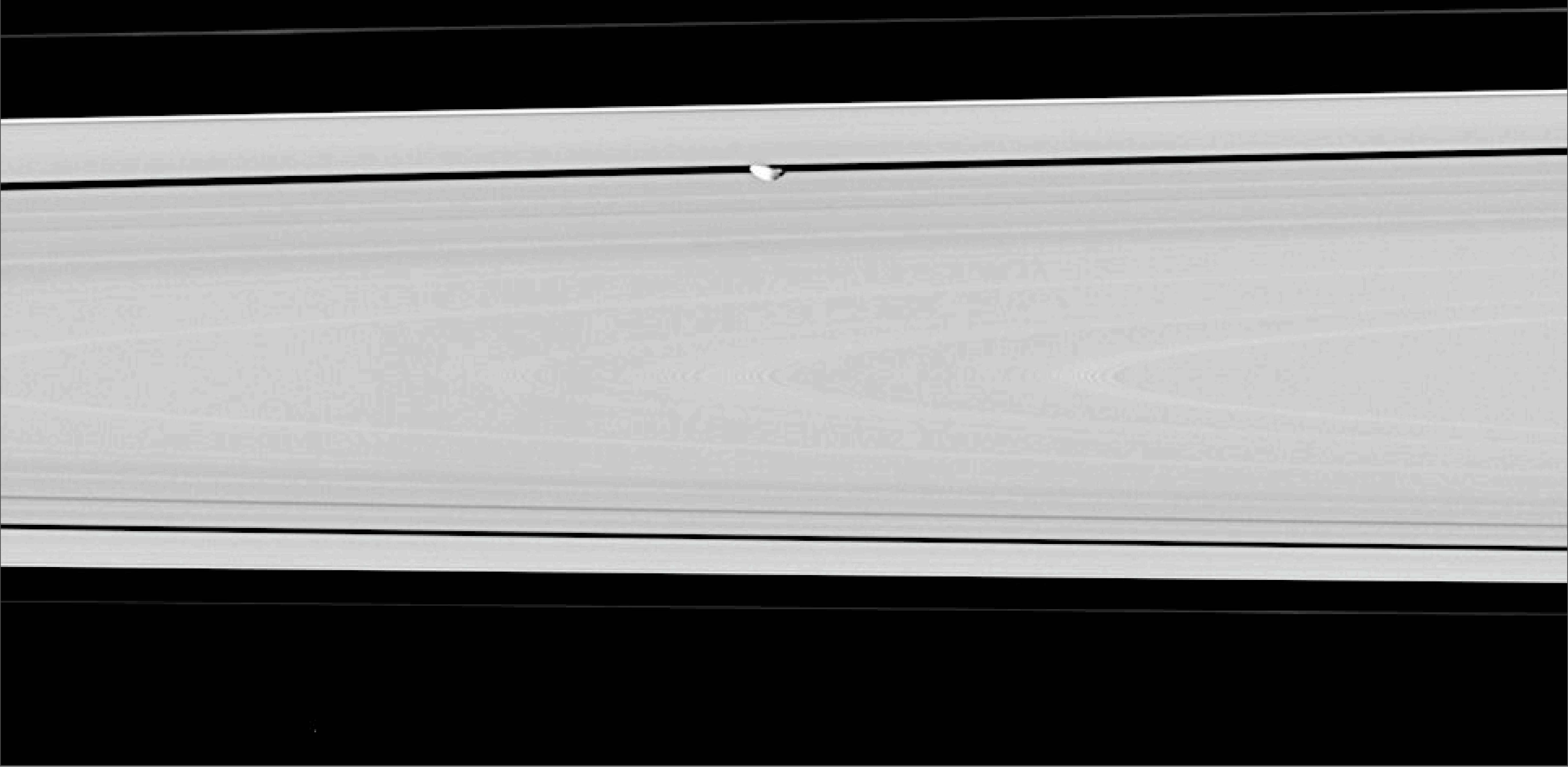
Gravitational Deflection

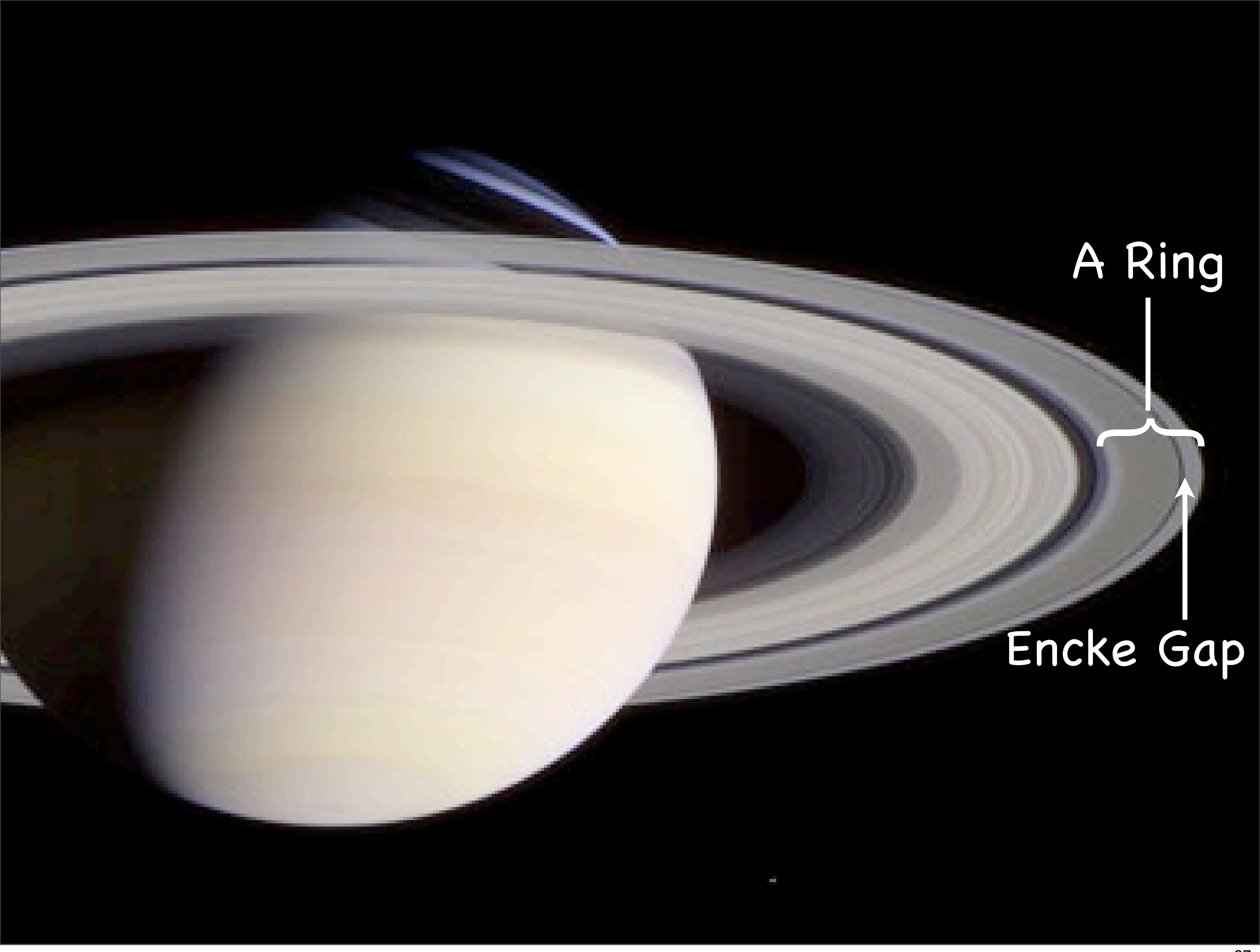


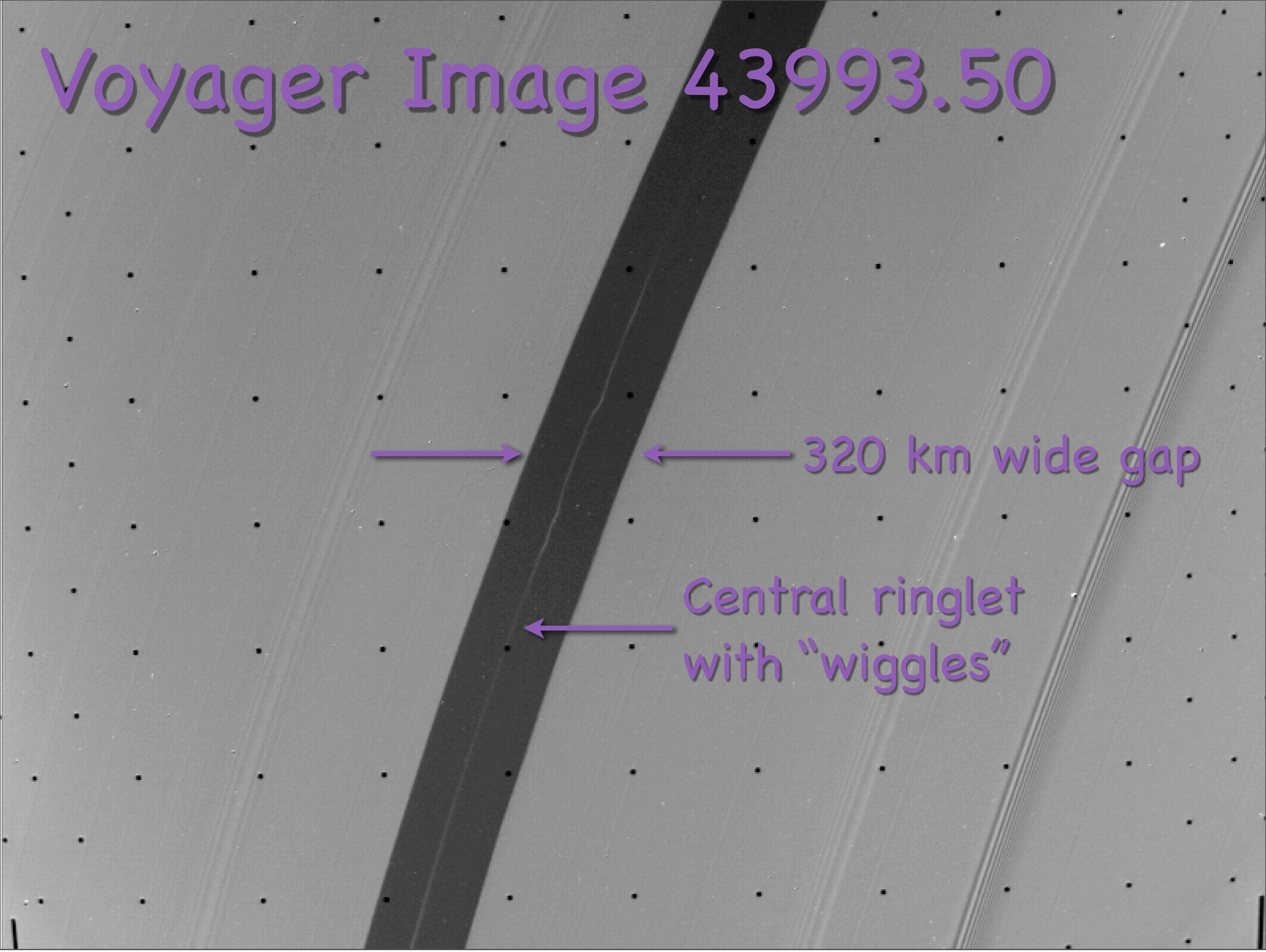
- Period $T = 2\pi/\kappa_R$
- $\Delta\theta = T\,|n_R - n_M| \cong 2\pi\ \Delta n/n \cong 3\pi\ \Delta a/a$
- Wavelength = $a\ \Delta\theta$ = $\boxed{3\pi\ \Delta a}$

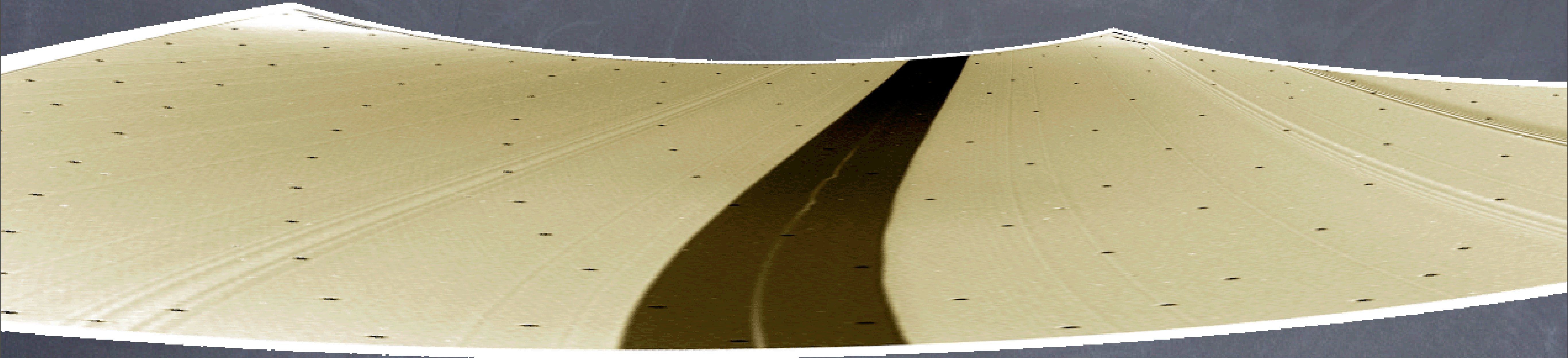
Real-World Example:

The Encke Gap and the Discovery of Pan

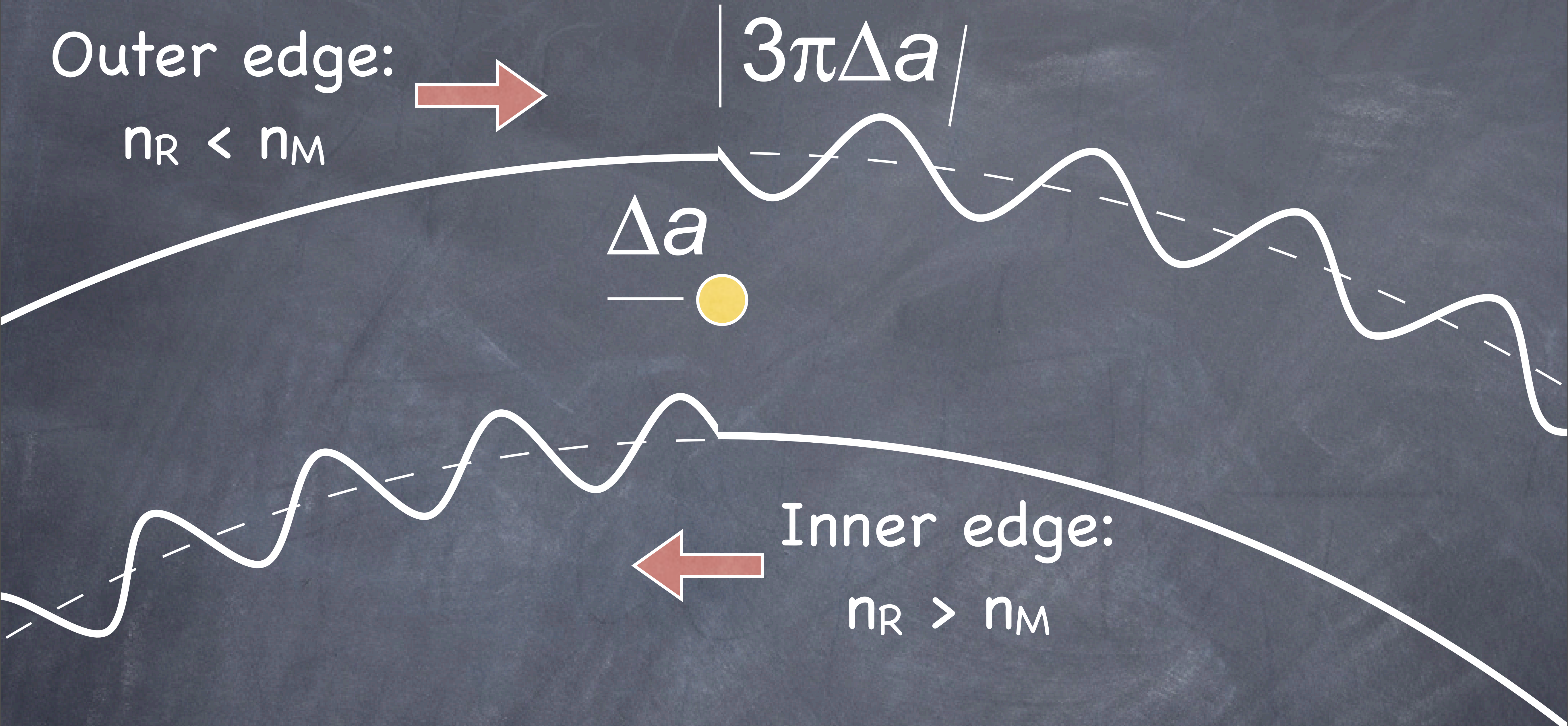



A Ring
Encke Gap


Voyager Image 43993.50
320 km wide gap
Central ringlet
with "wiggles"

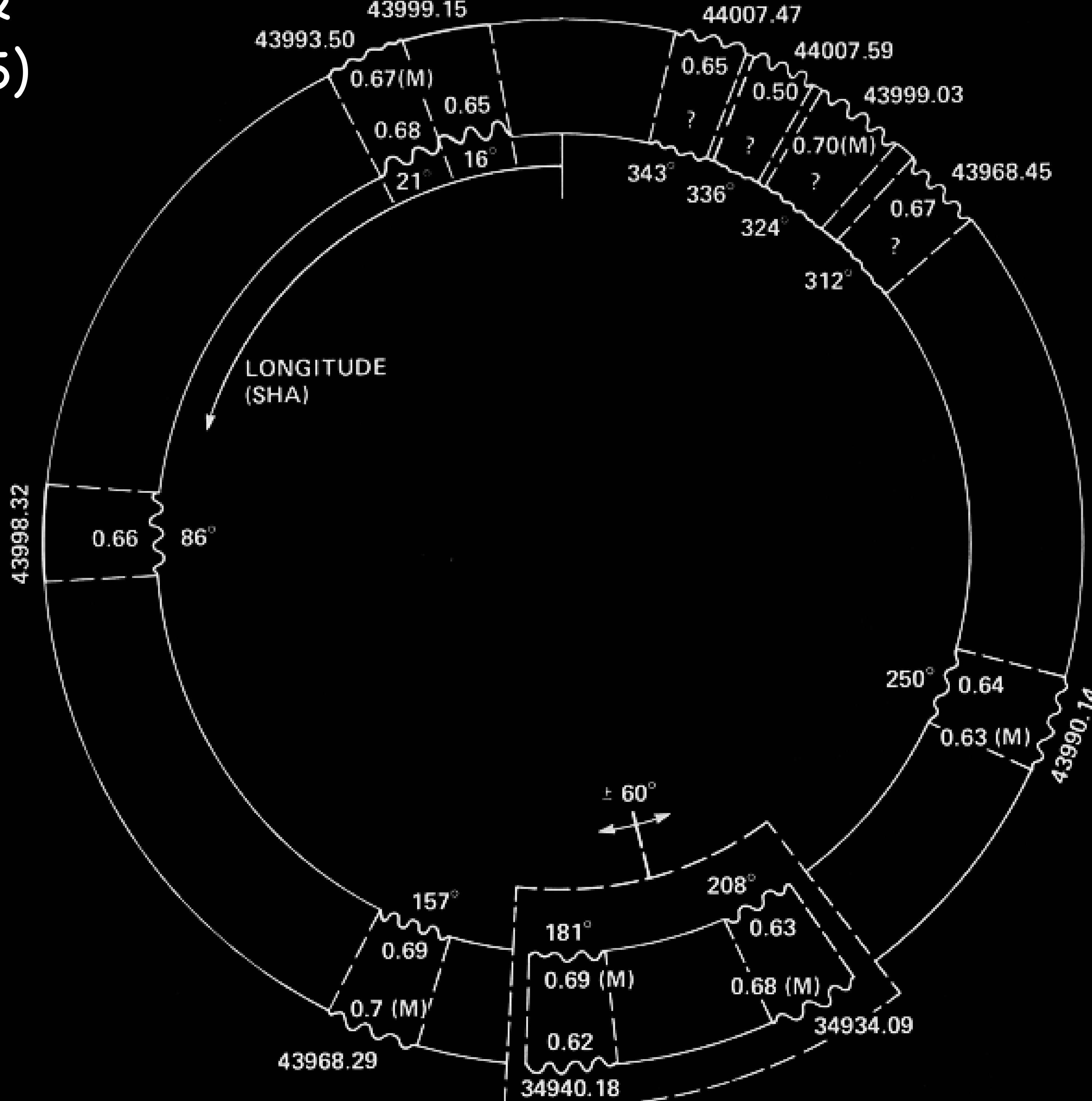


- "Eyeball" analysis of a photographic print.
  - by Jeff Cuzzi, Phoenix Airport, 1985.
- Discovery of a wavy edge.
- Implies that there is a moon in the Encke Gap!
- Wavelength ~ 1500 km implies that the moon is ~ 150 km away, near the middle of the gap.
- Amplitude ~ 5 km implies moon is ~ 10 km in radius.



- A wavy edge should lead the moon on the inner edge; trail it on the outer.
- Collisions may damp the pattern with increasing distance from the moon.

- From Cuzzi & Scargle (1985)
- Searched all fine-resolution Voyager images.
- Isolated moon within a 20° "box" that was not imaged well.



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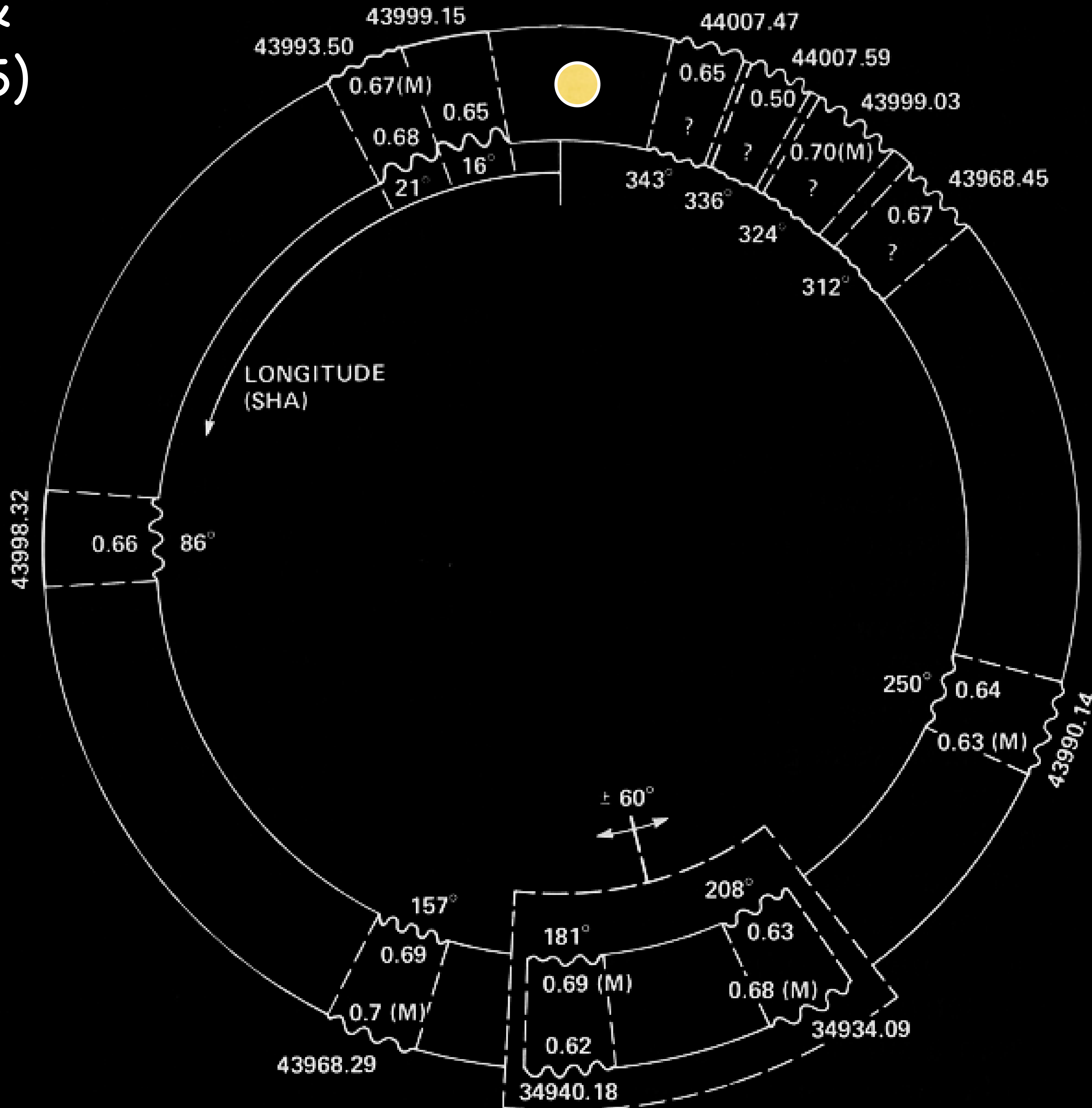
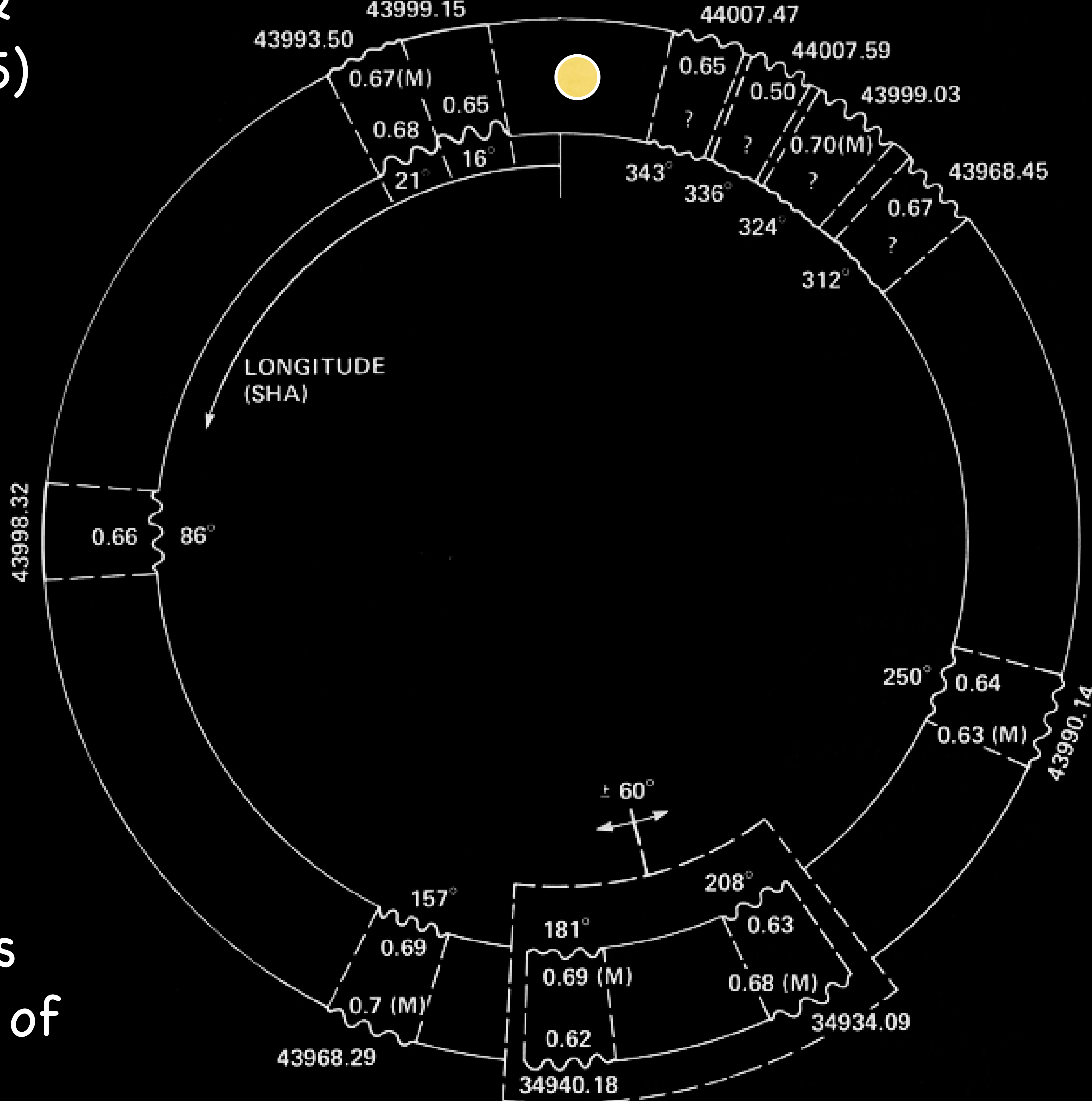
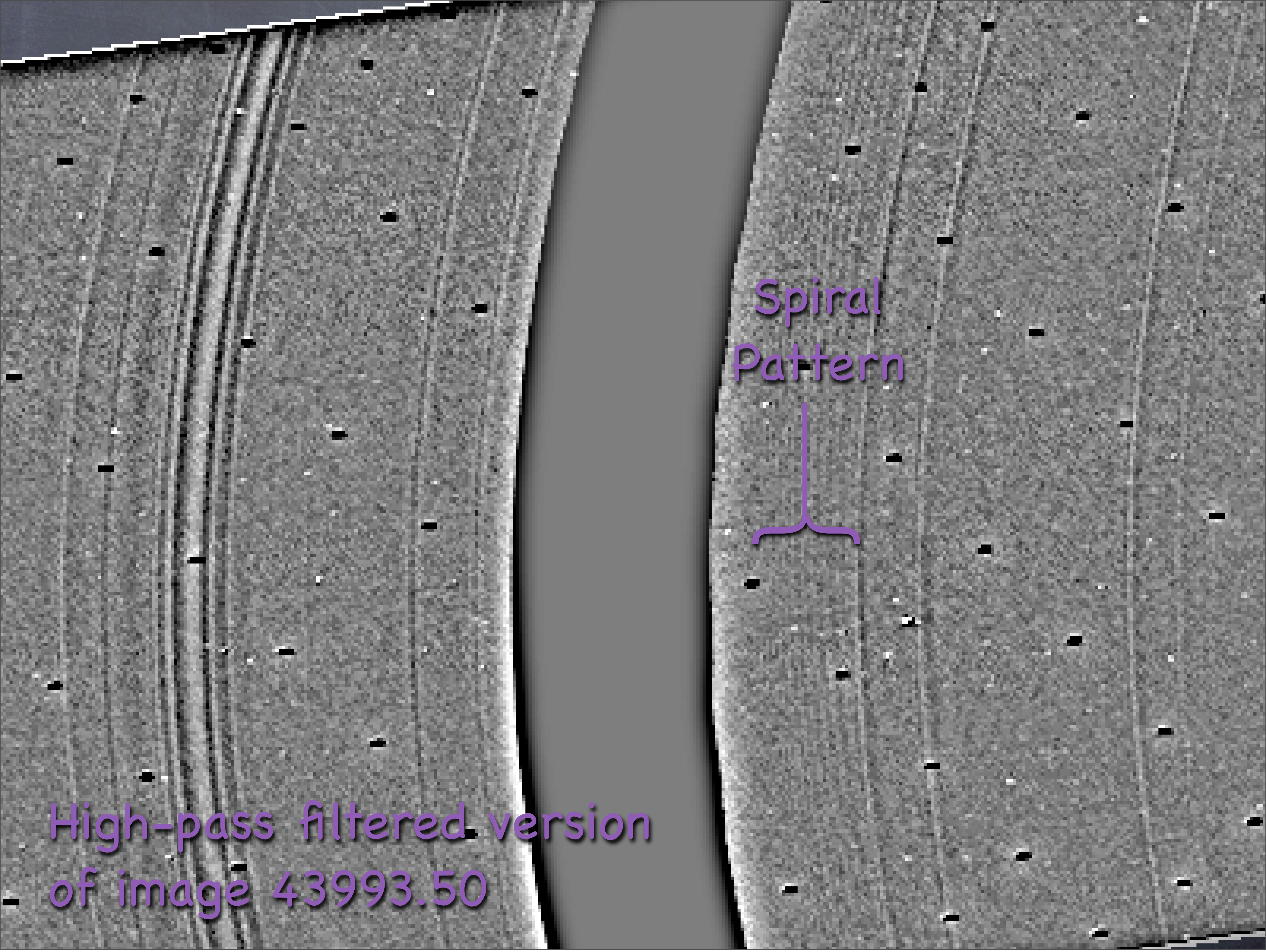


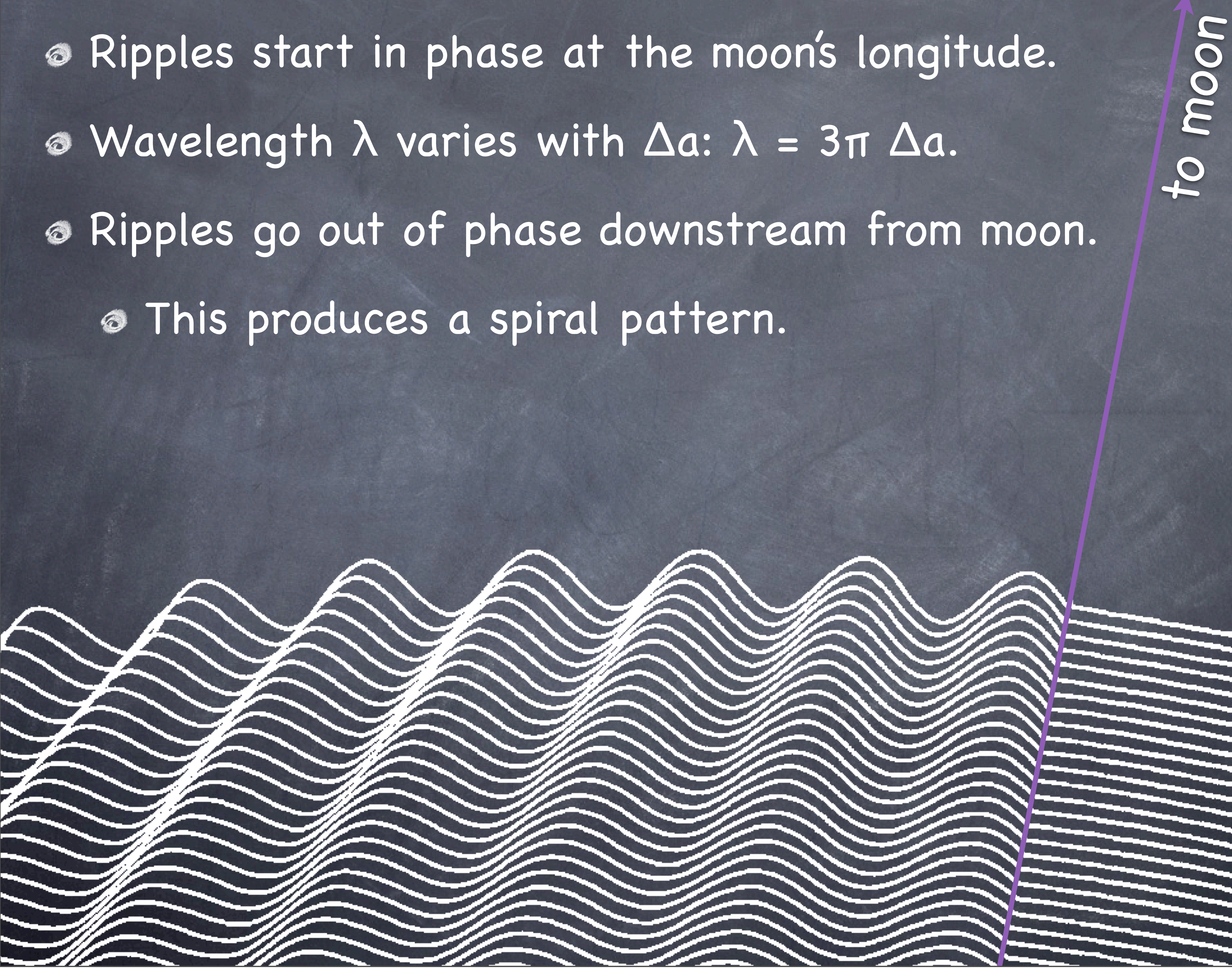
- From Cuzzi & Scargle (1985)
- Searched all fine-resolution Voyager images.
- Isolated moon within a 20° "box" that was not imaged well.
- ...but this was NOT the end of the story!



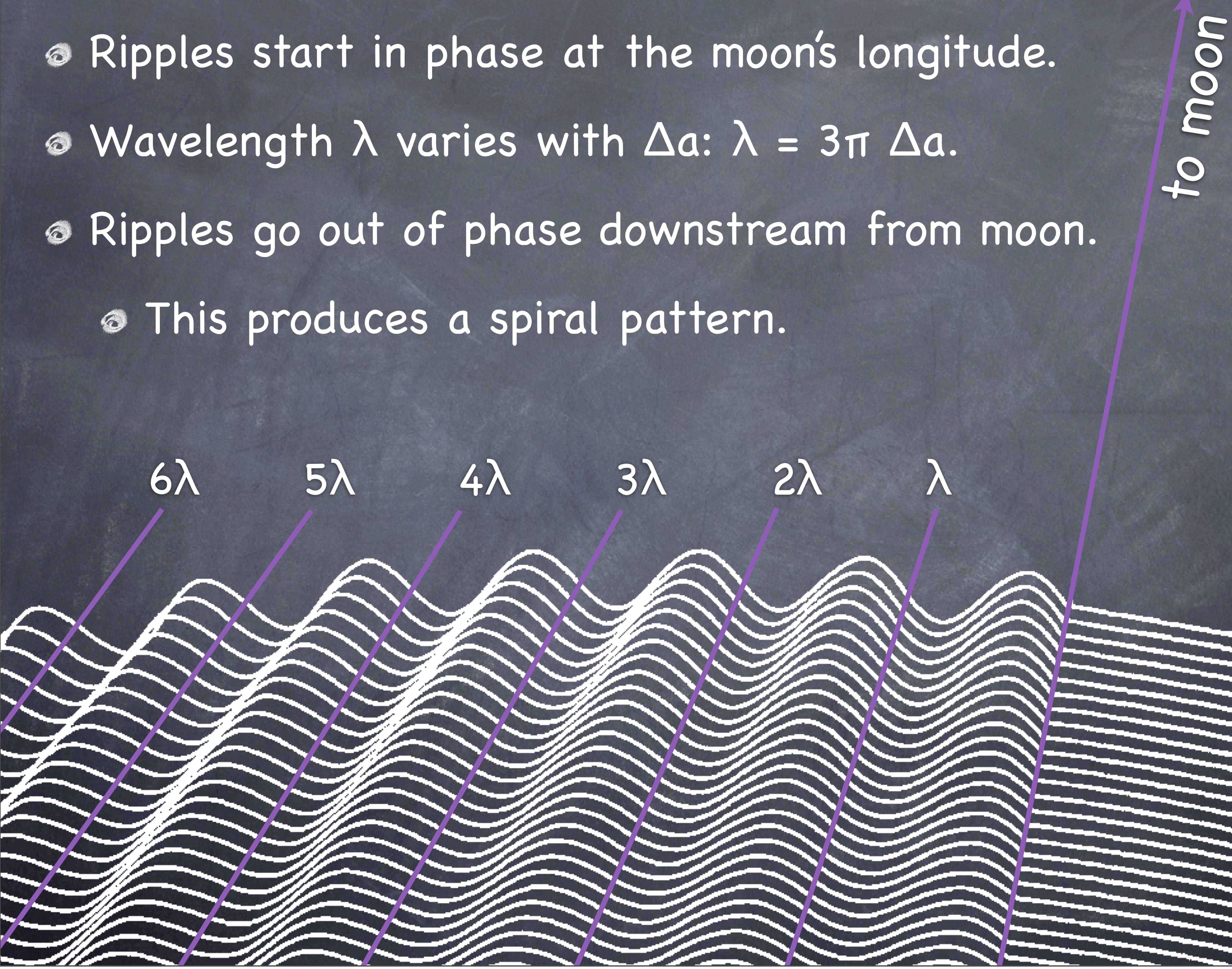
Moonlet Wakes

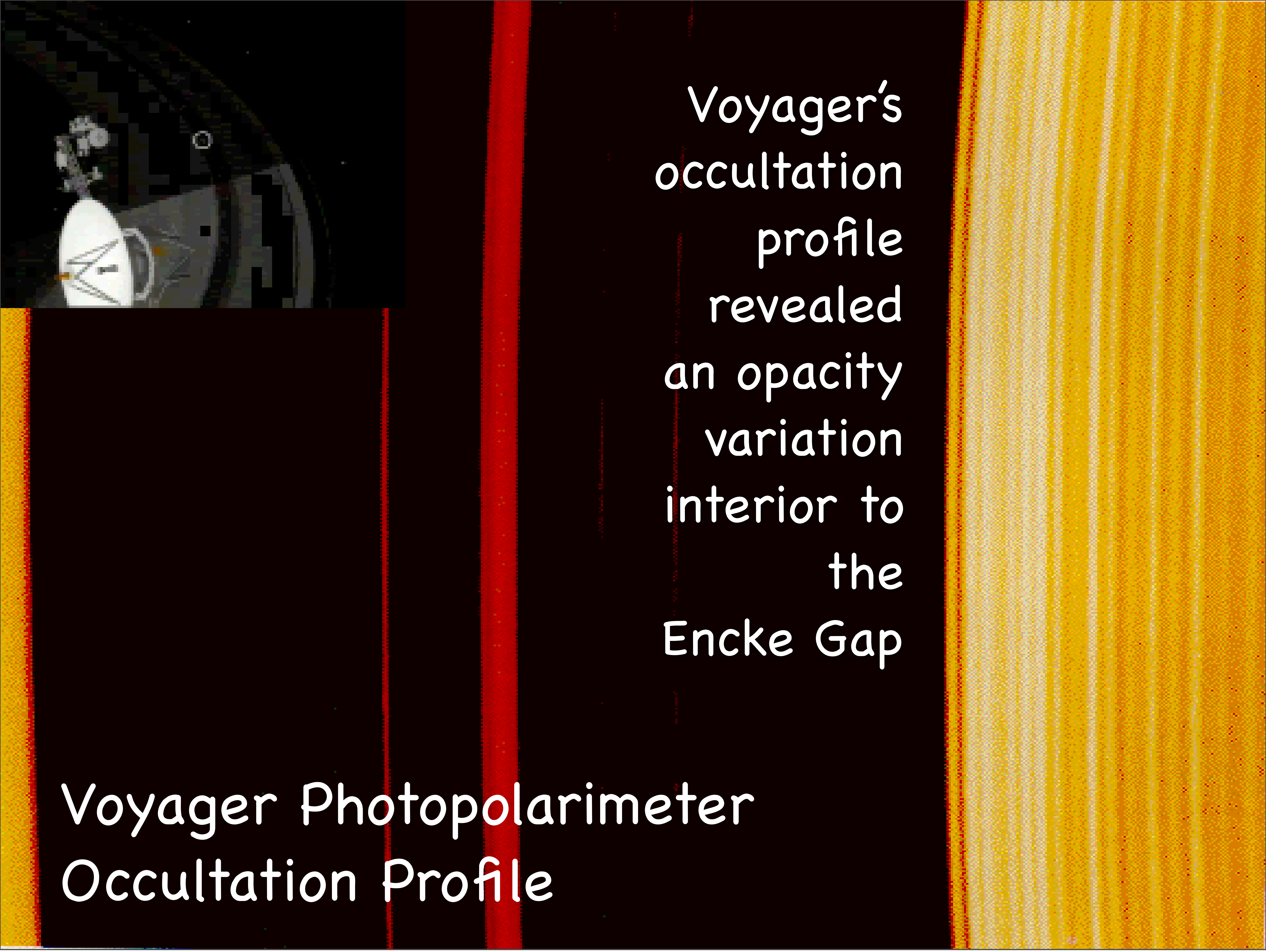

Spiral
Pattern
High-pass filtered version
of image 43993.50

- Ripples start in phase at the moon's longitude.
- Wavelength λ varies with Δa: $\lambda = 3\pi\,\Delta a$.
- Ripples go out of phase downstream from moon.
  - This produces a spiral pattern.

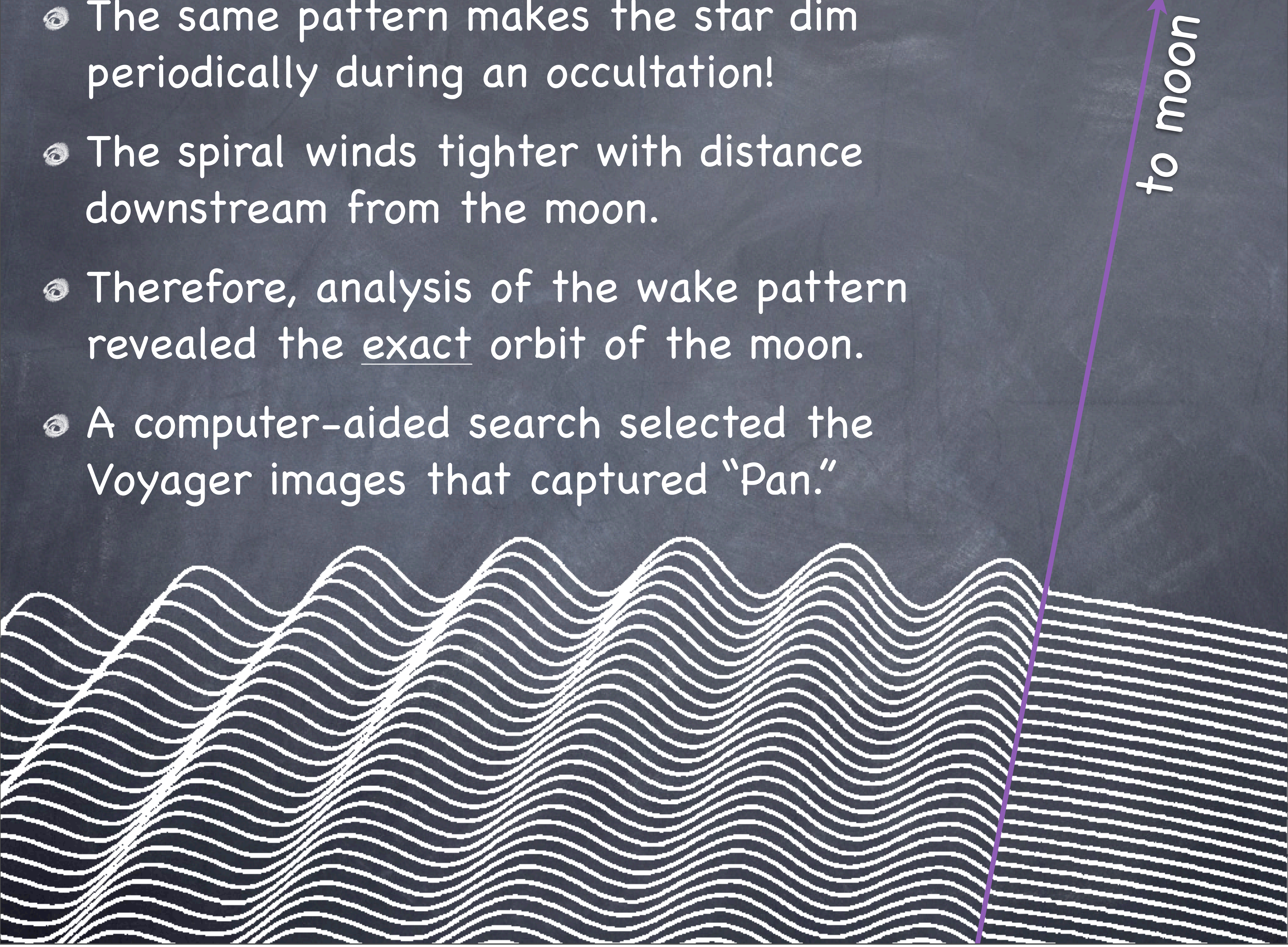


- Ripples start in phase at the moon's longitude.
- Wavelength λ varies with Δa: $\lambda = 3\pi\, \Delta a$.
- Ripples go out of phase downstream from moon.
  - This produces a spiral pattern.




Voyager's occultation profile revealed an opacity variation interior to the Encke Gap
Voyager Photopolarimeter Occultation Profile

- The same pattern makes the star dim periodically during an occultation!
- The spiral winds tighter with distance downstream from the moon.
- Therefore, analysis of the wake pattern revealed the <u>exact</u> orbit of the moon.
- A computer-aided search selected the Voyager images that captured "Pan."



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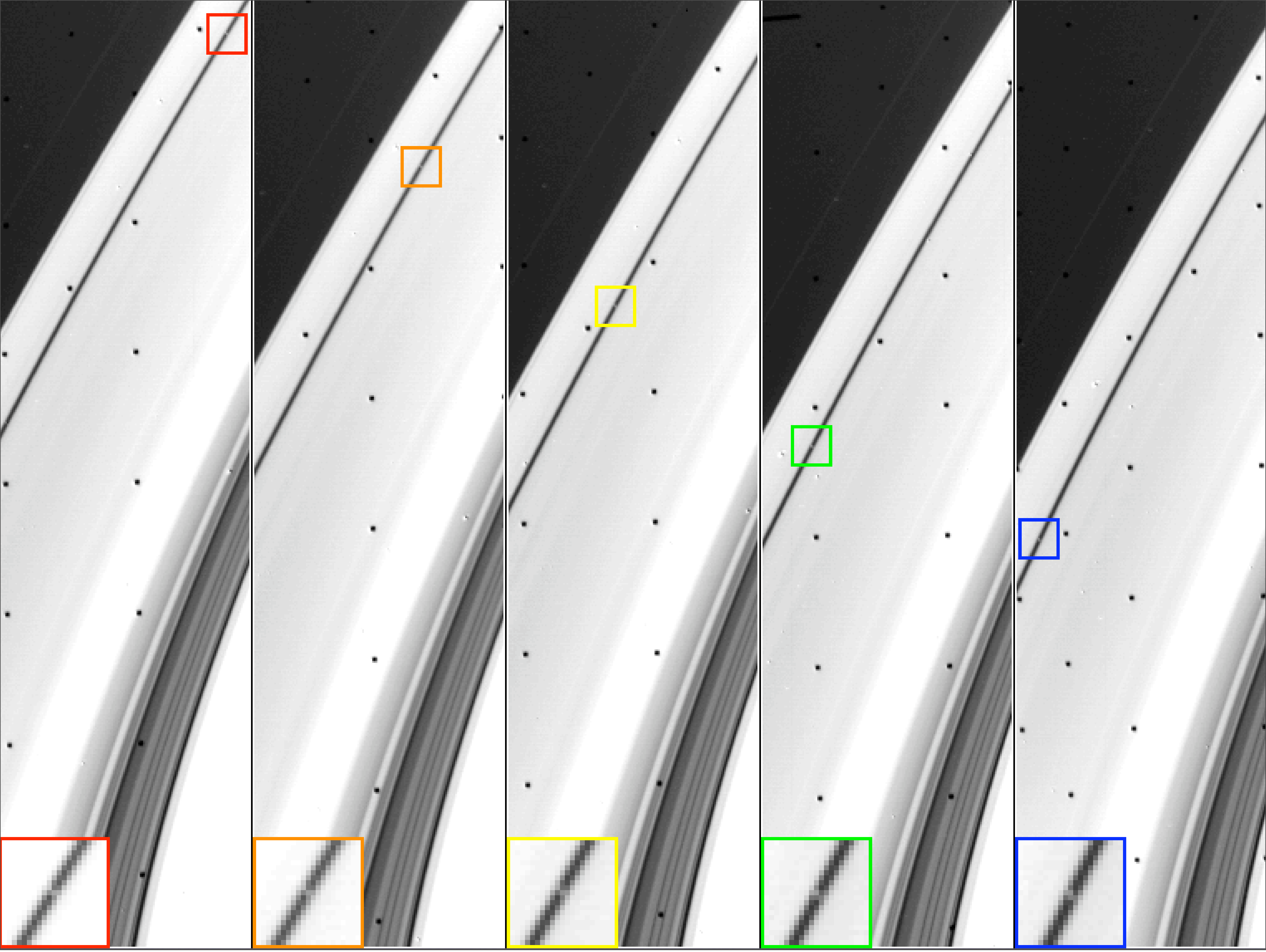
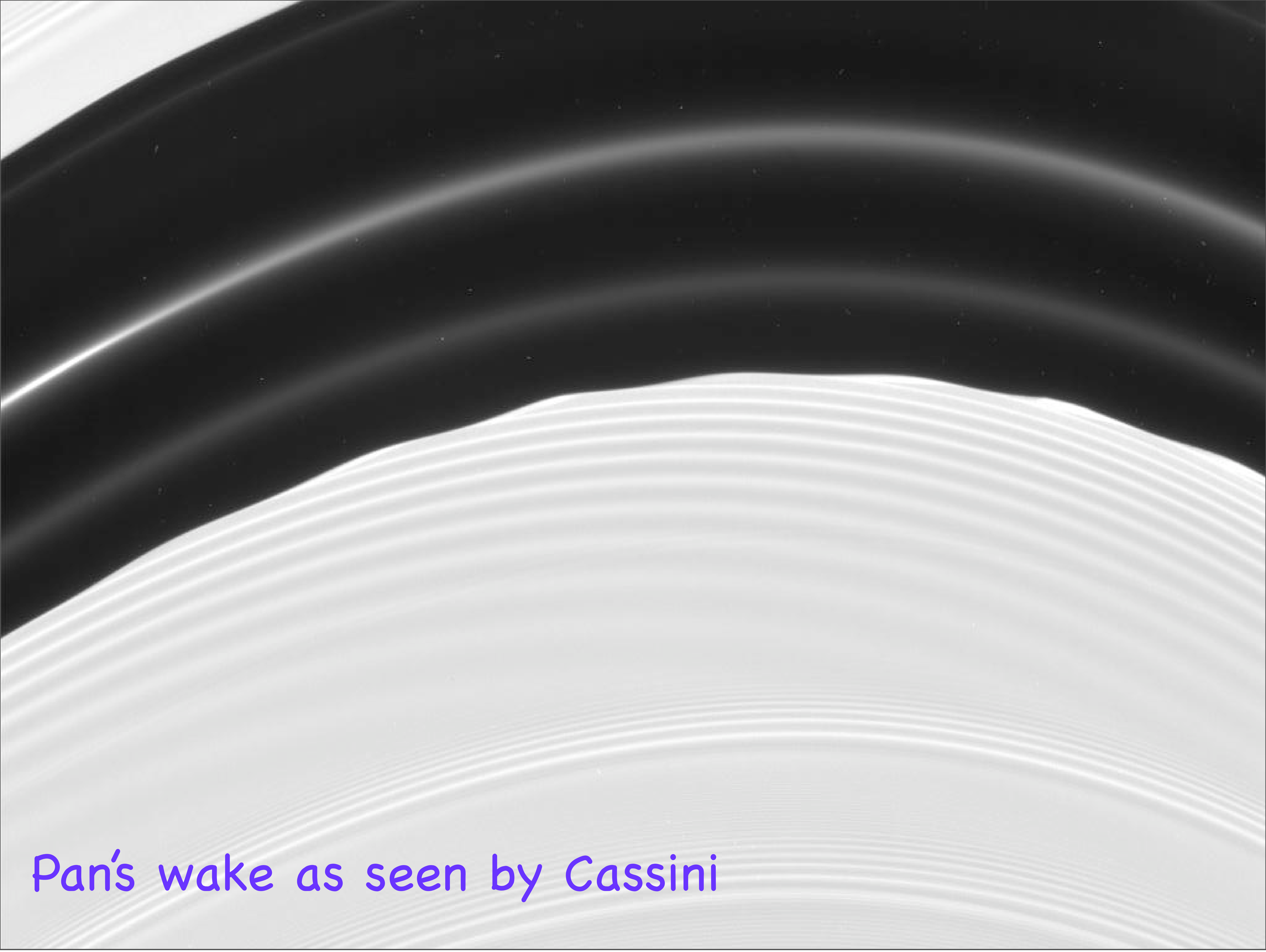
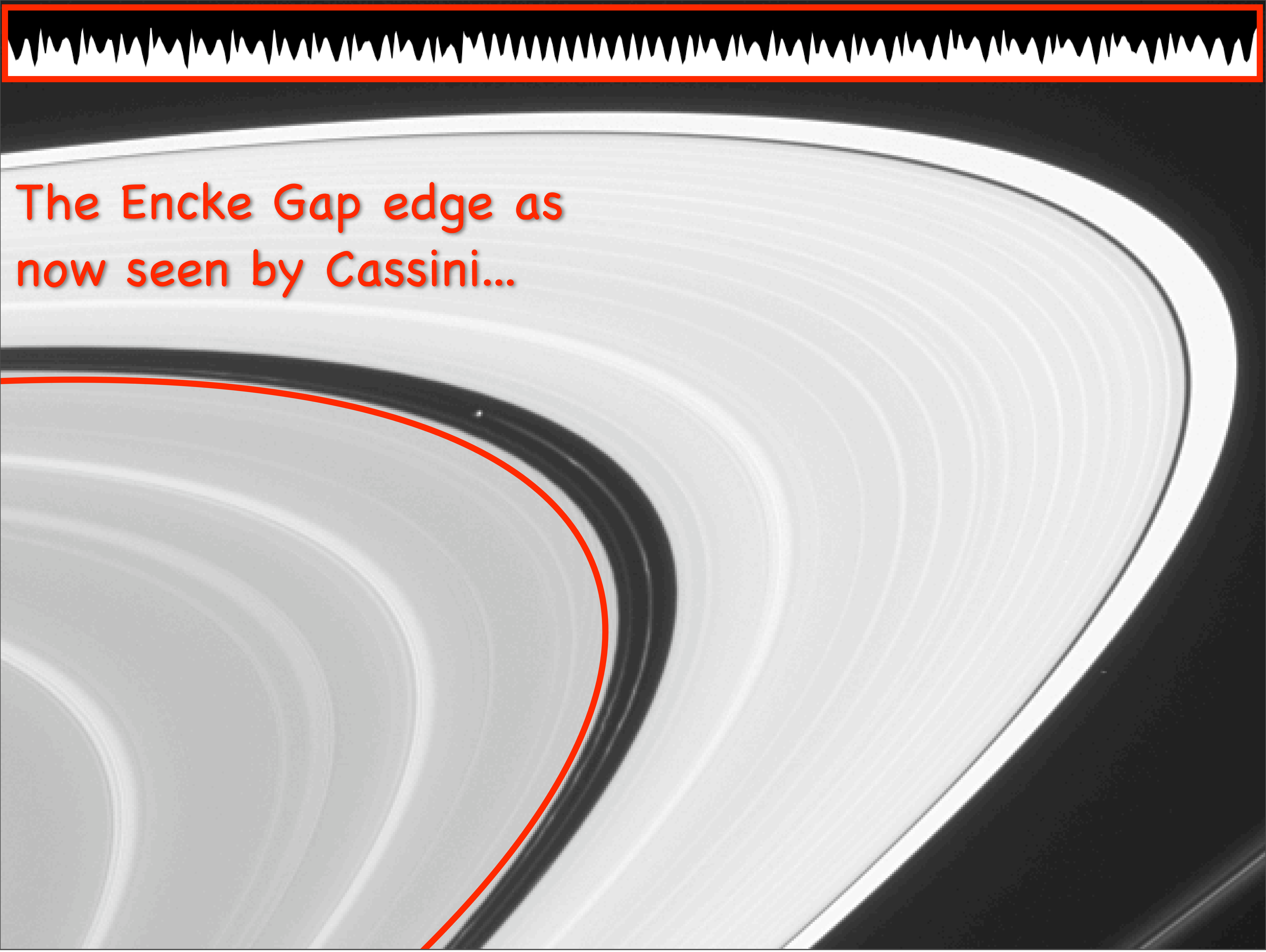
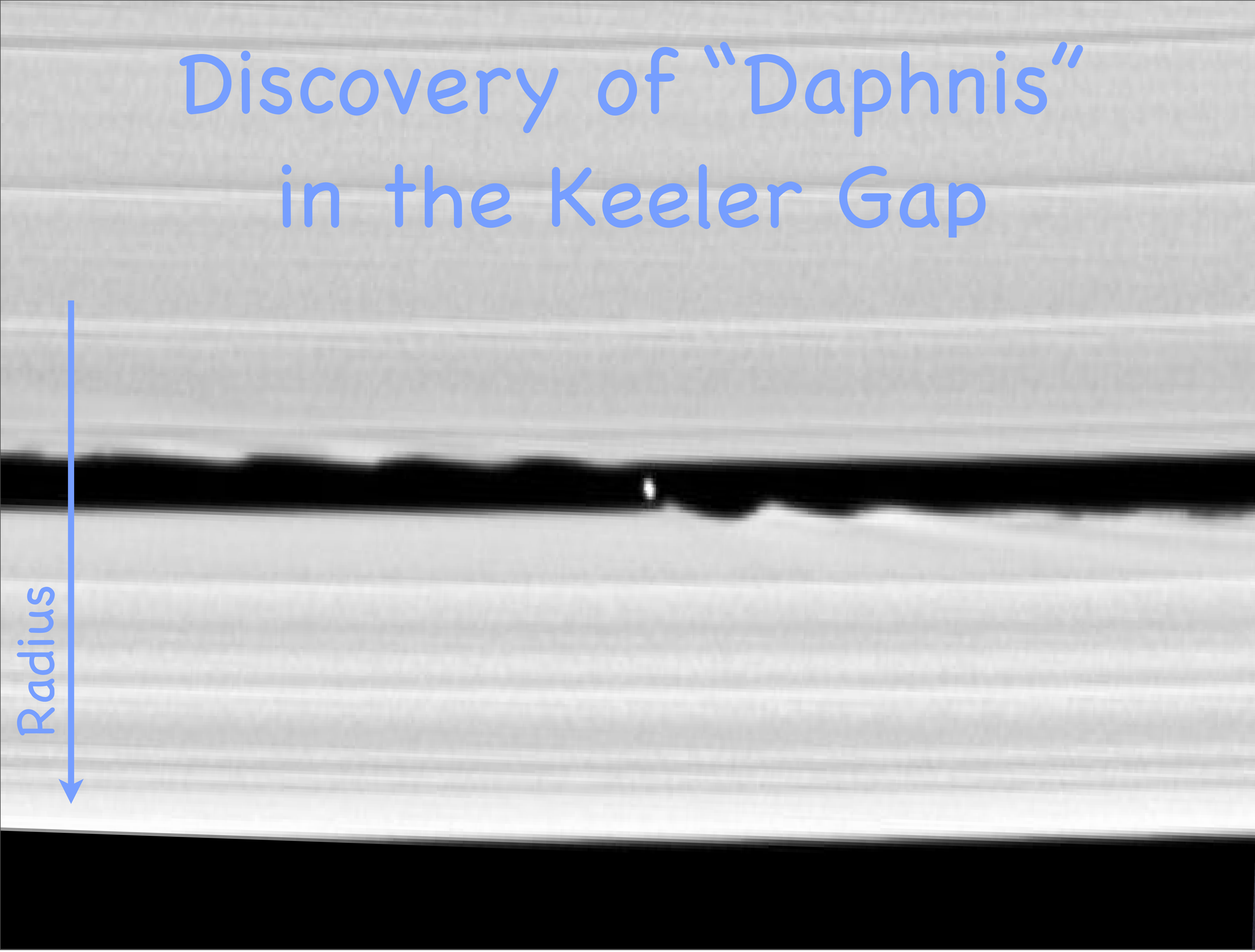


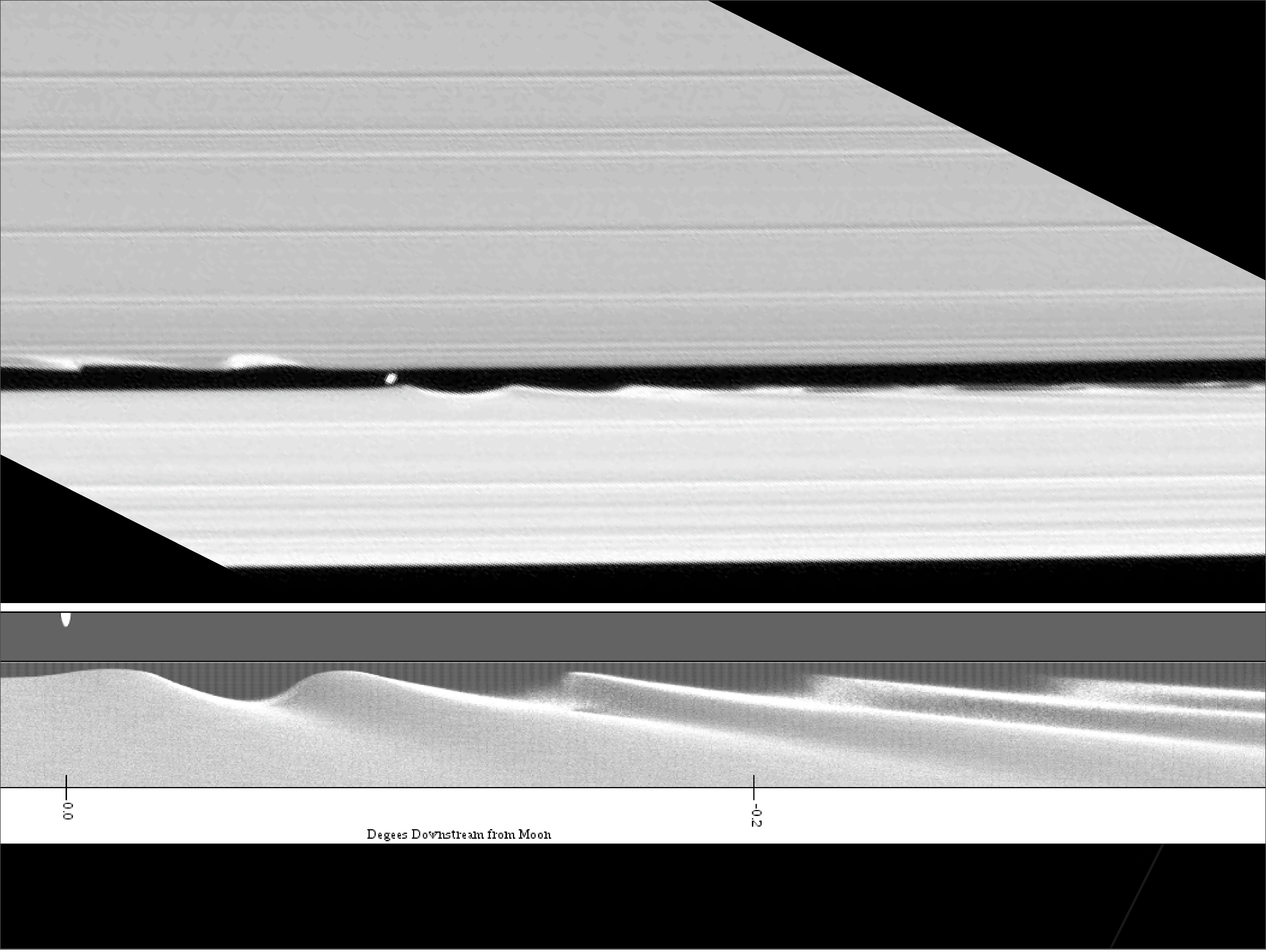


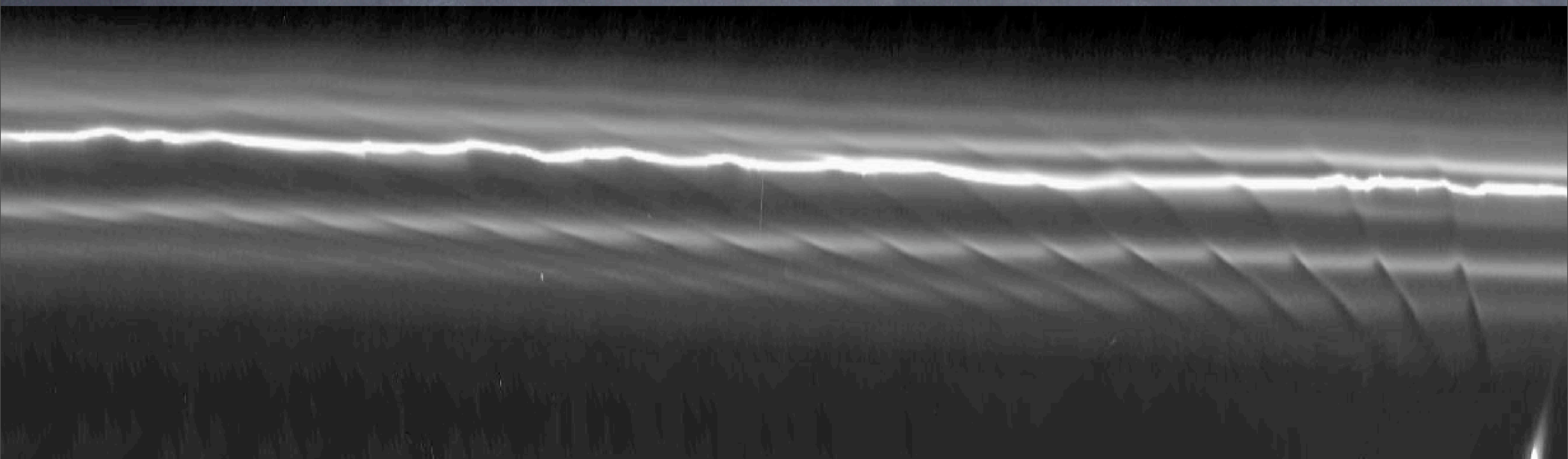
Pan's wake as seen by Cassini


The Encke Gap edge as
now seen by Cassini...

Discovery of "Daphnis" in the Keeler Gap

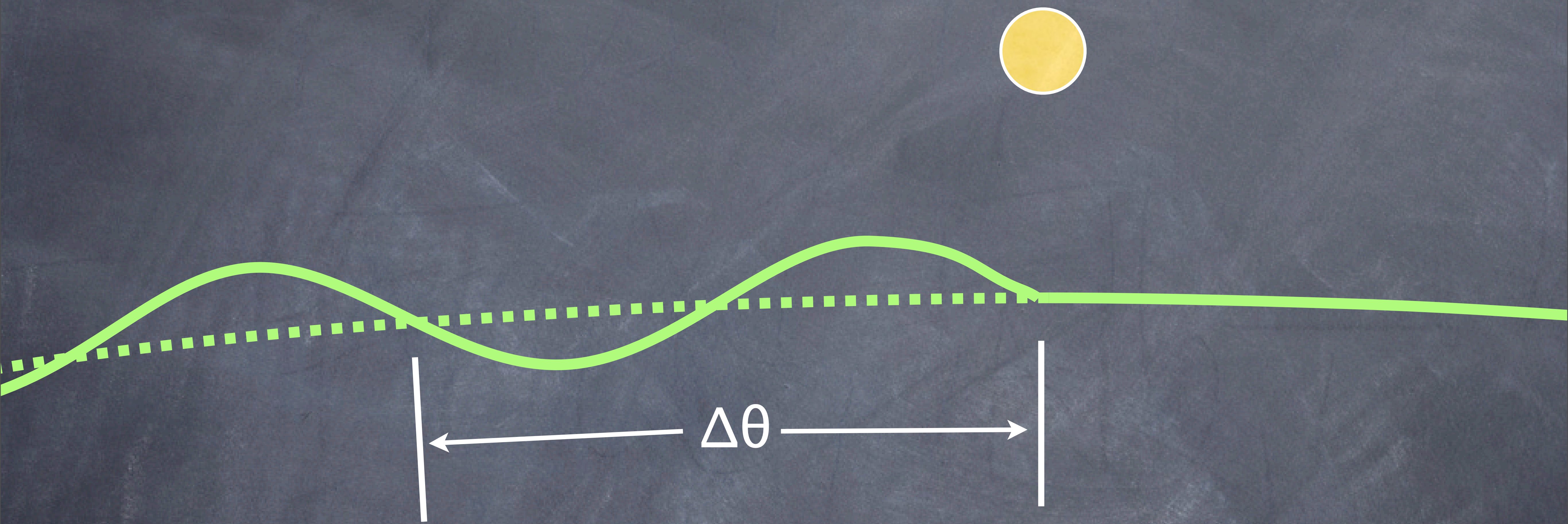



0.0
-0.2
Degees Downstream from Moon



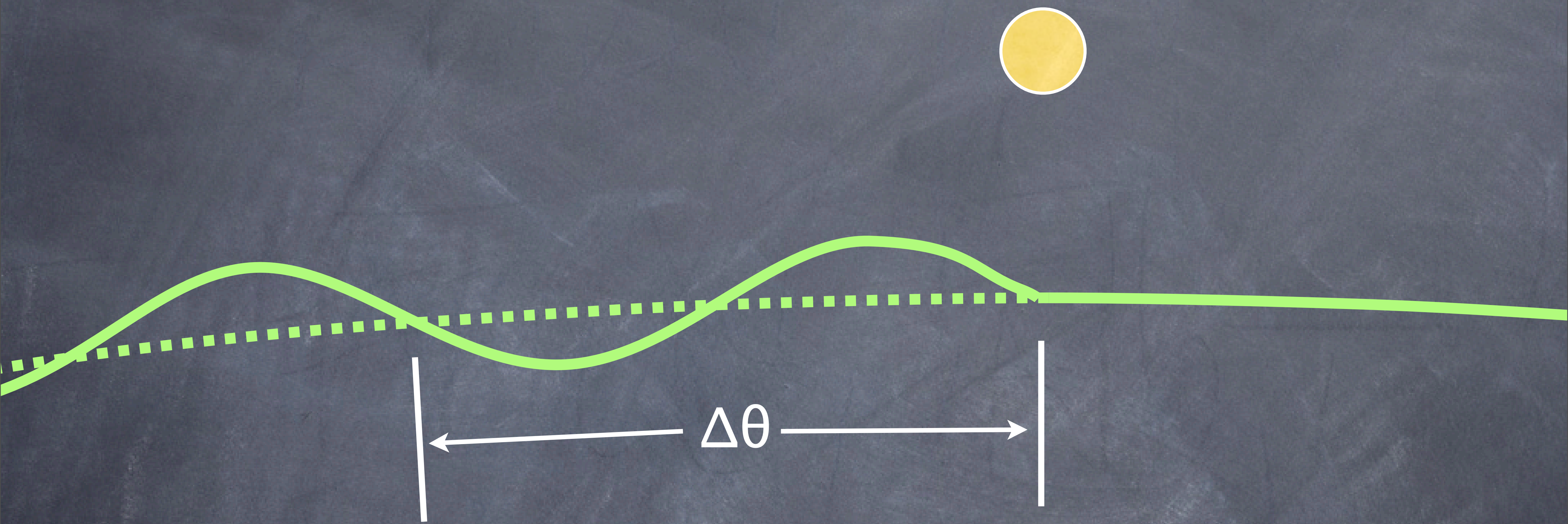
Prometheus produces a “wake” pattern much like Pan

Gravitational Deflection



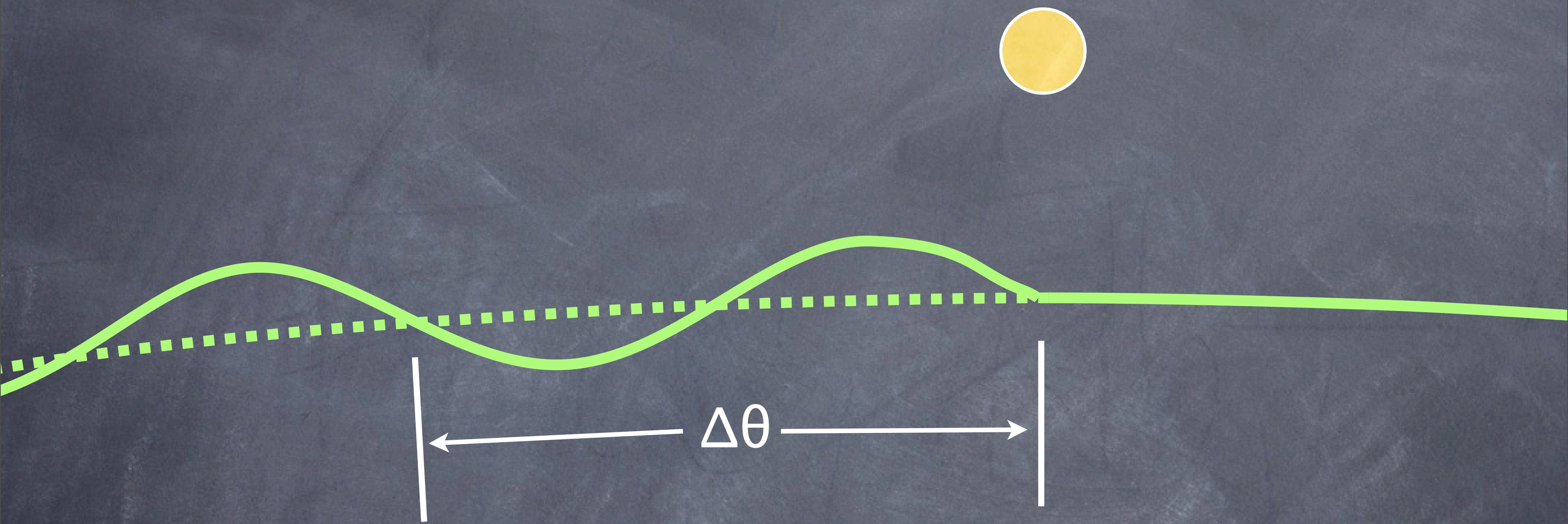
Top View,
Frame Rotating with Moon (n_M)

Gravitational Deflection



Top View,
Frame Rotating with Moon (n_M)

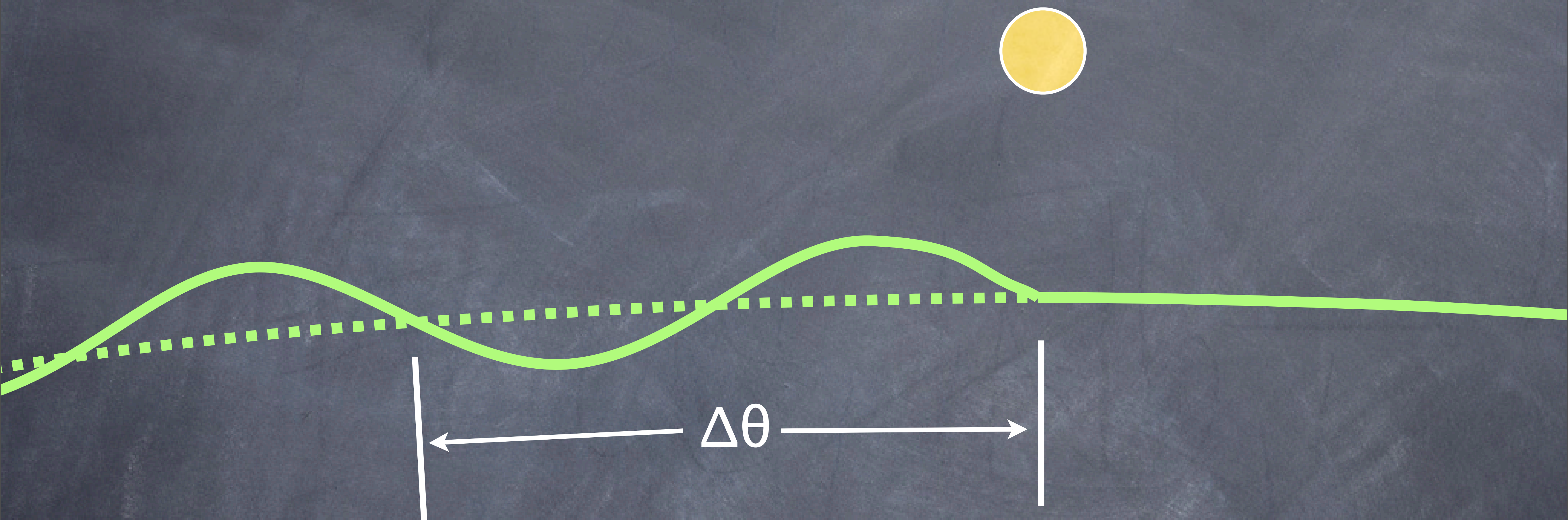
Gravitational Deflection



Question: What if $\Delta\theta = 2\pi/p$ for integer p?

Top View,
Frame Rotating with Moon (n_M)

Gravitational Deflection



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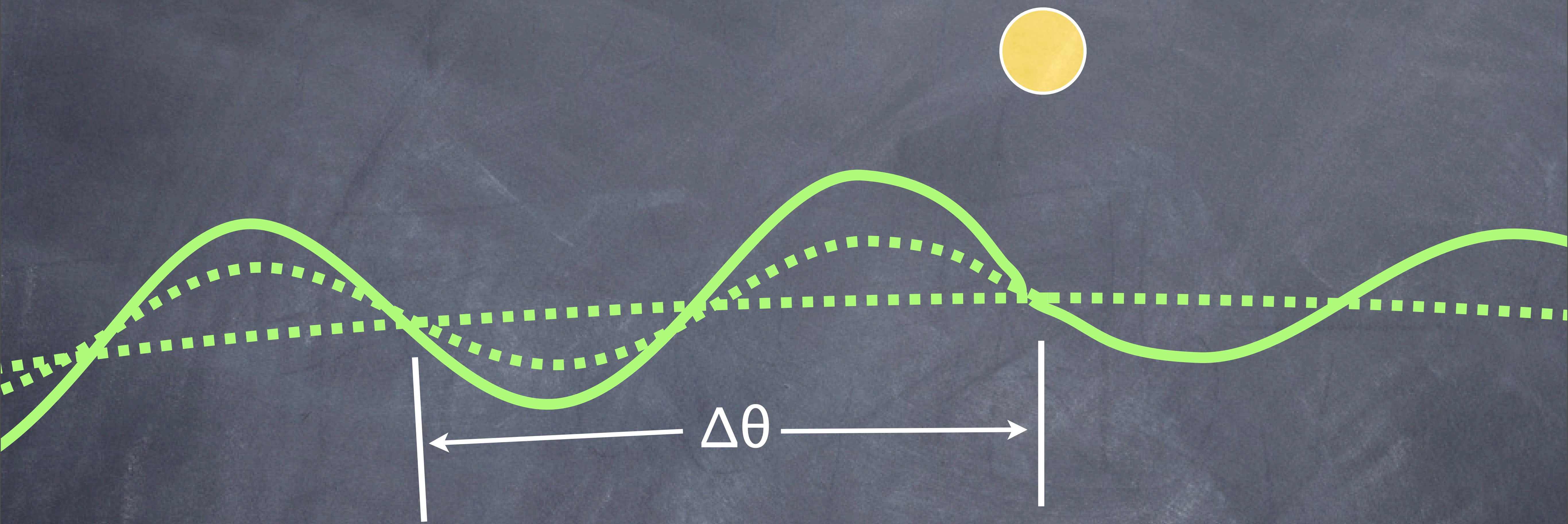
Top View,
Frame Rotating with Moon (n_M)

Gravitational Deflection



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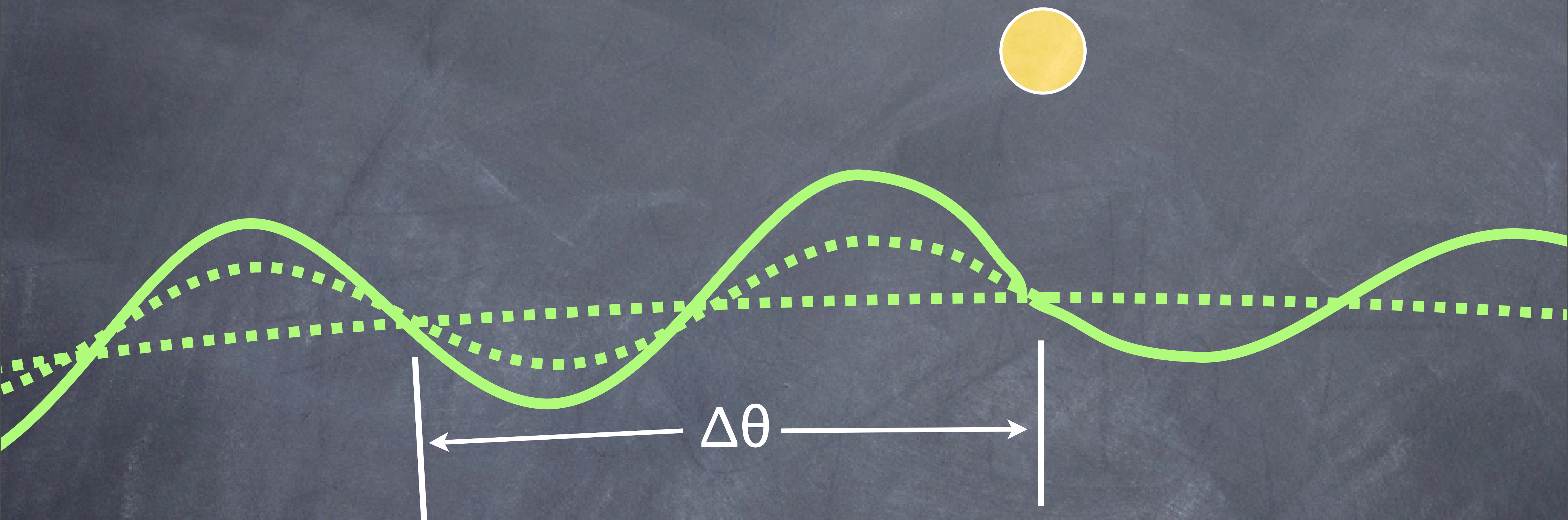
Top View,
Frame Rotating with Moon (n_M)

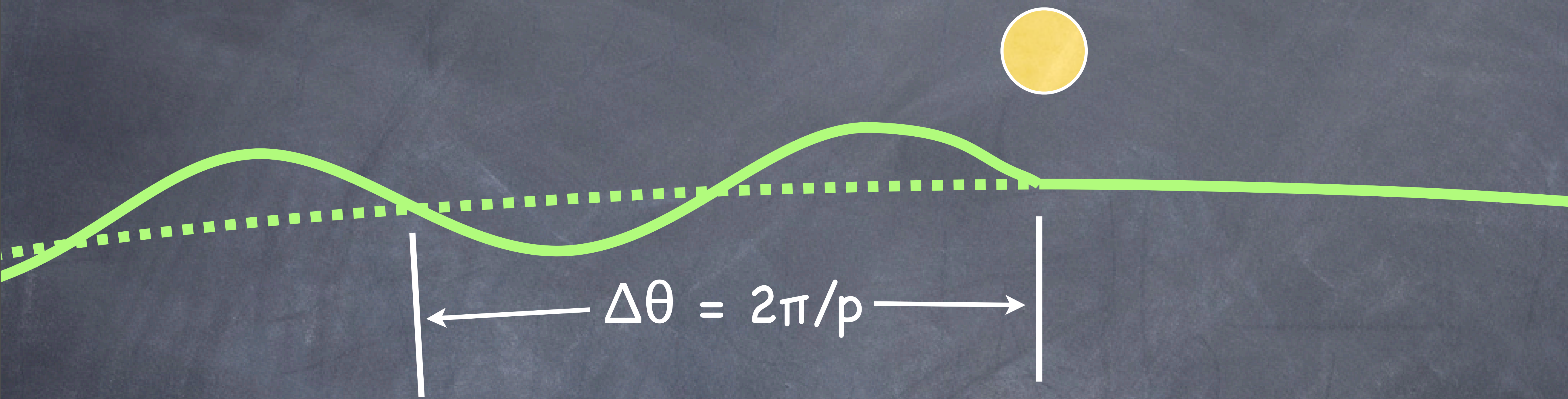
Gravitational Deflection



Question: What if $\Delta\theta = 2\pi/p$ for integer p?
Answer: Resonance!

Top View,
Frame Rotating with Moon (n_M)

Lindblad Resonances



- Epicyclic period of ring particle $T = 2\pi/\kappa_R$.
- In this period, the moon shifts $T\,|n_R - n_M| = 2\pi/p$.

$$p\,|n_R - n_M| = \kappa_R$$

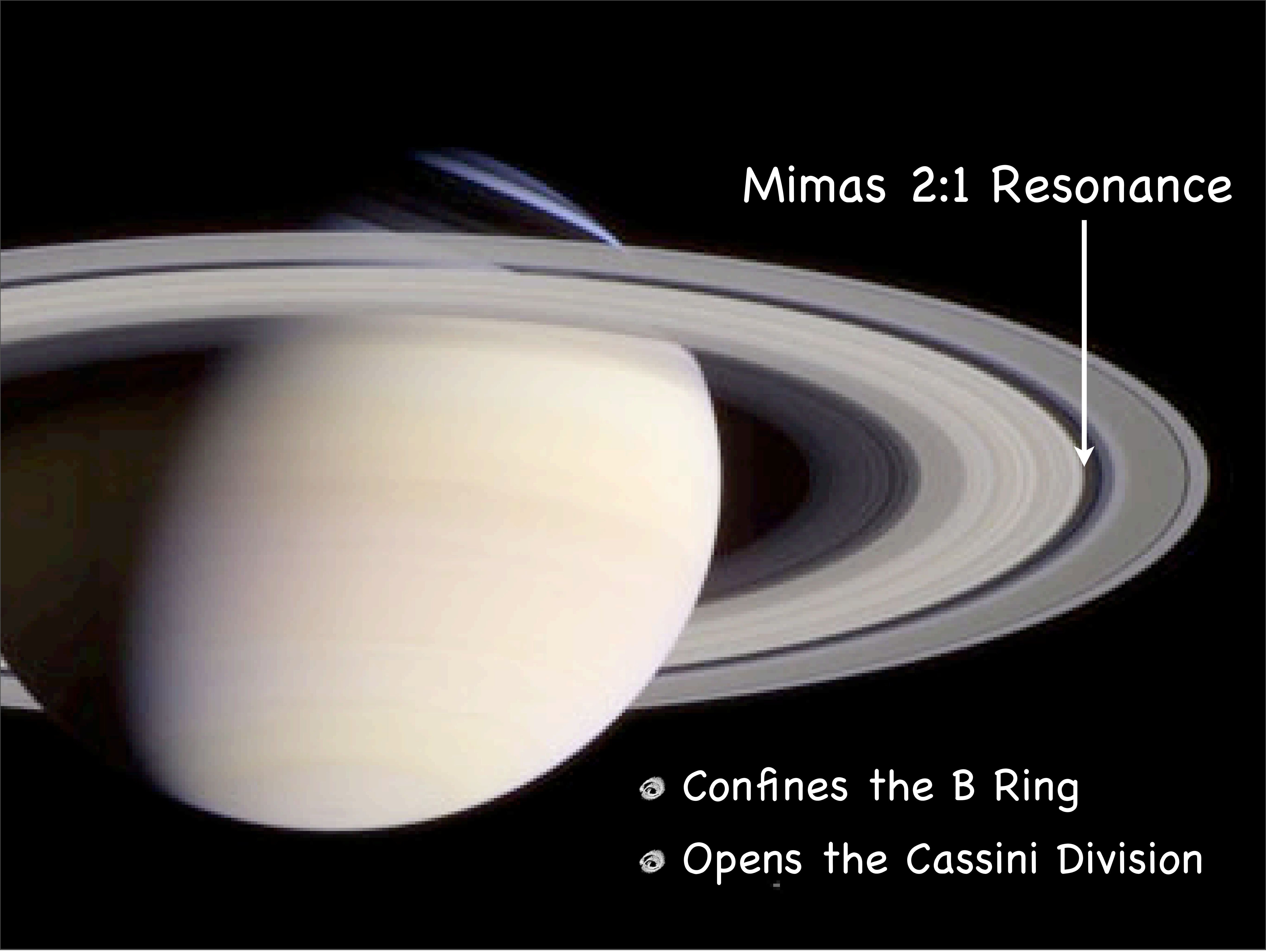
- Can be written in other forms.

Lindblad Resonances

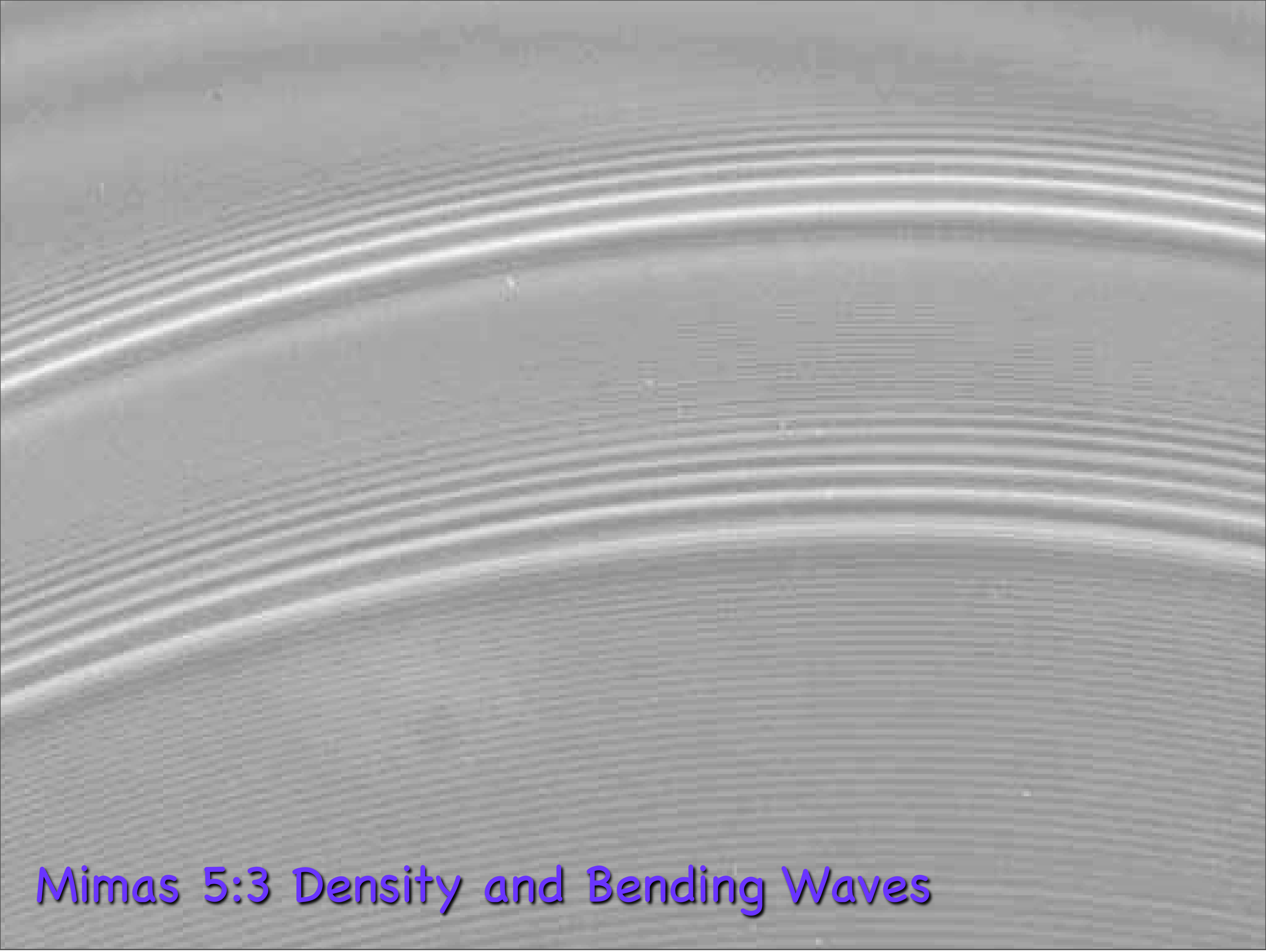
- Vertical resonances are perfectly analogous:

$$p\,|n_R - n_M| = \nu_R$$

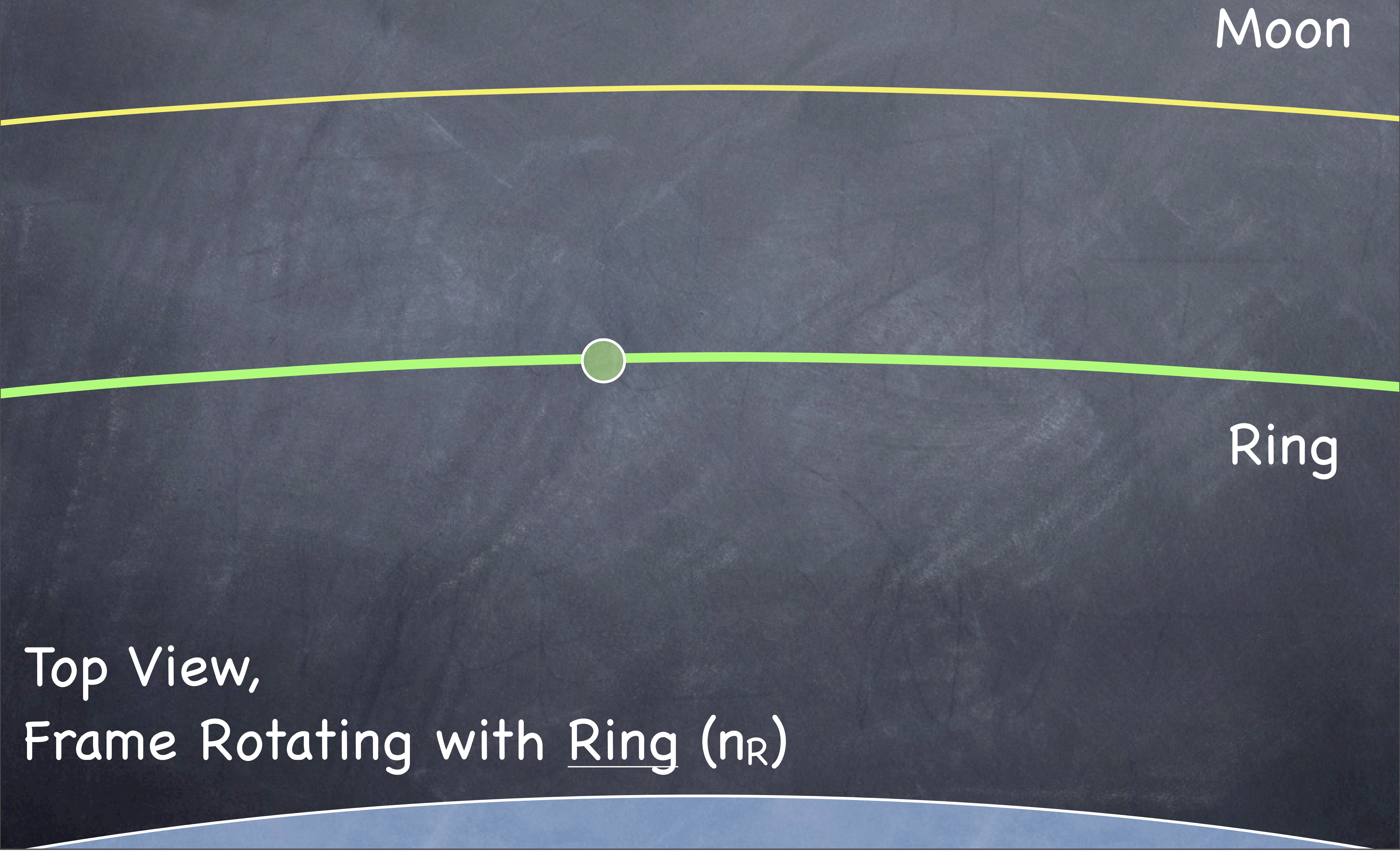
- These can lead to ...
  - Sharp ring edges.
  - Gaps.
  - Density and bending waves.


Mimas 2:1 Resonance
Confines the B Ring
Opens the Cassini Division


Atlas 7:6 Resonance
Confines the A Ring


Mimas 5:3 Density and Bending Waves

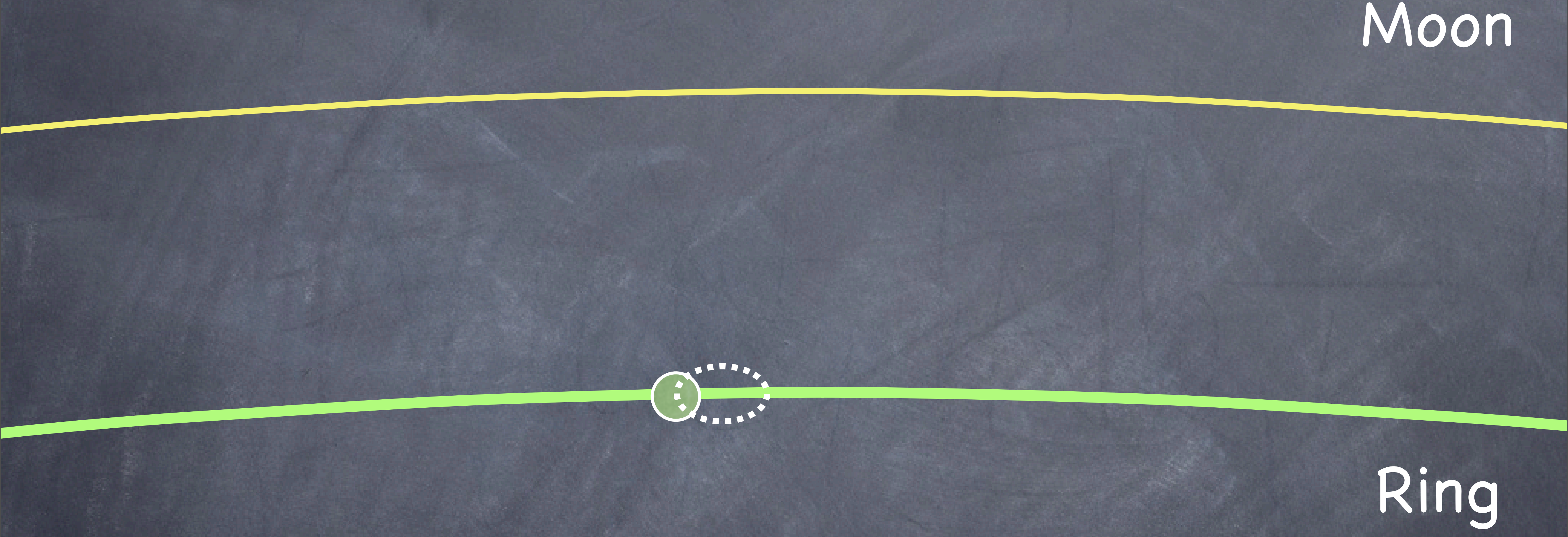
Ring-Moon Interactions #2



Top View,

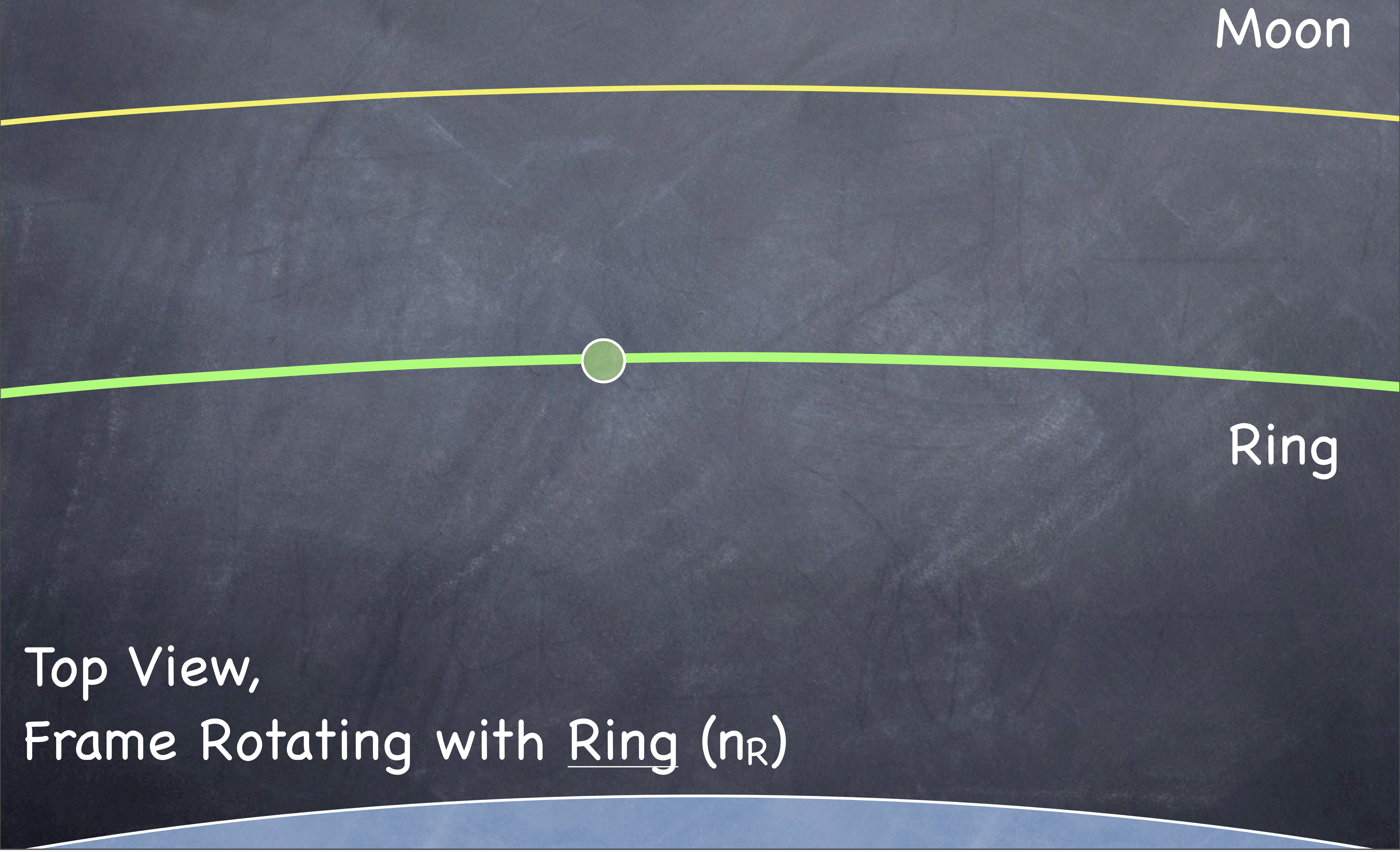
Frame Rotating with <u>Ring</u> (n_R)

Ring-Moon Interactions #2



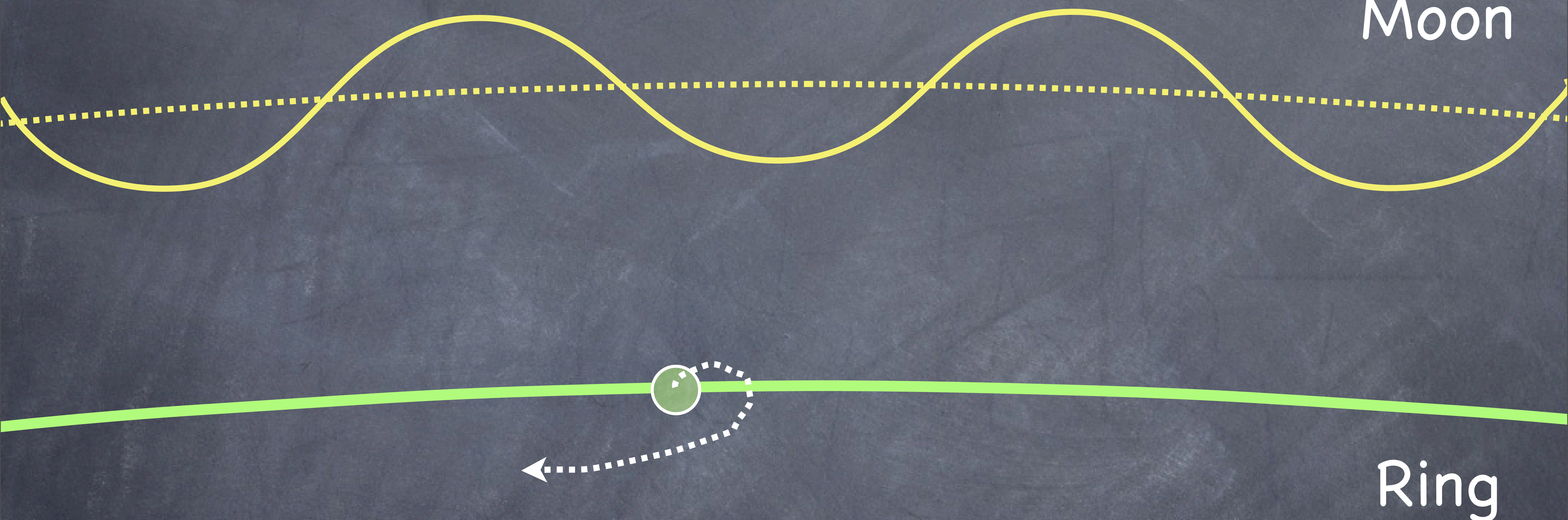
Top View,
Frame Rotating with Ring (n_R)

Ring-Moon Interactions #2



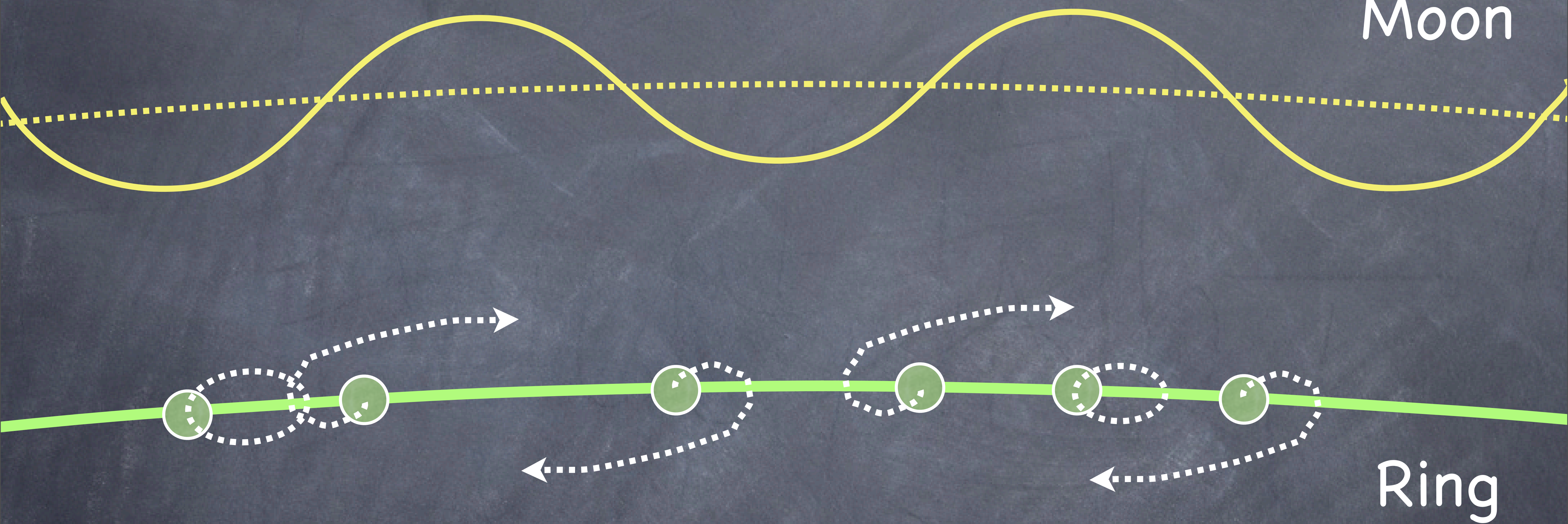
Top View,
Frame Rotating with Ring (n_R)

Ring-Moon Interactions #2



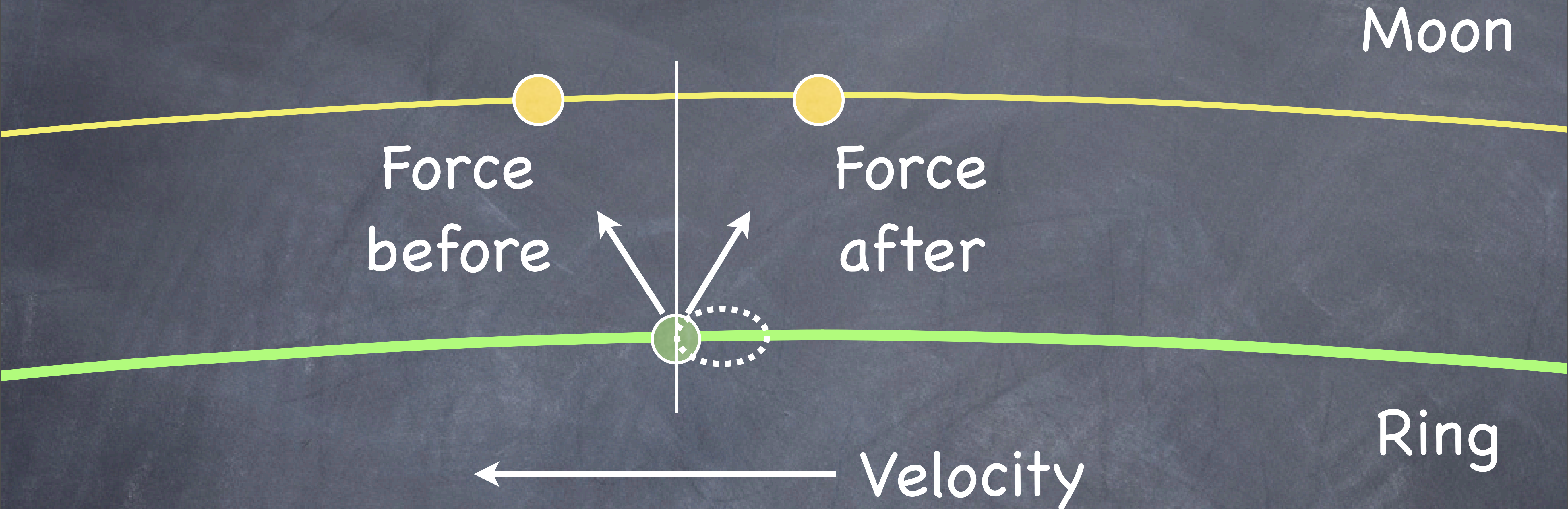
Top View,
Frame Rotating with Ring (n_R)

Ring-Moon Interactions #2



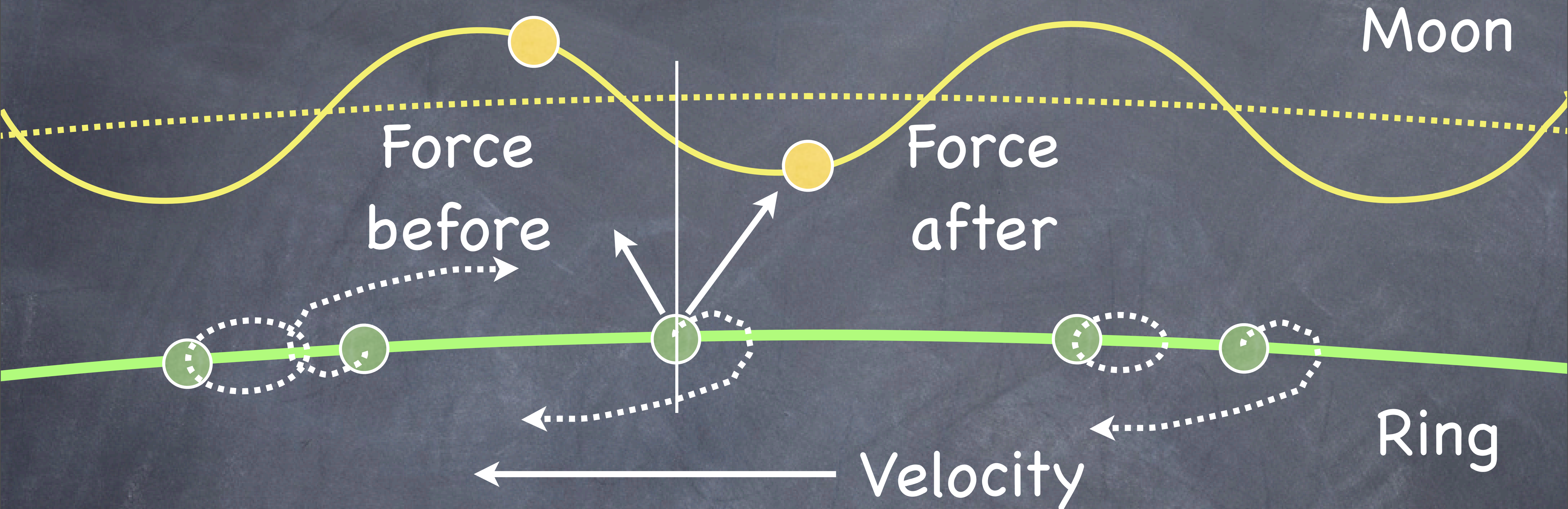
Top View,
Frame Rotating with Ring (n_R)

Orbital Energy Exchange



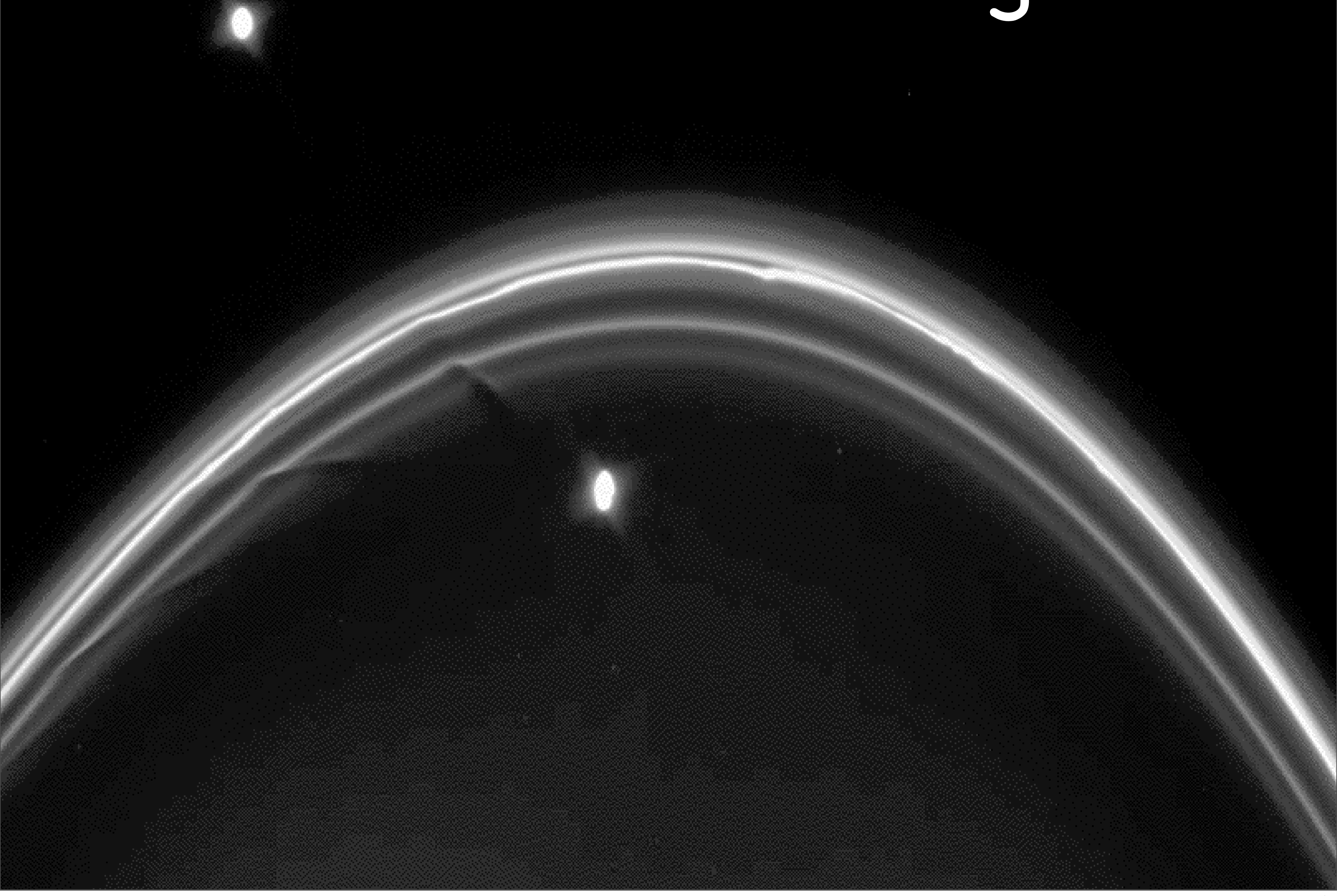
- Force x Distance = Work.
- Work before encounter cancels work after.
- With no net change in energy, semimajor axis is conserved.

Orbital Energy Exchange

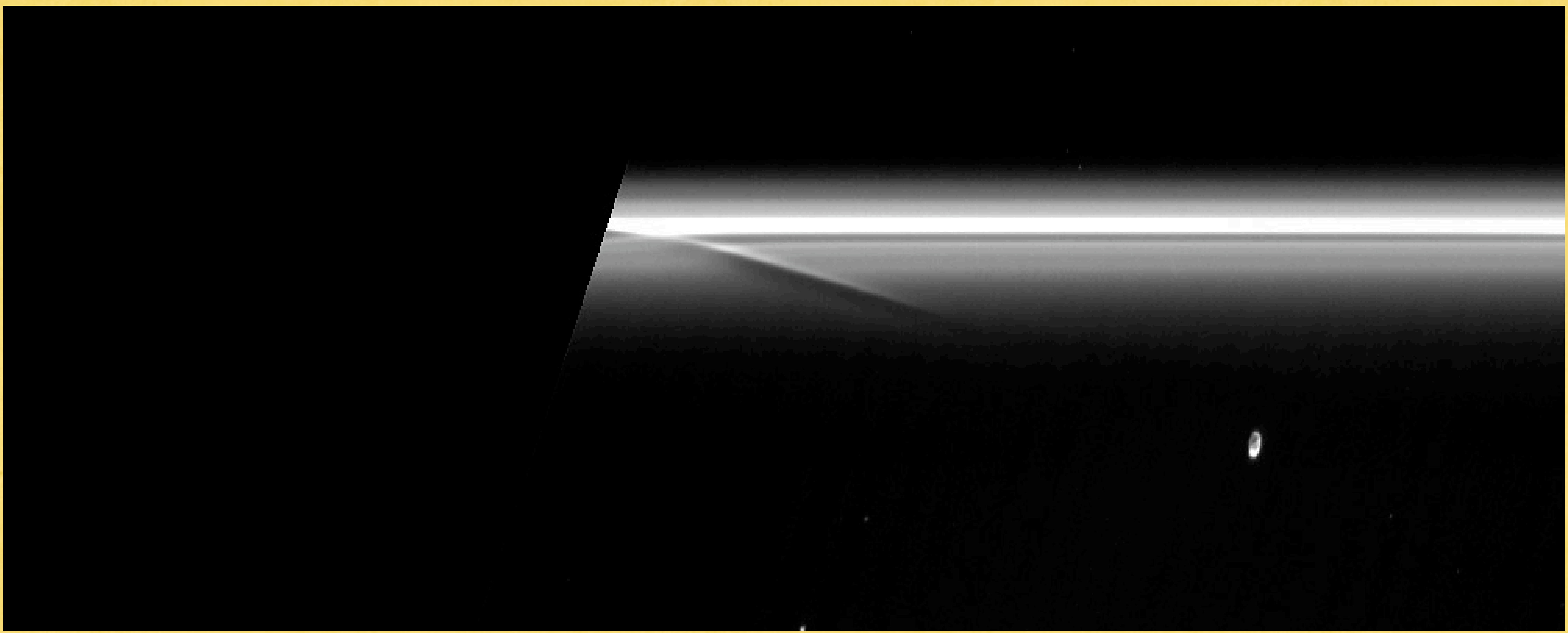


- Work after encounter is larger than work before.
- Net work is negative, so semimajor axis decreases and mean motion increases.
- Ring bodies no longer have the same mean motion.

Back to the F Ring...

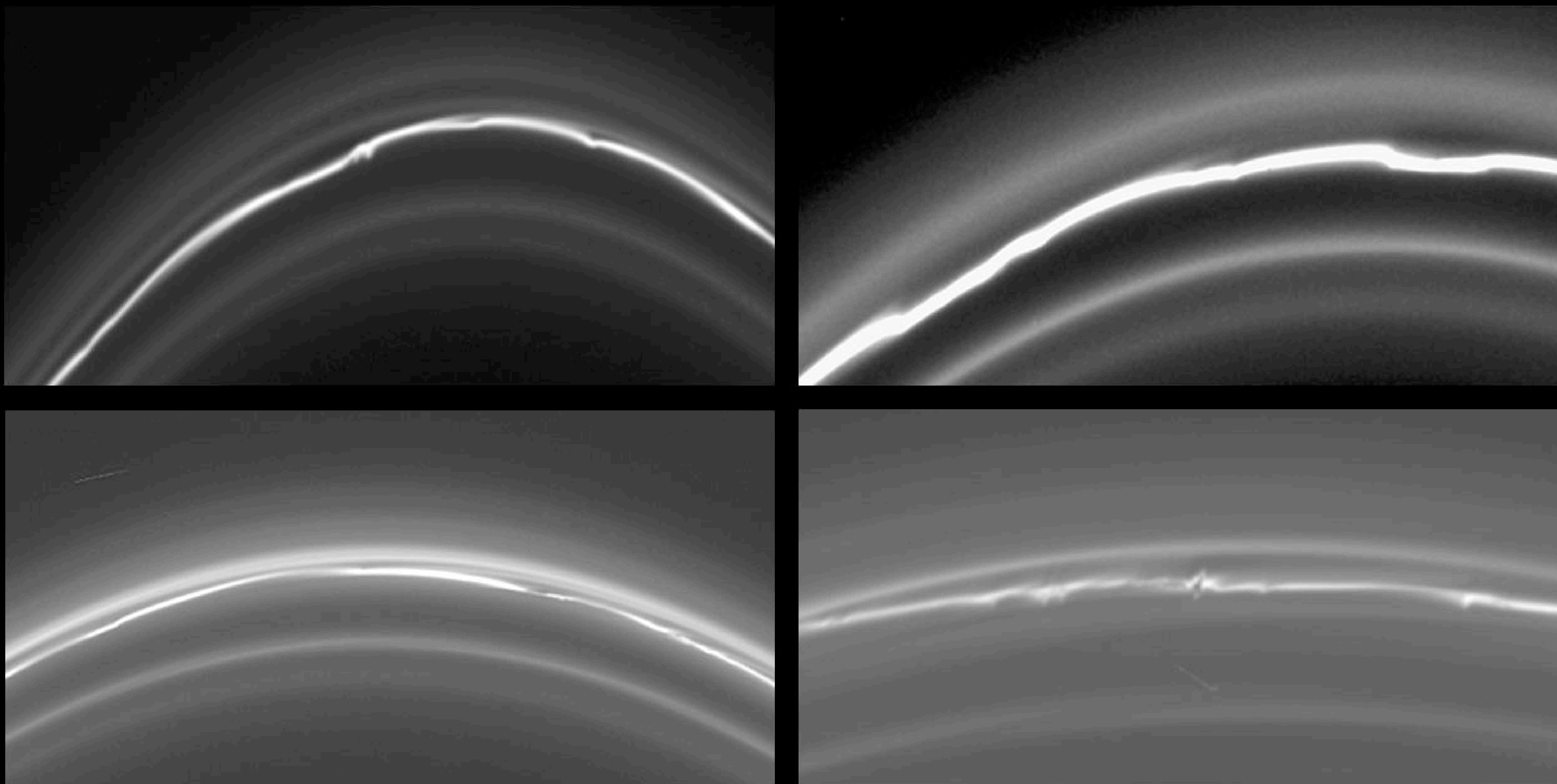


Pandora Perturbs the Ring



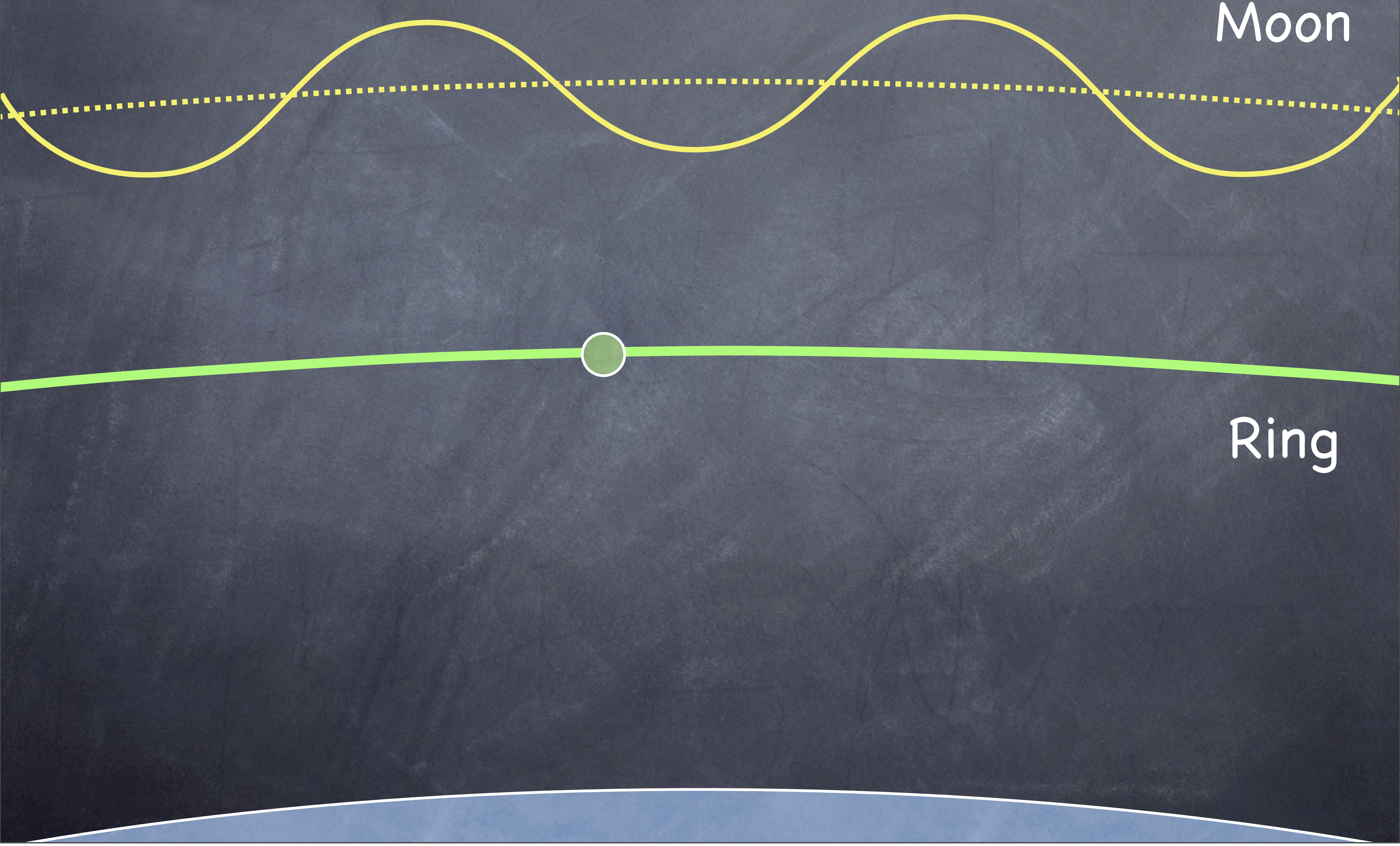
Pandora Perturbs the Ring

Pandora and Prometheus:

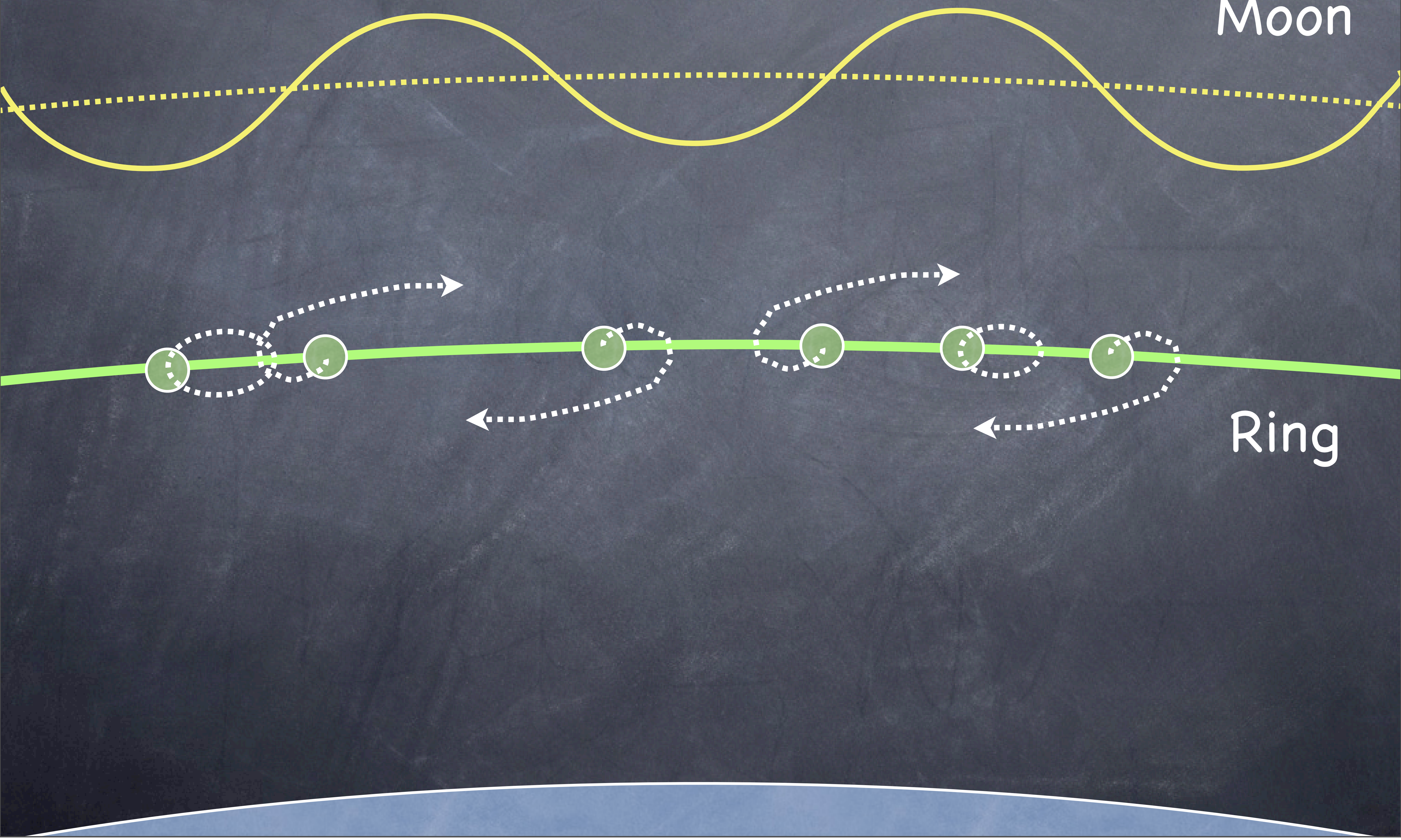


“Shepherds” or “Wolves”?

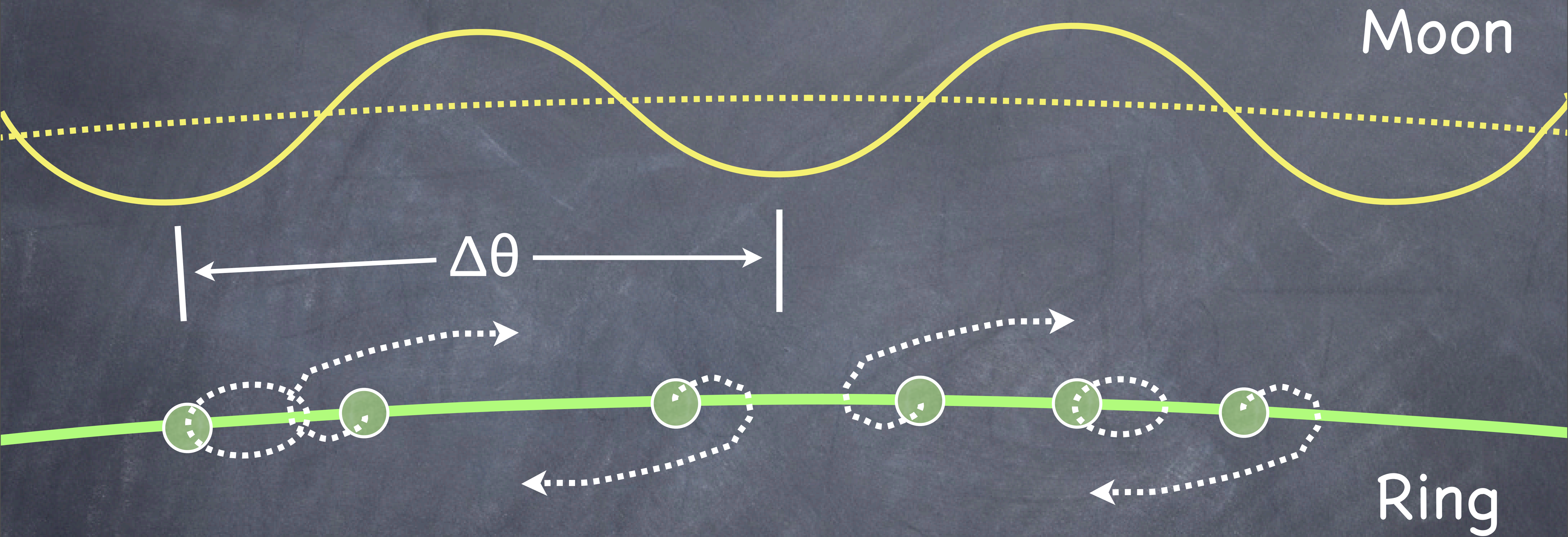
Orbital Energy Exchange



Orbital Energy Exchange



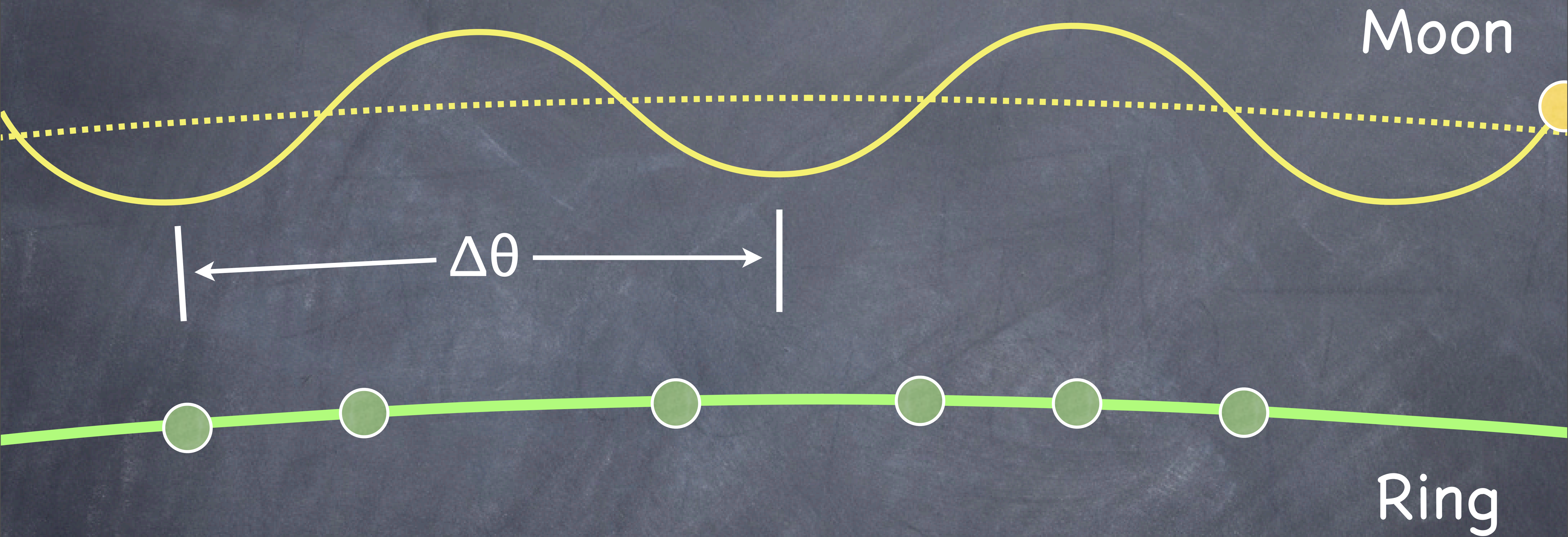
Orbital Energy Exchange



Questions: What if $\Delta\theta = 2\pi/p$ for integer p?

What if perturbation is smaller?

Corotation Resonances



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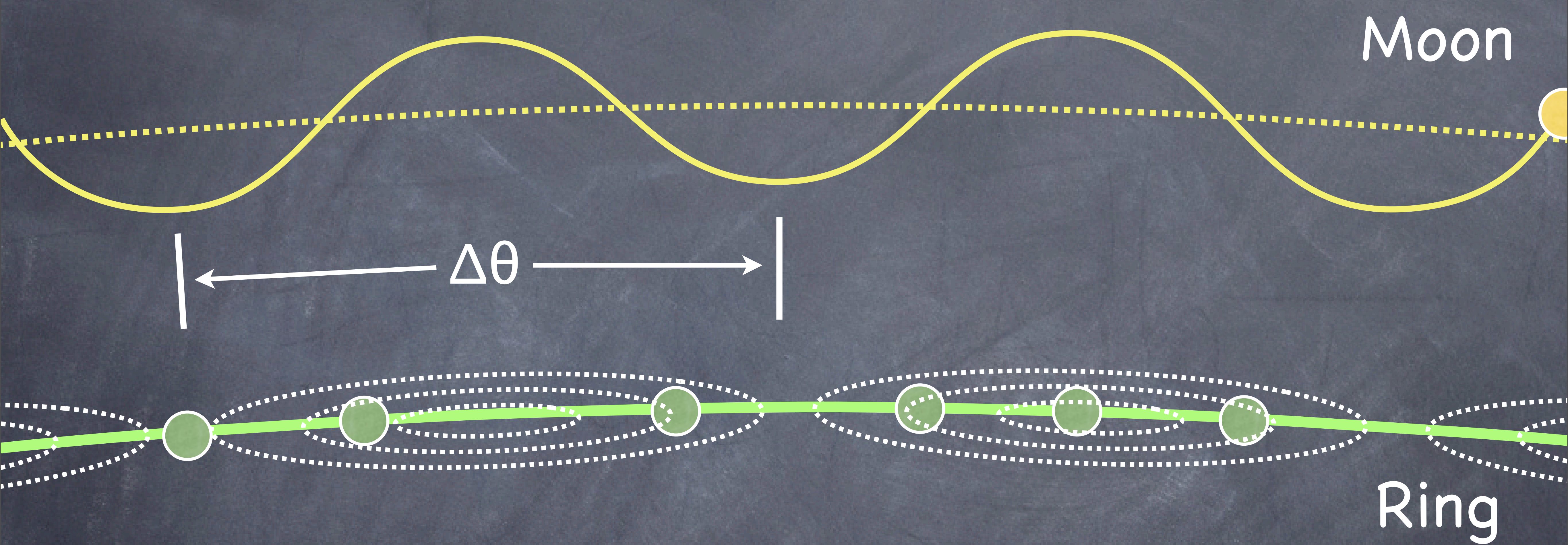
Corotation Resonances

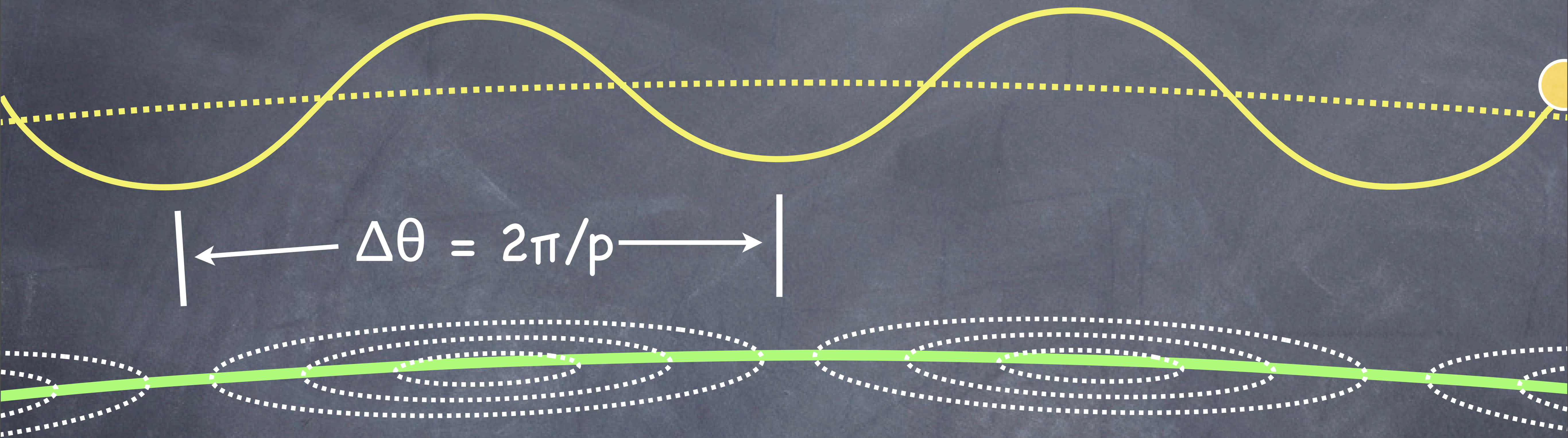


Questions: What if $\Delta\theta = 2\pi/p$ for integer p?

What if perturbation is smaller?

Answer: Stable Resonant Confinement!

Corotation Resonances



- Epicyclic period of <u>moon</u> $T = 2\pi/\kappa_M$.
- In this period, the <u>ring</u> shifts $T\,|n_R - n_M| = 2\pi/p$.

$$p\,|n_R - n_M| = \kappa_M$$

- Compare to Lindblad: $p\,|n_R - n_M| = \underline{\kappa_R}$

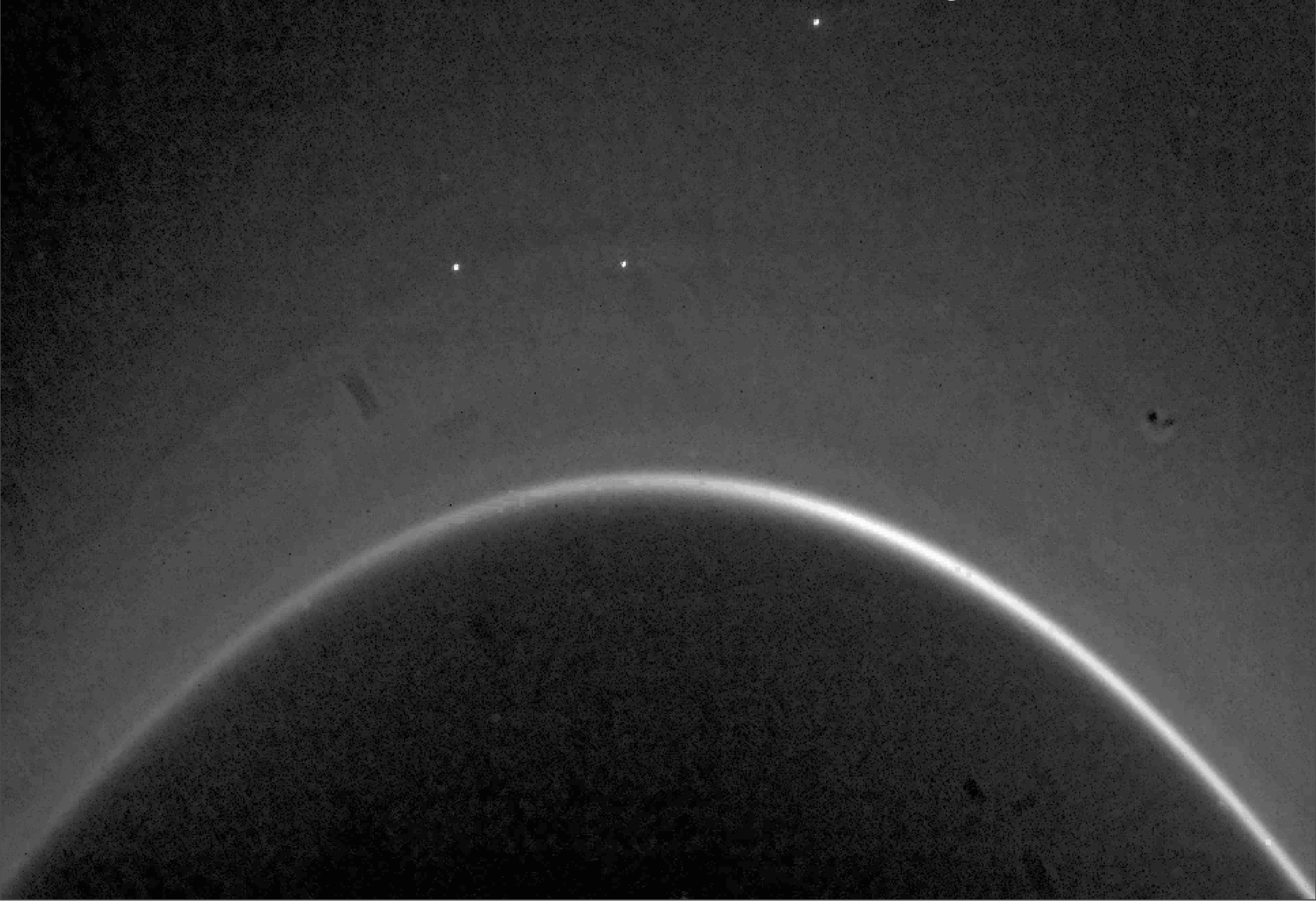
Corotation Resonances

- Vertical resonances are almost perfectly analogous:

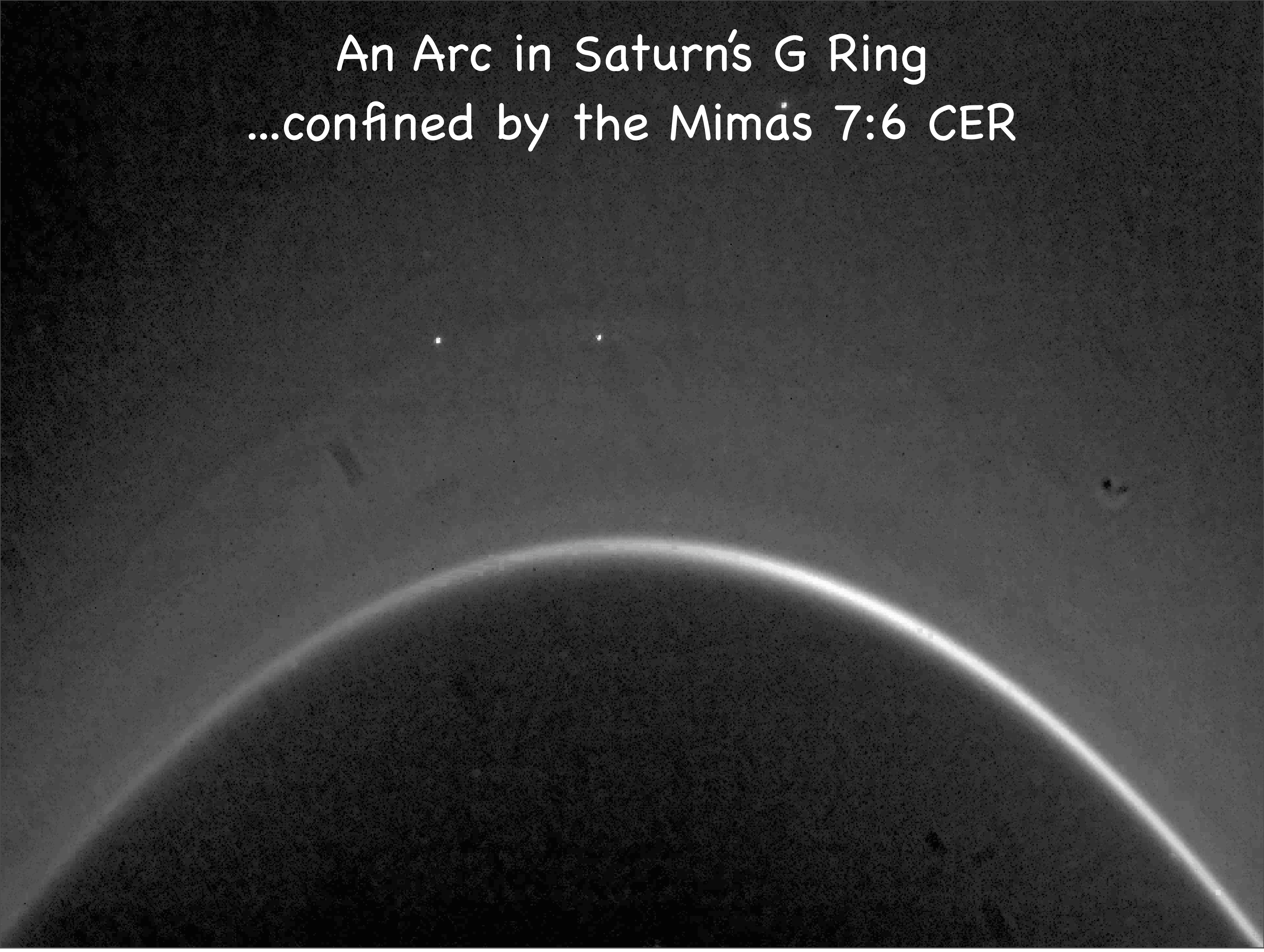
$$p\ |n_R - n_M| = \nu_M/2$$

- Why the difference?
    - An inclined moon has two close approaches per orbit rather than one!
- These can lead to ...
    - Confinement of clumps and arcs.

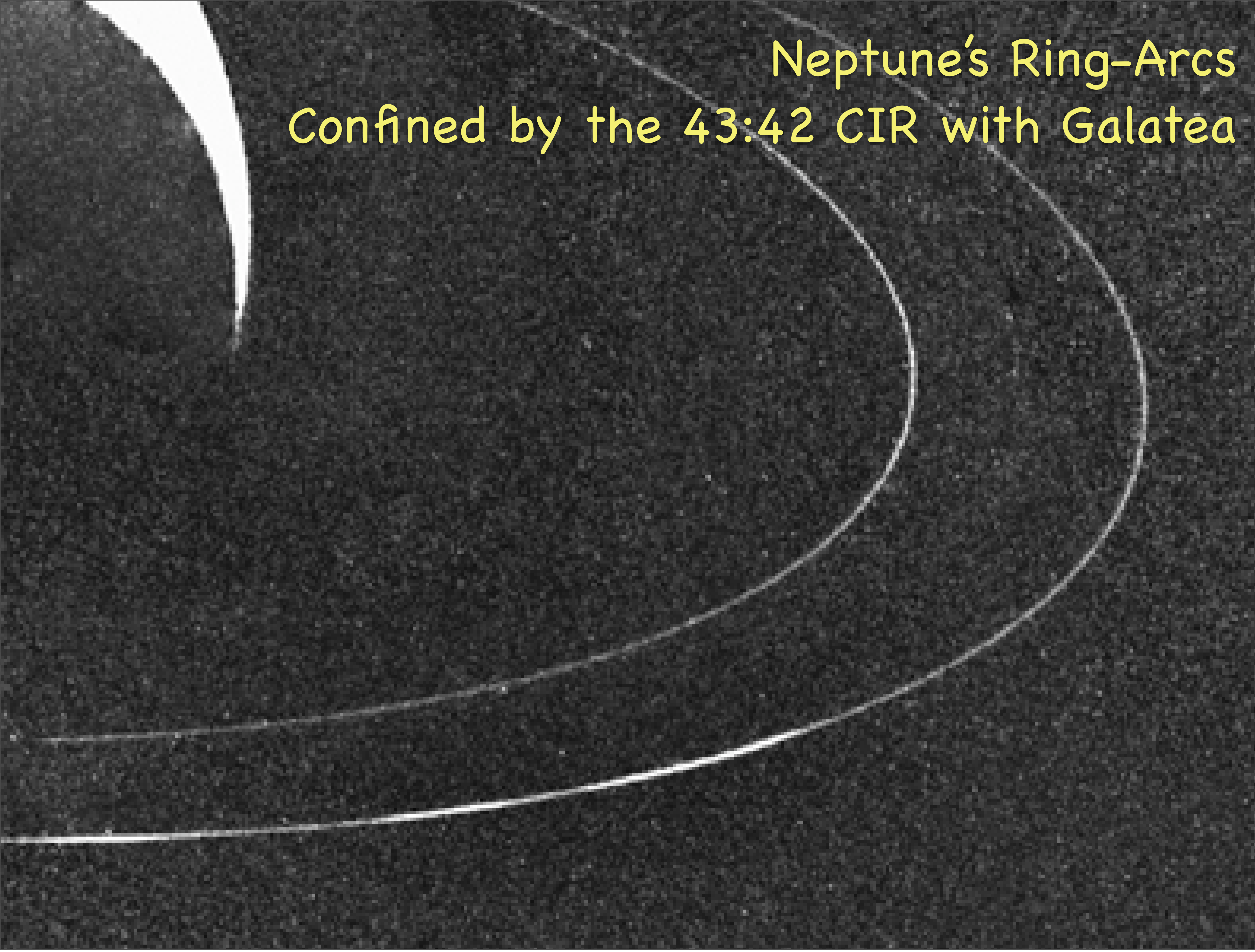
An Arc in Saturn's G Ring

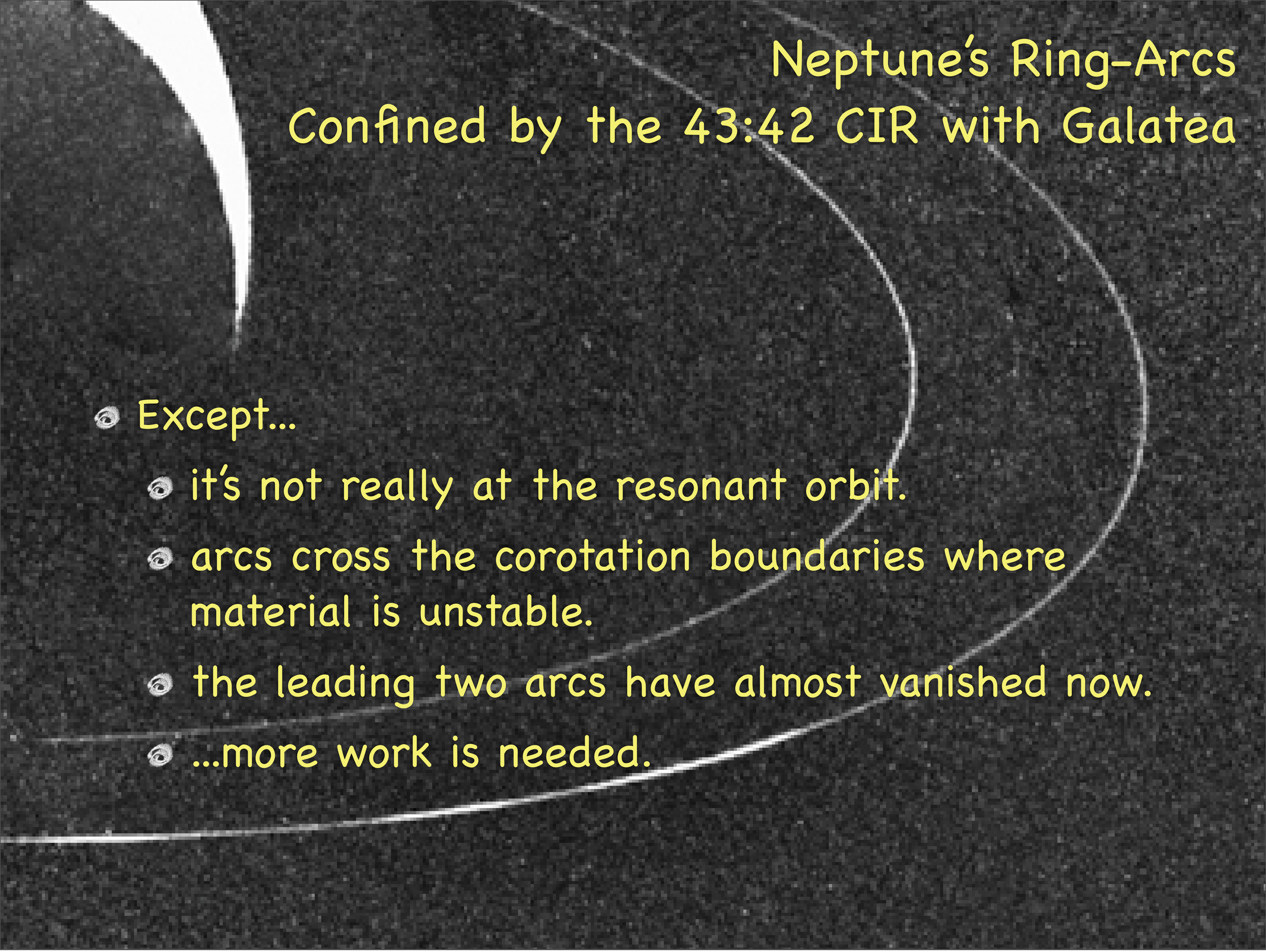


An Arc in Saturn's G Ring
...confined by the Mimas 7:6 CER



Neptune's Ring-Arcs
Confined by the 43:42 CIR with Galatea

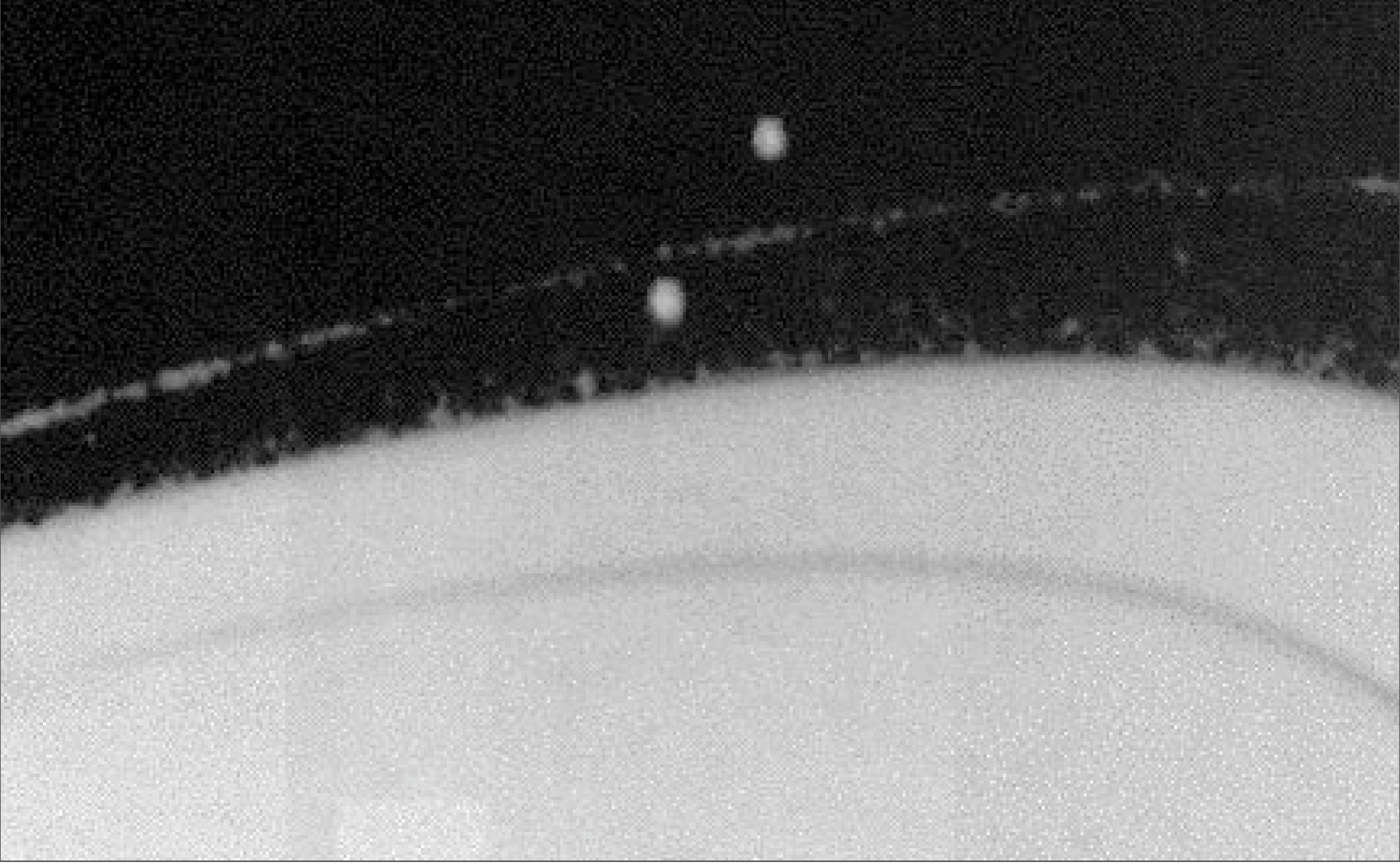




Neptune's Ring-Arcs Confined by the 43:42 CIR with Galatea

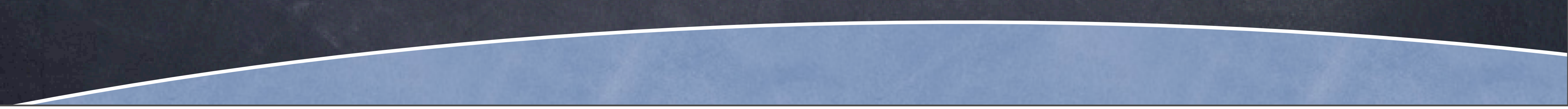
- Except...
  - it's not really at the resonant orbit.
  - arcs cross the corotation boundaries where material is unstable.
  - the leading two arcs have almost vanished now.
  - ...more work is needed.

What is “Shepherding”?

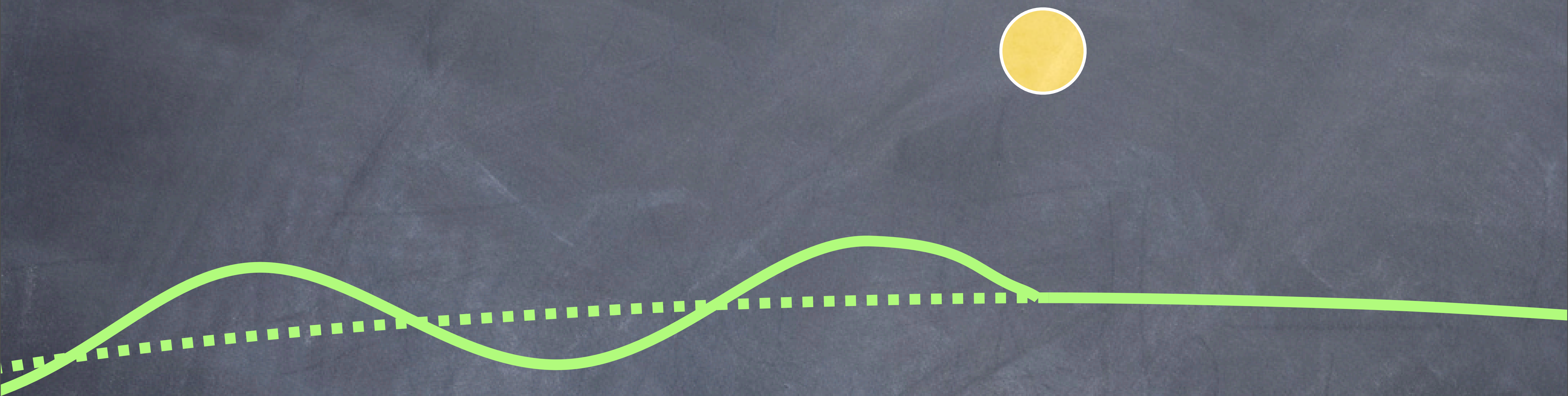


“Traditional” Shepherding

- Particle approaches with $e_R = 0$.
- Particle departs with $e_R > 0$.



"Traditional" Shepherding



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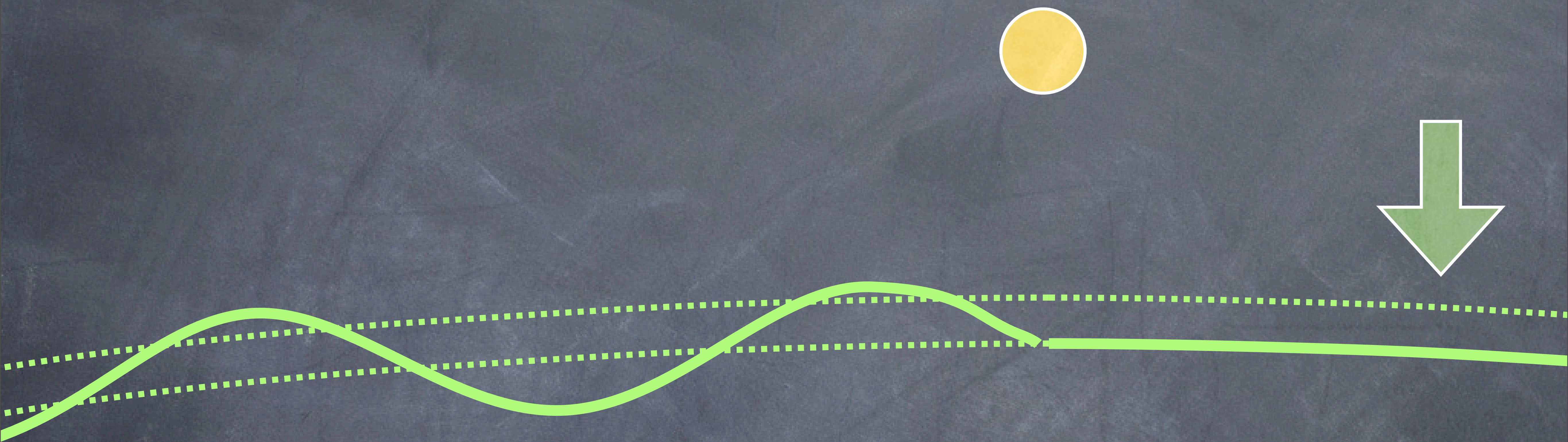
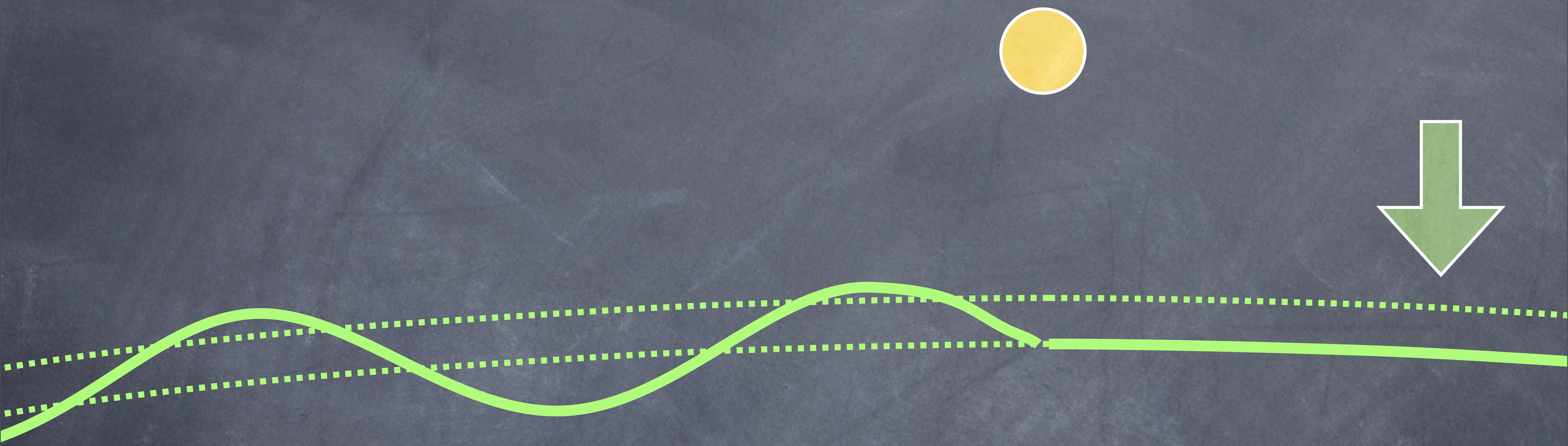
"Traditional" Shepherding



- Particle approaches with $e_R = 0$.
- Particle departs with $e_R > 0$.
- If e_R is damped before the next passage, then conservation laws require Δa to increase very slightly.

“Traditional” Shepherding



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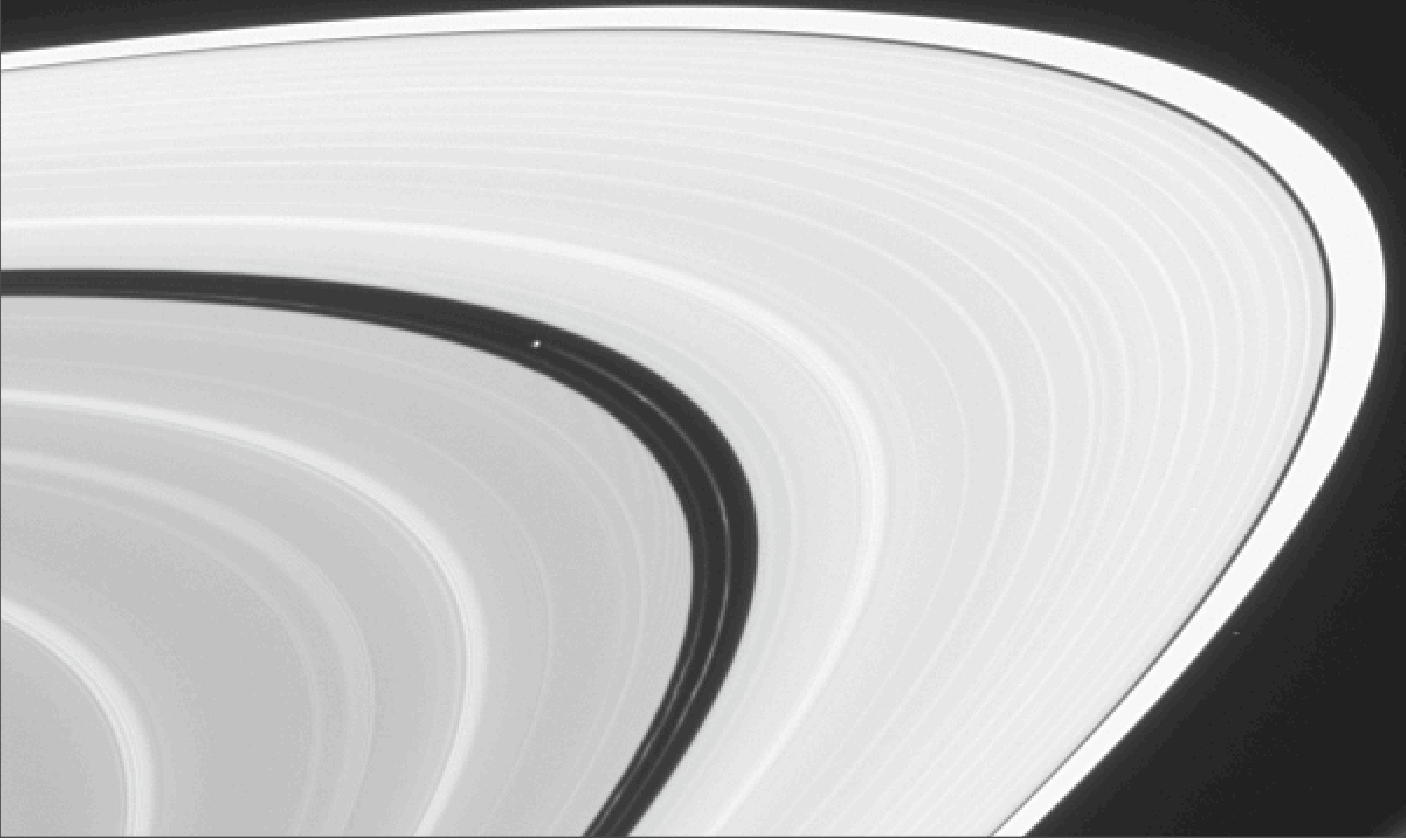
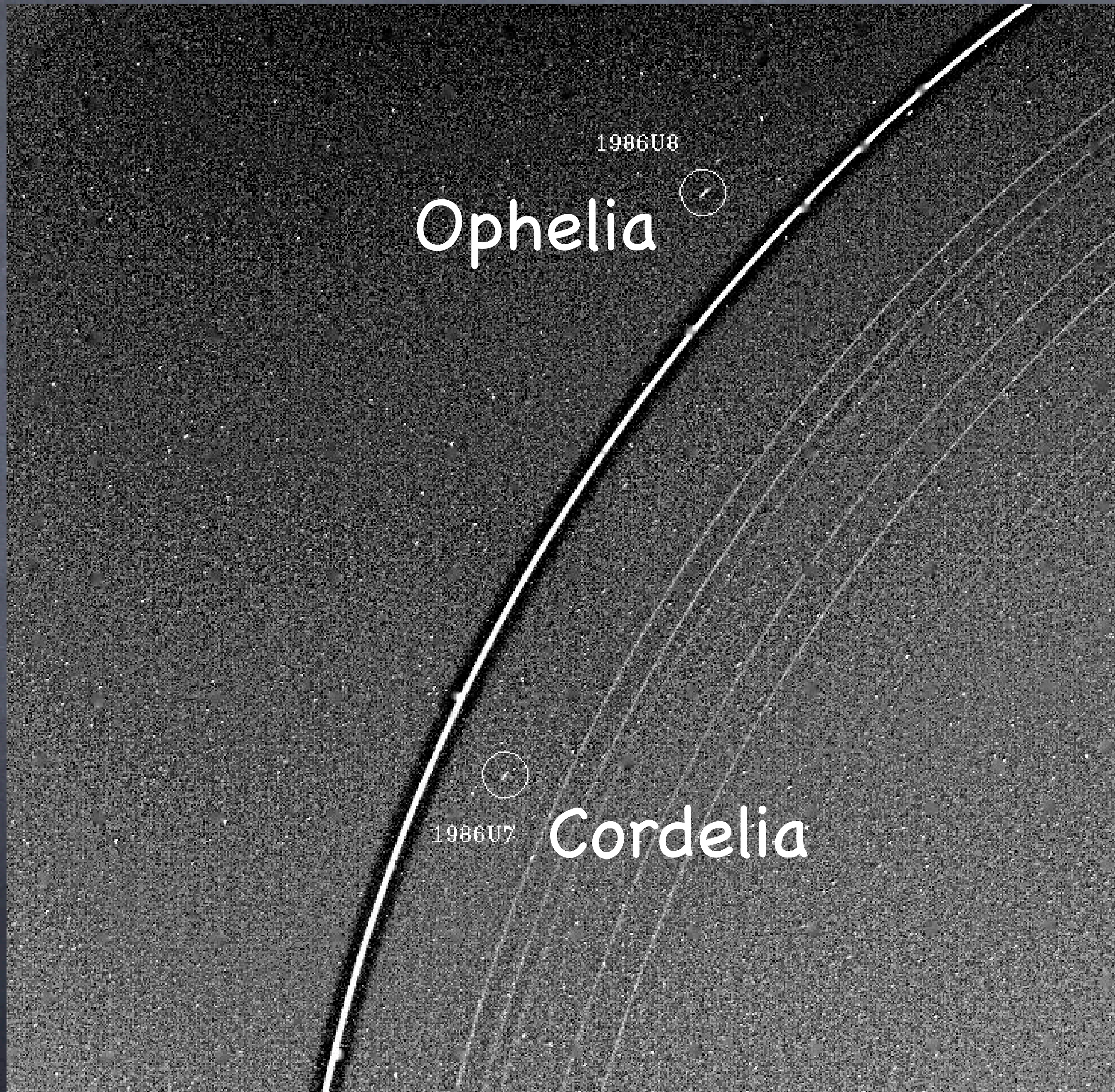
Case #1: Overlapping Resonances



Case #2: Lindblad Resonances



Case #3: Gravitational Stirring

Case #3: Gravitational Stirring

Case #4: None of the Above?