Hohmann Transfer

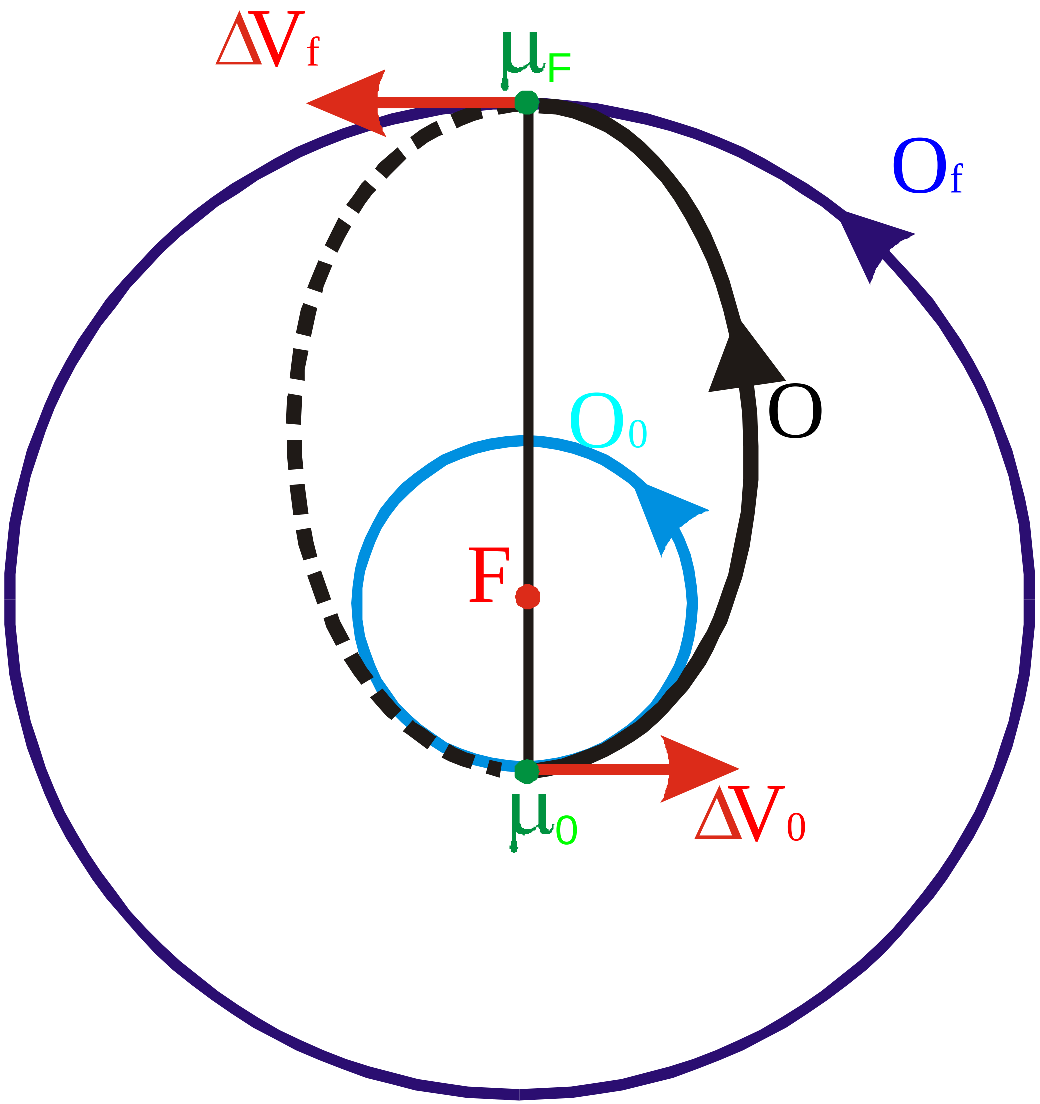
i) At the initial orbit

$$\Delta V_0 = V_0 \left| \sqrt{\frac{2\left(R_f / R_0\right)}{\left(R_f / R_0\right) + 1}} - 1 \right| ;$$

ii) At the final orbit:

$$\Delta V_f = V_0 \left| 1 - \sqrt{\frac{2}{\left(R_f / R_0\right) + 1}} \right| \sqrt{\left(R_0 / R_f\right)}$$

Generalized to include the circular-elliptic transfer, the elliptic-elliptic-co-axial and out-of-plane transfers
Analytical proof in Barrar (1963)



The Bi-elliptic Transfer

A. A first impulse ΔV_0 is applied in the initial orbit that makes the spacecraft goes to na elliptic orbit with periapsis R_0 and apoapsis R ($R > R_f$);

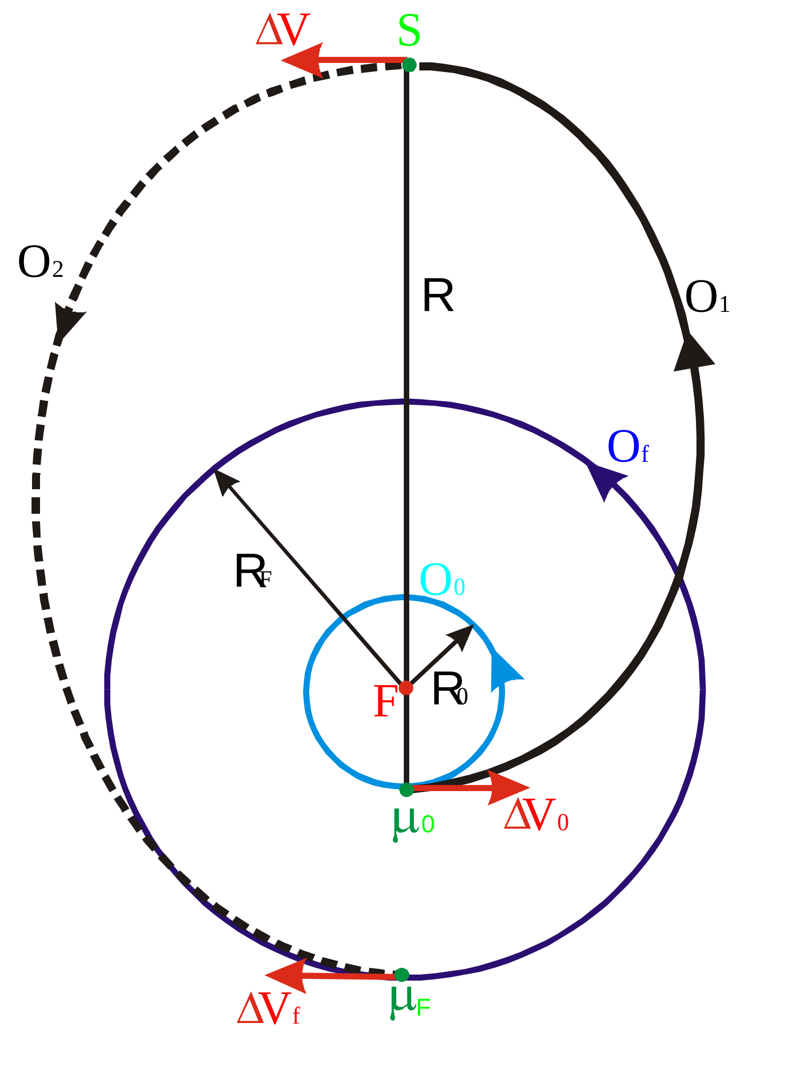
B. When the spacecraft is at the apoapsis, a second impulse ΔV is applied when the spacecraft is at the periapsis to circularize the orbit;

C. A third impulse is applied to circularize the orbit.

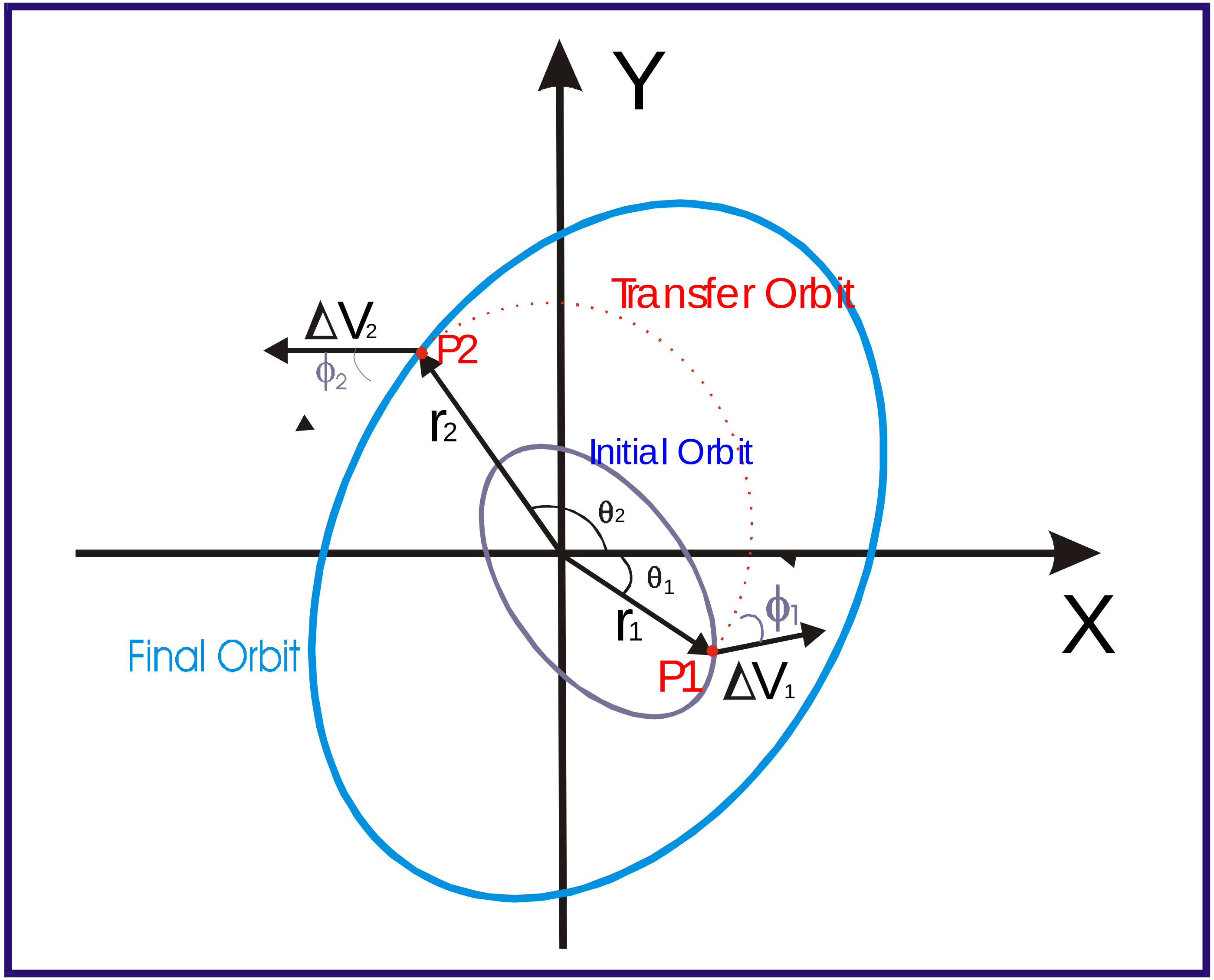
* **$R_f / R_0 \cong 55$ (Earth-Moon)**
* **Bi-Parabolic is the limit**

Hoelker and Silver (1959):

Better for $R_f / R_0 > 11.94$.



General Planar Transfer

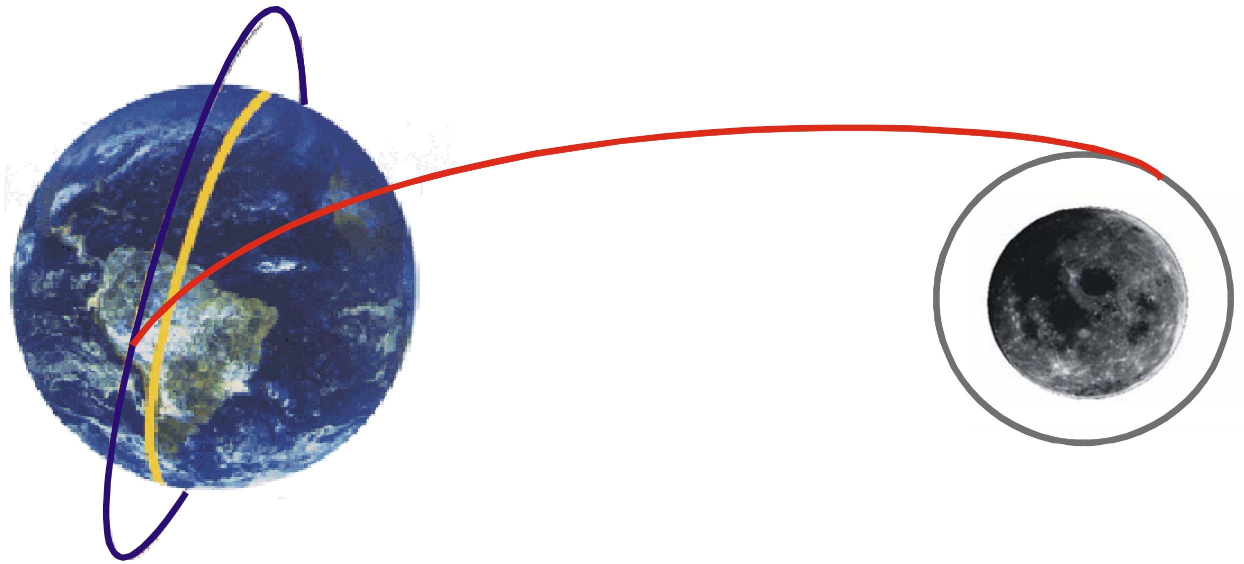


Bi-Impulsive Transfer, can be extended to 3-D.

Patched Conic

The patched conic method divede the trajectories in two parts:

1. The first leg neglects the effect of the Moon and any method (Holmann, bi-elliptic, etc.) can be used to transfer the spacecraft to an orbit that crosses the Moon's path;
2. When the spacecraft reaches a position where the Moon's gravity field dominates its motion, the Earth's effects are neglected and orbit is studied as a Keplerian lunar orbit.



Options for Dynamics Actuators and Optimization Methods

Dynamics:

* Two-Body Problem
* Two-Body Perturbed Problem
* Three-body Problem (in particular the restricted version of this problem)
* N-Bodies Problem

Actuators (control):

* Impulsive system (ΔV)
* Continuous system

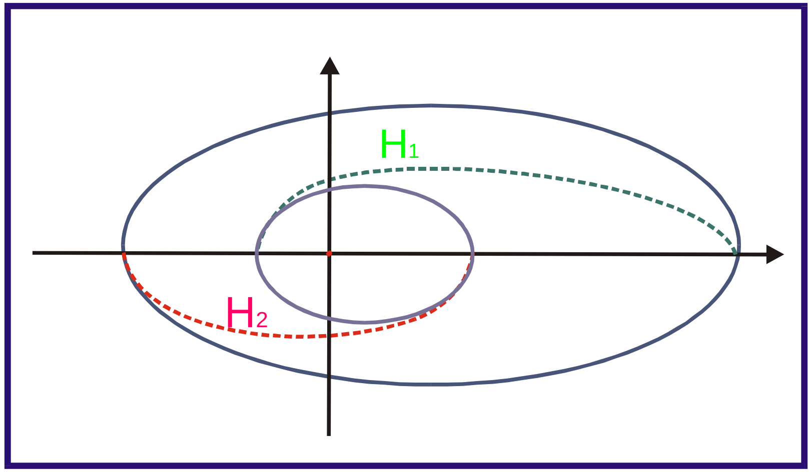
Optimization methods:

* Direct methods (search of parameters that minimizes a certain objective function)
* Indirect method (first-order necessary conditions are used)
* Hybrid approach (first-order necessary conditions are written and transformed in a search of parameters)

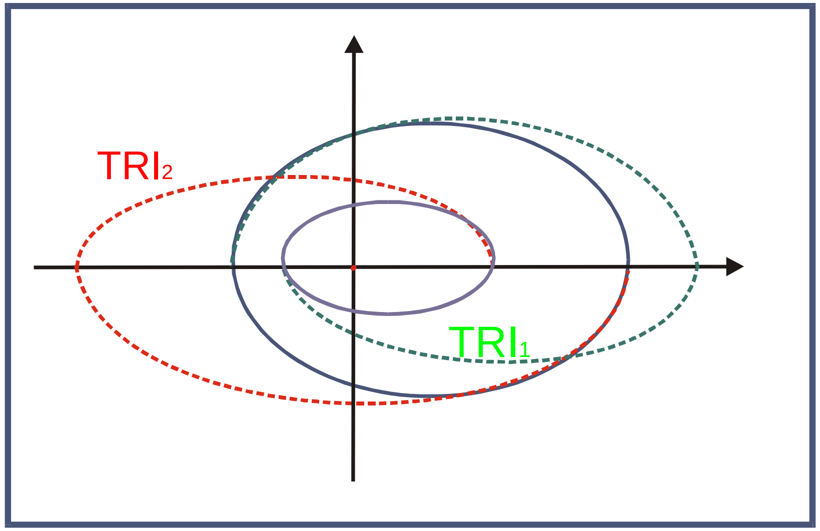
TRANSFERS BETWEEN TWO CO-AXIAL ELLIPTIC ORBIT

(USING 2 OR 3 IMPULSES) CASE 1: ALIGNED ORBITS

* **Optimal solution is hohmann type (impulse applied at the apsis);**
* **The best two-impulse transfer is the one that uses the most distant apsis (H_1);**
* **TRI_1 is better than TRI_2;**
* **Best H vs Best TRI depends on the initial and final orbits;**



H_1: To apoapsis
H_2: To periapsis

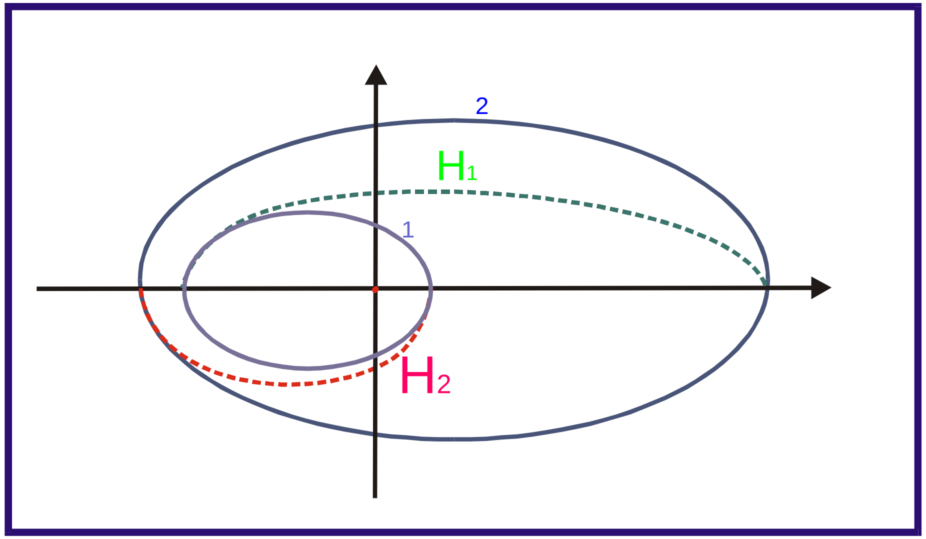


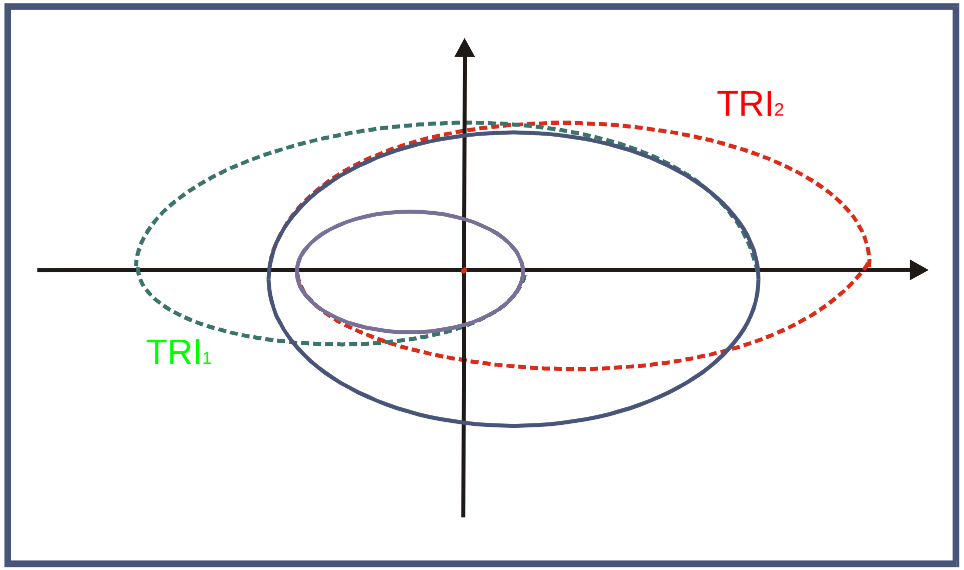
TRI_1: To apoapsis
TRI_2: To periapsis

TRANSFERS BETWEEN TWO CO-AXIAL ELLIPTIC ORBITS

(USING 2 OR 3 IMPULSES) CASE 2: OPPOSITE ORBITS

* **Optimal solution is hohmann type (impulse applied at the apsis);**
* **The best two-impulse transfer is the one that uses the most distant apsis (H_1);**
* **TRI_1 x TRI_2 depends on the initial and final orbits;**
* **Best H vs Best TRI depends on the initial and final orbits;**





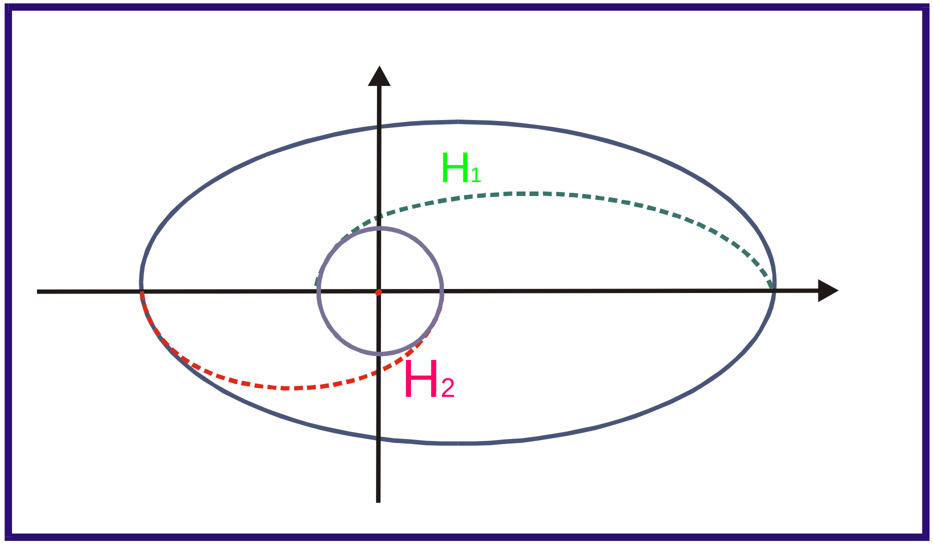
H_1: To apoapsis
H_2: To periapsis

TRI_1: To periapsis
TRI_2: To apoapsis

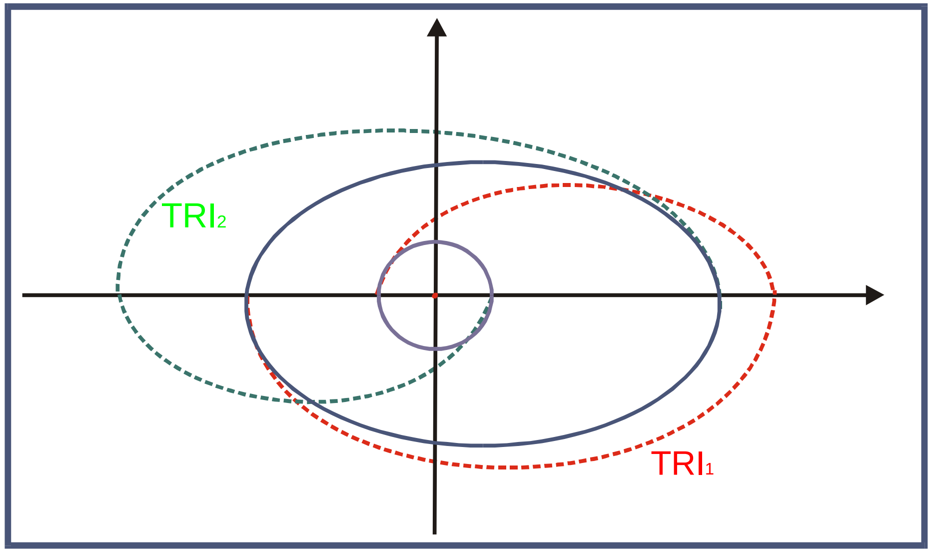
TRANSFERS CIRCULAR - ELLIPTIC ORBITS

(USING 2 OR 3 IMPULSES)

* There are two choices for each type of transfer (using 2 or 3 impulses);
* H_2 is better than H_1;
* TRI_1 is better and faster then TRI_2;
* H_2 x TRI_1 depends on the initial and final orbits.



H_1: To apoapsis
H_2: To periapsis



TRI_1: To periapsis
TRI_2: To apoapsis

EQUATIONS TO MINIMIZE TOTAL ΔV

$$D = \frac{\mu}{C}; \quad k = e\mathrm{Cos}(\omega); \quad h = e\mathrm{Sin}(\omega)$$

$$\Delta V_{r1} = \left(D_1 k_1 - D_0 k_0\right)\mathrm{Sin}\left(\theta_1\right) - \left(D_1 h_1 - D_0 h_0\right)\mathrm{Cos}\left(\theta_1\right)$$
$$\Delta V_{t1} = D_1 - D_0 + \left(D_1 k_1 - D_0 k_0\right)\mathrm{Cos}\left(\theta_1\right) + \left(D_1 h_1 - D_0 h_0\right)\mathrm{Sin}\left(\theta_1\right)$$
$$\Delta V_{r2} = \left(D_2 k_2 - D_1 k_1\right)\mathrm{Sin}\left(\theta_2\right) - \left(D_2 h_2 - D_1 h_1\right)\mathrm{Cos}\left(\theta_2\right)$$
$$\Delta V_{t2} = D_2 - D_1 + \left(D_2 k_2 - D_1 k_1\right)\mathrm{Cos}\left(\theta_2\right) + \left(D_2 h_2 - D_1 h_1\right)\mathrm{Sin}\left(\theta_2\right)$$

$$g_1 = D_0^2\left(1 + k_0\mathrm{Cos}\left(\theta_1\right) + h_0\mathrm{Sin}\left(\theta_1\right)\right) - D_1^2\left(1 + k_1\mathrm{Cos}\left(\theta_1\right) + h_1\mathrm{Sin}\left(\theta_1\right)\right) = 0$$
$$g_2 = D_2^2\left(1 + k_2\mathrm{Cos}\left(\theta_2\right) + h_2\mathrm{Sin}\left(\theta_2\right)\right) - D_1^2\left(1 + k_1\mathrm{Cos}\left(\theta_2\right) + h_1\mathrm{Sin}\left(\theta_2\right)\right) = 0$$

$$k_1 = -\mathrm{Csc}\left(\theta_1 - \theta_2\right)\left[\left(\left(\frac{D_0^2}{D_1^2}\right)\left(1 + k_0\mathrm{Cos}\left(\theta_1\right) + h_0\mathrm{Sin}\left(\theta_1\right)\right) - 1\right)\mathrm{Sin}\left(\theta_2\right) - \ldots\right.$$
$$\left.\ldots - \left(\left(\frac{D_2^2}{D_1^2}\right)\left(1 + k_2\mathrm{Cos}\left(\theta_2\right) + h_2\mathrm{Sin}\left(\theta_2\right)\right) - 1\right)\mathrm{Sin}\left(\theta_1\right)\right]$$

Space Trajectories

We show space trajectories from one body back to the same body and to the Lagrangian points.

The mathematical model is the restricted three-body problem. Earth-Sun and the Earth-Moon.

Five families of transfer orbits are found.

The problem of sending a spacecraft from the Earth to the Lagrangian points L4 and L5 is treated.

Two transfer orbits from the Earth to L4 and to L5 are found.

Numerical integration is extended beyond the points and it is found, the spacecraft passes near the Lagrangian points L3, L4 and L5 and comes back to the neighborhood of the Earth.

In general, the orbits found here can be applied to:

* Transfers between any two points in the group formed by the Earth and the Lagrangian points L3, L4, L5 with near-zero ΔV;

*

* Make a tour to the Lagrangian points for reconnaissance purposes with near-zero ΔV for the entire tour;

*

* Build a cycler transportation system linking all the points involved or only two of them.

The Three-Body Lambert’s Problem

This problem can be formulated as:

"Find an orbit (in the three-body context) that makes a spacecraft to leave a given point A and goes to another given point B".

The problem becomes the Lambert's three-body problem:

"Find an orbit (in the three-body problem context) that makes a spacecraft to leave a given point A and go to another given point B, arriving there after a specified time of flight". By varying the time of flight, it is possible to find a family of orbits.

The Solution of the TPBVP

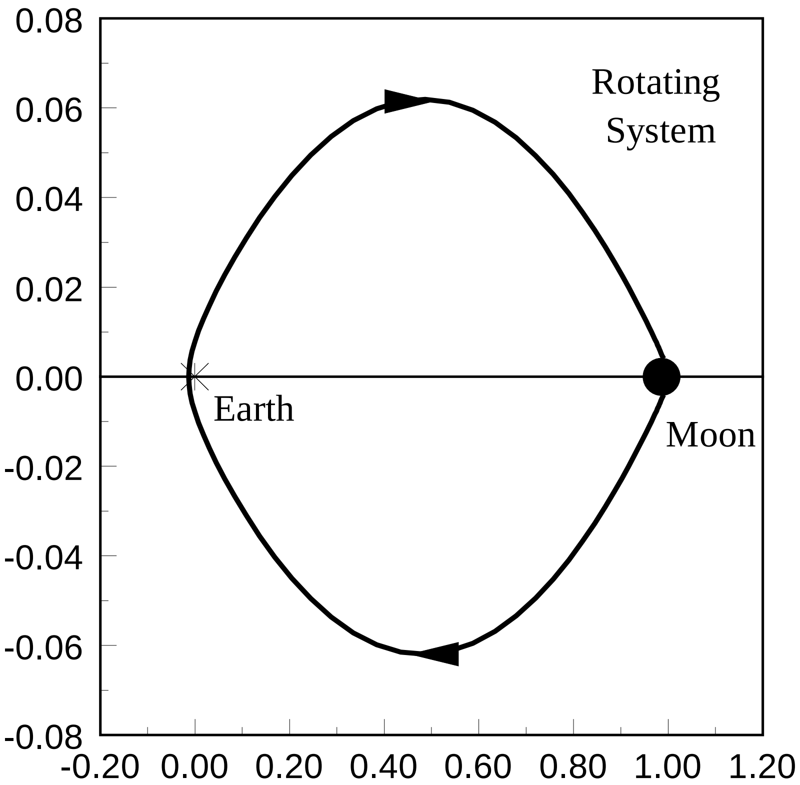
The following steps are used:

* Guess a initial velocity $\vec{v}_i$, so together with the position $\vec{r}_i$, the initial state is known;
*
* Guess a final regularized time τ_f and integrate the regularized equations of motion from $\tau_0 = 0$ until τ_f;
*

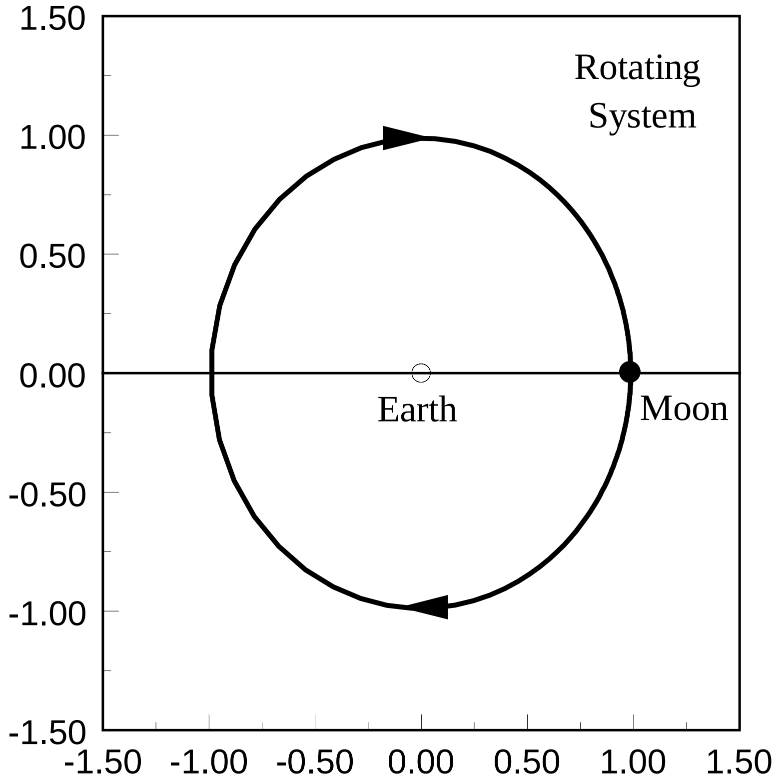
* Check the final position $\vec{r}_f$ obtained with the prescribed final position and the final real time with the specified time of flight.
*

If there is an agreement the solution is found. Not, an increment in the initial guessed velocity $\vec{v}_i$ and in the guessed final regularized time τ_f is made and the process goes back to step i).

Trajectories from the Moon to the Moon

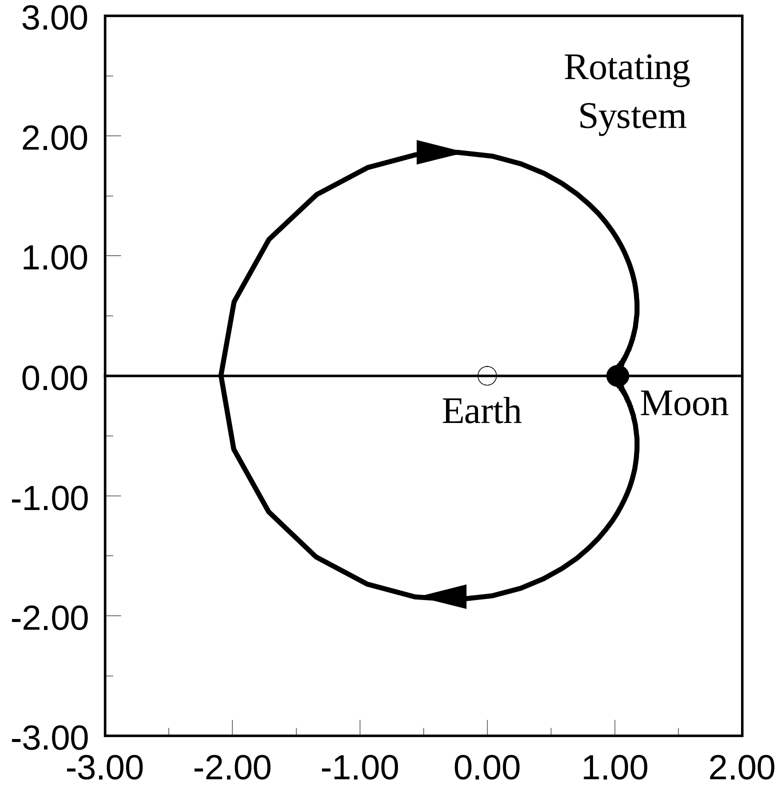


Hyperbolic Transfer Orbit (t = 1.74 days)



Elliptic Transfer Orbit (t = 13.48 days)

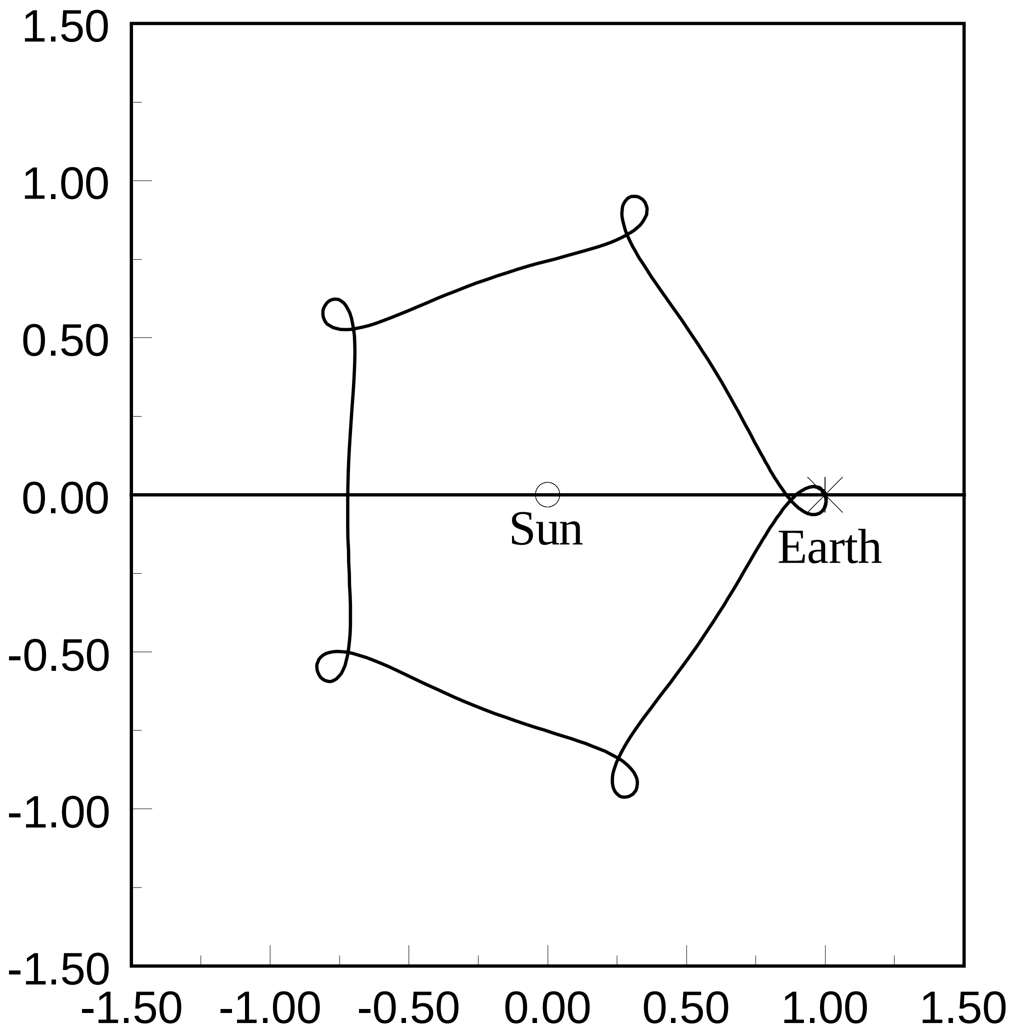
Trajectories from the Moon to the Moon



Elliptic Transfer Orbit (t = 24.78 days)

Transfer Orbits with Minimum ΔV

The two-body solution is used as the first guess and a trial and error technique (in the initial velocity) is used to find the solution. The ΔV for escape velocity from the Earth is 0.3735 canonical units. The ΔV found in this transfer orbit is 0.3839 canonical units.



Transfer Orbit with Minimum ΔV from the Earth Back to the Earth, as Seen in the Rotating Frame.

Data for the transfer orbit with minimum ΔV from the Earth back to the Earth

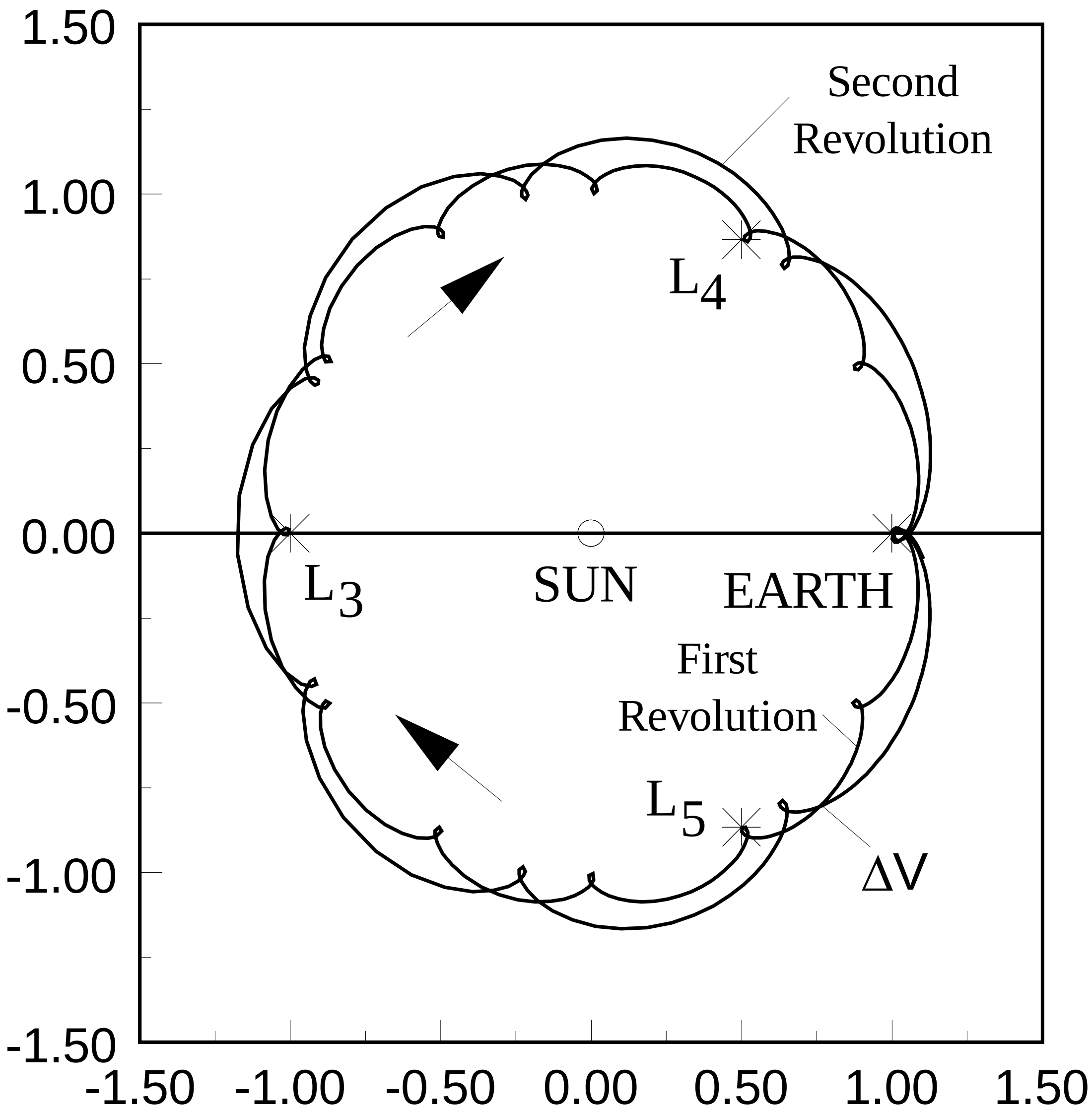
Position and velocity in the rotating frame in canonical units when leaving the Earth	$x = 0.999997$ $y = -0.000043$ $\dot{x} = 0.096957$ $\dot{y} = -0.371500$ $\Delta V = 0.383944$
Jacobi constant Regularized transfer time Canonical transfer time Transfer time in years	$J = -1.495886$ $T_r = 56.049850$ $T_c = 25.094343$ $T_y = 3.993889$

Transfers Earth-lagrangian points: results

The "SHORT-5-4" Orbit

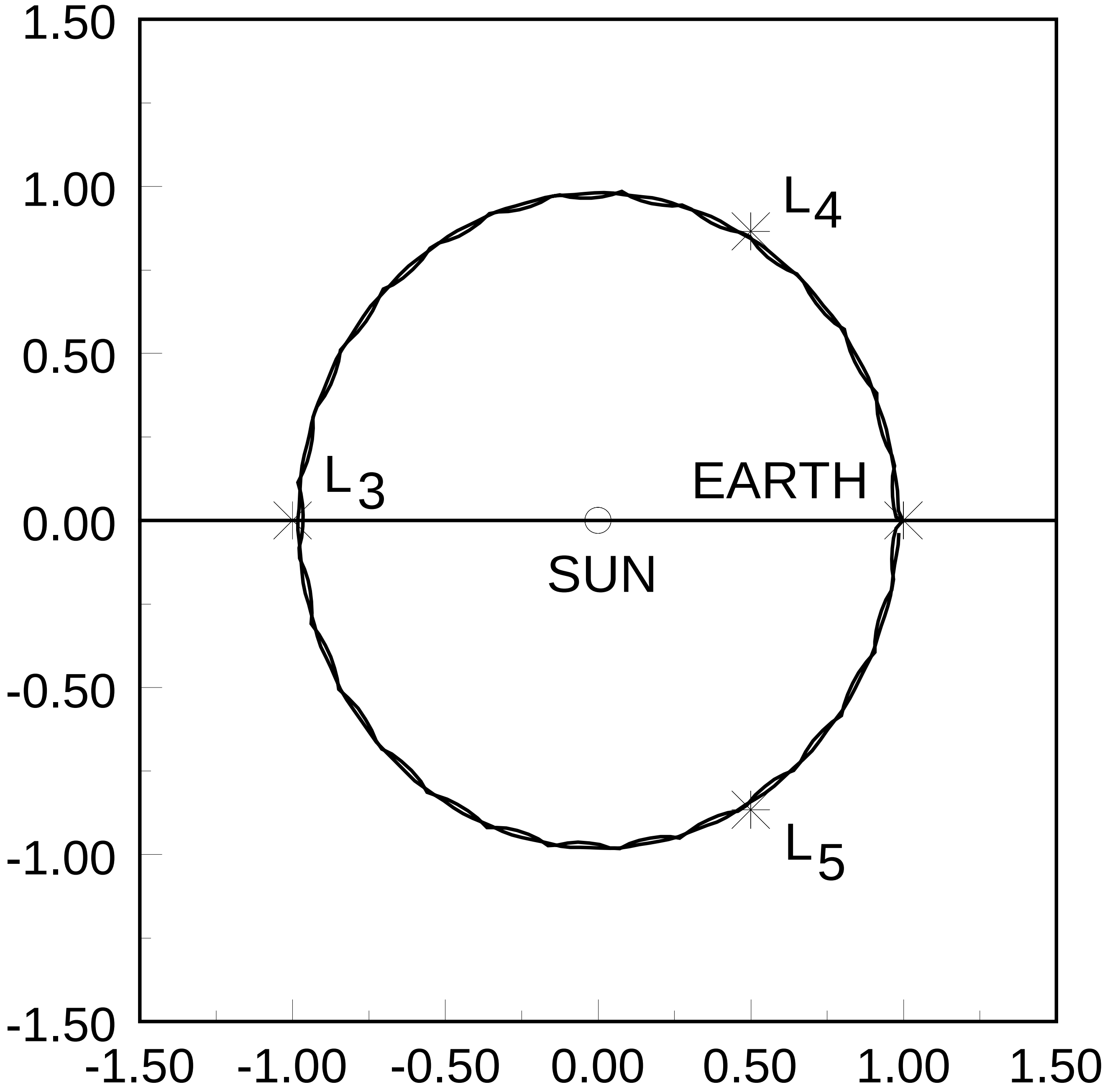
- A shorter time is required. Total tour is about 13 years. The legs connecting L4 and L5 to the Earth has about 2.1 years each;
- It also has closer approaches to the Lagrangian points visited, compared to the "LONG" transfers;
- After the first close approach this orbit continues in the same direction. The second trajectory is similar to the first one. There are 12 "crossing points", candidates for a one-burn maneuver which transfers the spacecraft between the trajectories. After this

maneuver the spacecraft starts again its journey to L5, L3, L4 and the Earth



The "LONG-4-5" Orbit

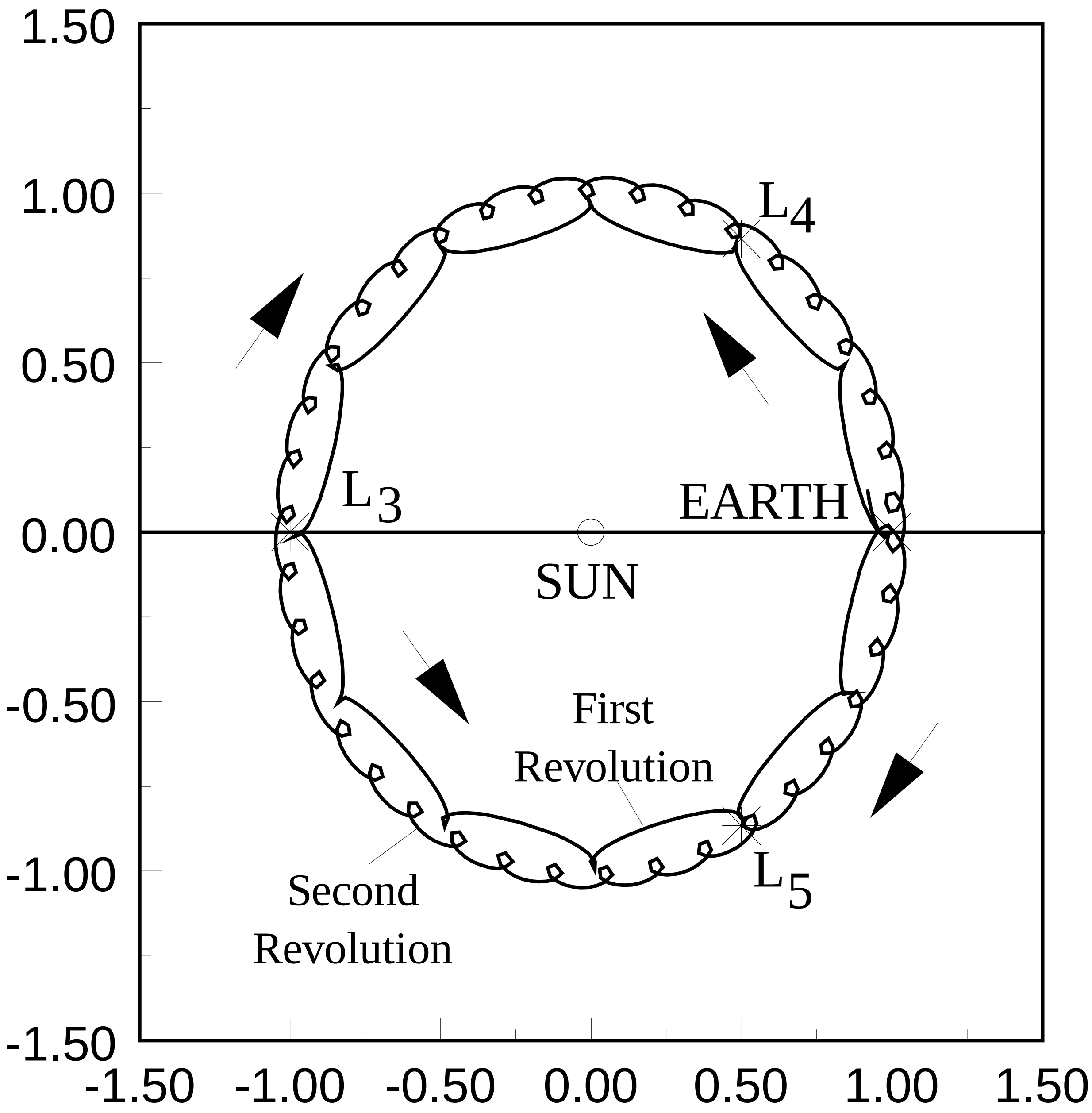
* It has the closest approach with the Earth at the end of the first revolution;
* Very close approaches to the Lagrangian points and the Earth again exist in at least two more revolutions, with no nominal corrections required. It makes this orbit the best one for a continuous cycler without nominal corrections;
* This orbit has the characteristic of reversing the direction of its motion after some of the "swing-by".



1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-1.50
-1.00
-0.50
0.00
0.50
1.00
1.50
L4
L3
EARTH
SUN
L5

The "SHORT-4-5" Orbit

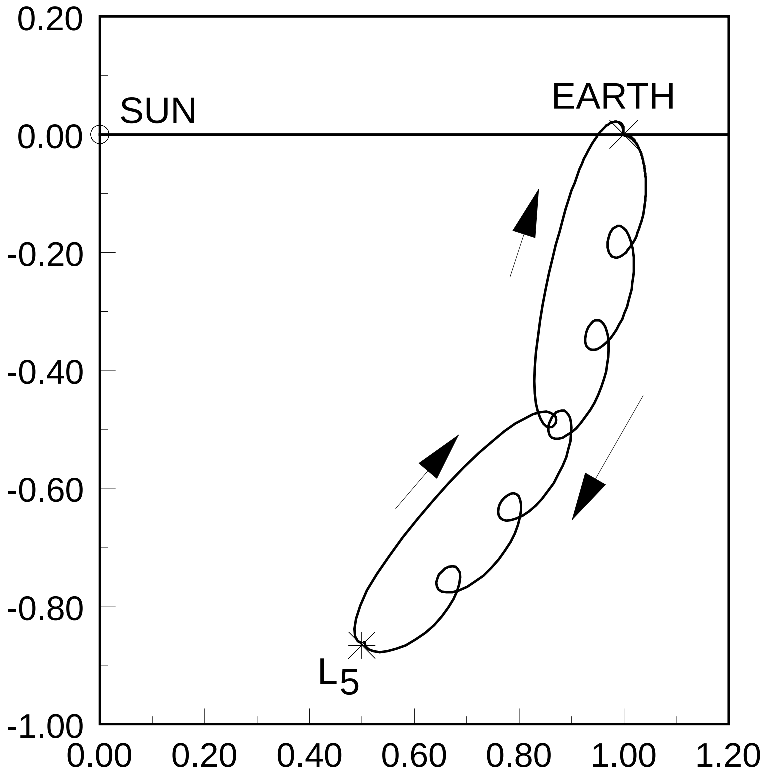
- After the first close approach the spacecraft starts a new tour in the reverse order. The first five revolutions have alternating directions of motion;
- It has the shortest transfer time (in the first revolution) of all orbits described. The period for an Earth-to-Earth trip is about 11 years and the legs connecting the Earth and the Lagrangian points L4 and L5 last about 1.8 years each way;
- It has the closest approaches to the Lagrangian points visited (during the first and second revolutions).


1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-1.50
-1.00
-0.50
0.00
0.50
1.00
1.50
L4
L3
EARTH
SUN
L5
First
Revolution
Second
Revolution

A Cycler Transportation System Between the Earth and the Lagrangian Points L4 and L5

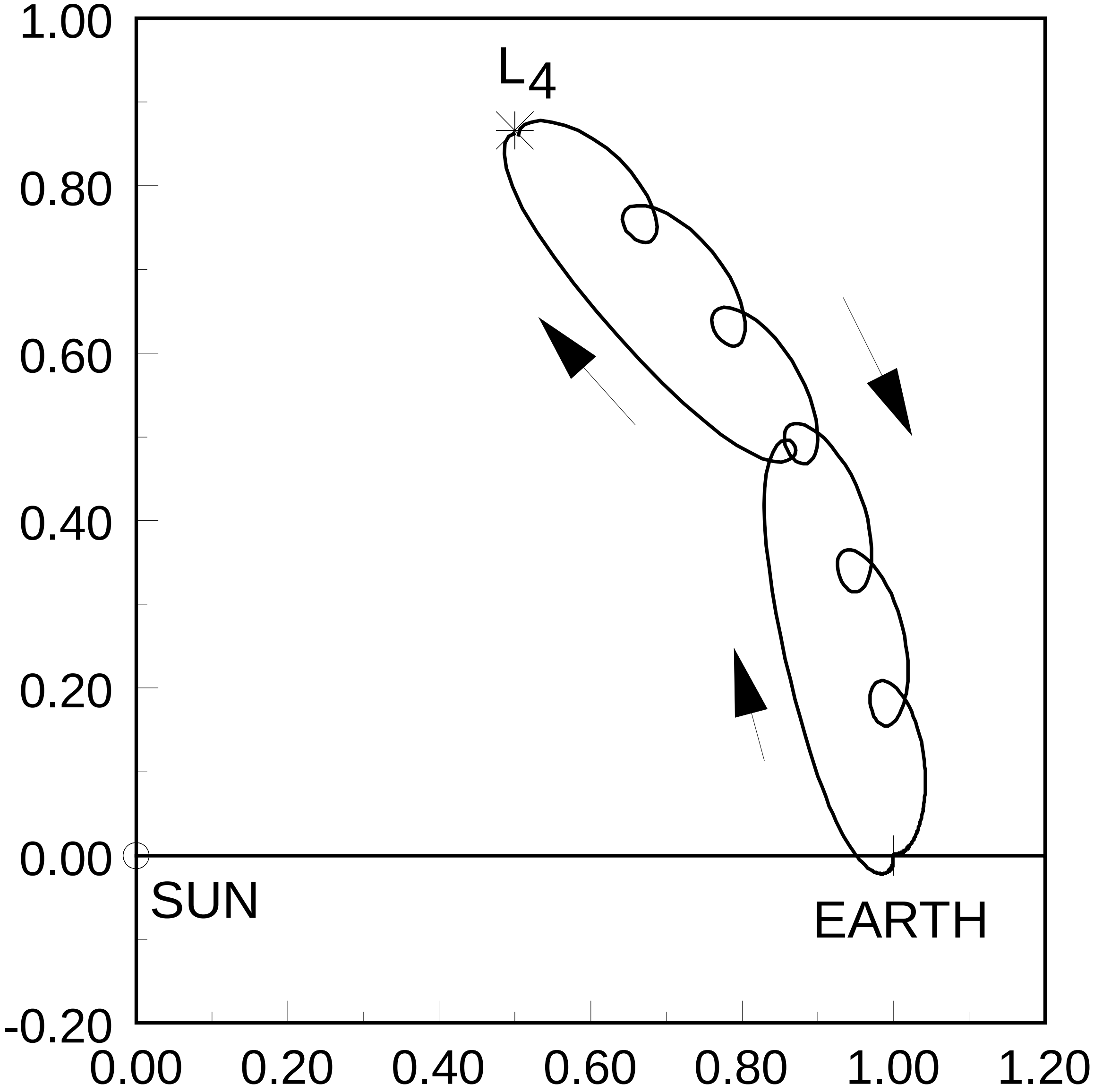
This "swing-by" can be used to build a cycler transportation system between the Earth and L5. If the spacecraft starts at L5 with zero velocity, it is possible to apply an impulse of 0.0274 (816 m/s) to get $V_x = -0.0271$ and $V_y = 0.0040$. So, the spacecraft follows one trajectory that is part of the SHORT-4-5. Then, it goes to the Earth, makes the "swing-by" and returns to L5, arriving there with $V_x = -0.0018$, $V_y = 0.0263$. Then, it is possible to apply an impulse $\Delta V = 0.0337$ (1003.8 m/s), such that its velocity goes to $V_x = -0.0271$, $V_y = 0.0040$ again and it starts the cycler one more time.

t = 0	The spacecraft leaves L5 from rest (as seen in the rotating frame) with an impulse of ΔV = 0.0274 (816 m/s)
t = 1.80 years	The spacecraft arrives at the Earth, makes a swing-by to reverse the sense of motion and it starts going back to L5
t = 7.62 years	The spacecraft arrives at L5. A new impulse of ΔV = 0.0377 (1003.8 m/s) is applied to send it back to the Earth and to start the cycler again



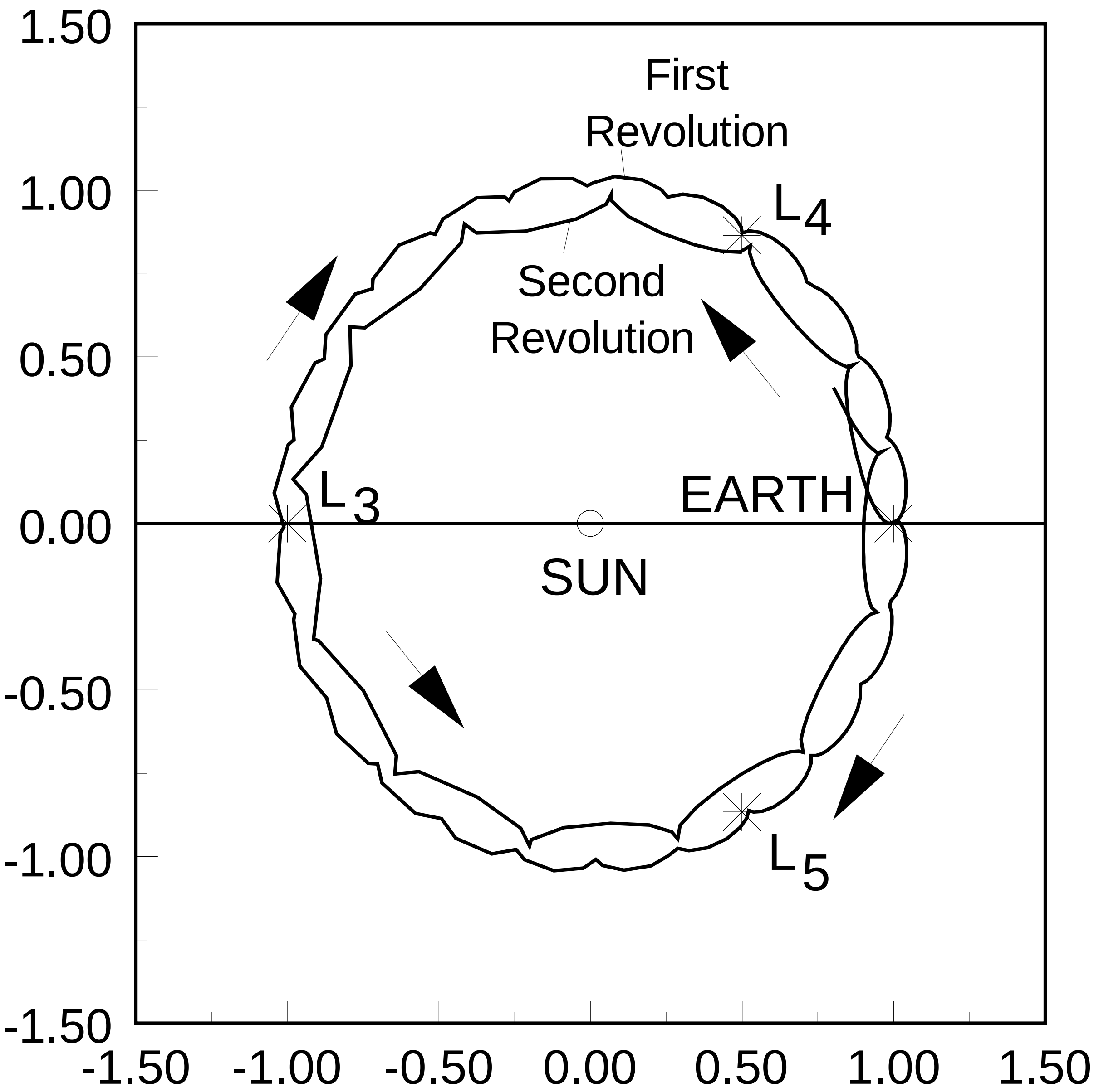
To reproduce this cycler system for the Lagrangian point L4 we can use the mirror image theorem. The time-line for a complete cycler is:

t = 0	The spacecraft leaves L4 from rest (as seen in the rotating frame) with an impulse of ΔV = 0.0274 (816 m/s)
t = 5.82 years	The spacecraft arrives at the Earth, makes a swing-by to reverse the sense of motion and it starts going back to L4
t = 7.62 years	The spacecraft arrives at L4. A new impulse of ΔV = 0.0377 (1003.8 m/s) is applied to send it back to the Earth and to start the cycler again


L4
SUN
EARTH
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0.00
0.20
0.40
0.60
0.80
1.00
1.20

The "LONG-5-4" Orbit

- This is the orbit with smaller residual velocity during the close approaches with the Lagrangian points;
- After completing the first revolution, the spacecraft makes a "swing-by" with the Earth, changes its direction of motion (as seen in the rotating frame) from "clock-wise" to "counter-clock-wise" and goes back to pass near L4, L3, L5 and the Earth, in a second revolution.

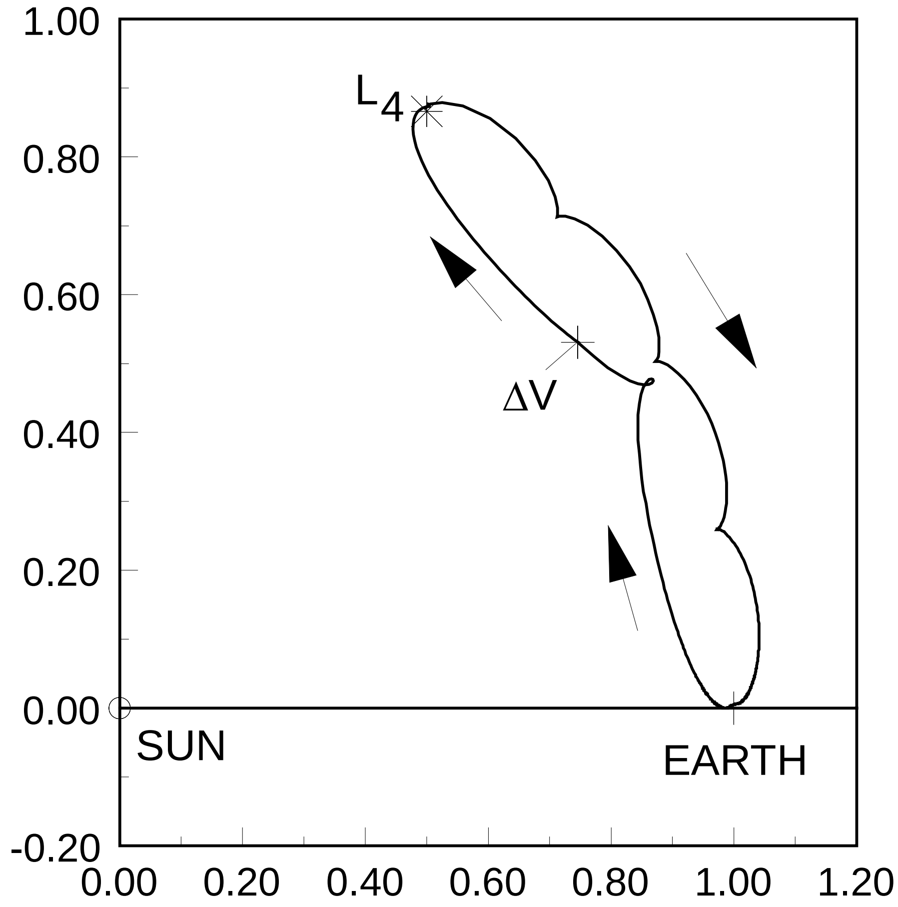


1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-1.50
-1.00
-0.50
0.00
0.50
1.00
1.50
First
Revolution
Second
Revolution
L4
L3
L5
EARTH
SUN

An Option for a Faster Cycler Transportation System Between the Earth and L5 or L4

The spacecraft leaves L4 (by applying an impulse such that V_X = 26.8 m/s and V_Y = 47.7 m/s, goes to the Earth, and returns to L4 with the impulse given by the Earth's swing-by. Next, an extra impulse is applied, to make a fine adjustment that allows M3 to arrive at L4. Then, after M3 arrives at L4, it is necessary to apply another impulse to reverse its motion and send it back to the Earth, following the same trajectory it did in the first revolution.

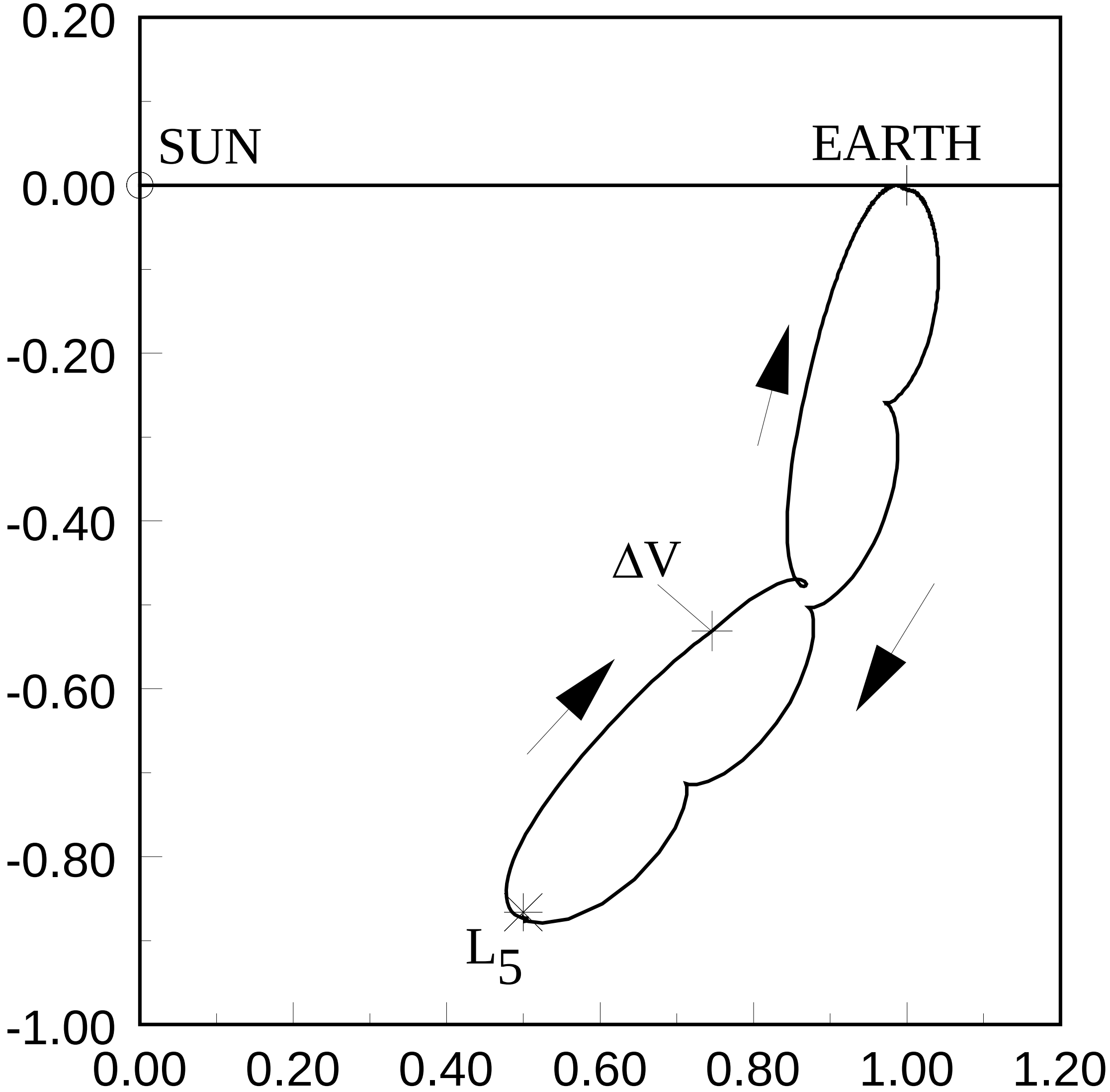
t = 0	The spacecraft leaves L4 from rest (as seen in the rotating frame) with an impulse of ΔV = 56.6 m/s
t = 4.07 years	The spacecraft arrives at the Earth, makes a "swing-by" with the Earth to reverse the sense of motion and it starts going back to L4
t = 5.33 years	An extra maneuver with ΔV = 0.02 (560 m/s) is performed to adjust the final arrival at L4
t = 5.86 years	The spacecraft arrives at L4. A new impulse with ΔV = 0.05 (1500 m/s) is applied to send it back to the Earth and to start the cycler again



The result is a trajectory that requires 4.0728 years for the Earth-bound trip, 1.7825 years for the L4-bound trip and about 2060 m/s per revolution in maneuvers. It is a little more expensive than the previous system (2060 x 1820 m/s), but it is faster (5.86 x 7.62 years).

A similar system can be build between the Earth and L5 by using the mirror image theorem. Note that the mirror image of the legs for an Earth-bound trip in now a L5-bound trip and the mirror image of the L4-bound leg is now the Earth-bound leg.

t = 0	The spacecraft leaves L5 from rest (as seen in the rotating frame) with an impulse of $\Delta V = 56.6$ m/s
t = 0.53 years	An extra maneuver with $\Delta V = 0.02$ (560 m/s) is performed to adjust the final arrival at the Earth
t = 1.79 years	The spacecraft arrives at the Earth, makes a "swing-by" with the Earth to reverse the sense of motion and it starts going back to L5
t = 5.86 years	The spacecraft arrives at L5. A new impulse with $\Delta V = 0.05$ (1500 m/s) is applied to send it back to the Earth and to start the cycler again



0.20
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
0.00
0.20
0.40
0.60
0.80
1.00
1.20
SUN
EARTH
ΔV
L5

Position, Velocity and Time for the passages by the Lagrangian points in Canonical Units (referred to the Rotating frame)

Point	x	y	R	V_x	V_y	V	t
Orbit "SHORT-5-4"							
Earth	-	-	-	0.0000	0.3737	0.3737	0.00
L_5	0.5007	-0.8696	0.0037	0.0103	0.0198	0.0223	13.30
L_3	-1.0026	0.0088	0.0092	0.0085	-0.0205	0.0222	40.61
L_4	0.5086	0.8671	0.0087	-0.0043	0.0230	0.0234	68.38
Earth	1.0054	0.0000	0.0054	0.0161	0.0373	0.0406	82.00
Orbit "LONG-5-4"							
Earth	-	-	-	0.0000	0.3729	0.3729	0.00
L_5	0.5223	-0.8666	0.0223	-0.0017	-0.0167	0.0168	26.64
L_3	-1.0272	0.0000	0.0272	-0.0066	0.0440	0.0449	80.07
L_4	0.5011	0.8732	0.0073	0.0009	0.0016	0.0019	130.75
Earth	1.0000	0.0050	0.0050	-0.0315	-0.0085	0.0326	156.34
Orbit "SHORT-4-5"							
Earth	-	-	-	0.0000	-0.3740	0.3740	0.00
L_4	0.5004	0.8635	0.0025	0.0240	-0.0112	0.0264	11.39
L_3	-0.9981	-0.0025	0.0031	0.0006	0.0265	0.0266	34.47
L_5	0.4985	-0.8617	0.0046	-0.0271	0.0040	0.0274	57.79
Earth	0.9999	-0.0008	0.0008	0.0773	-0.0452	0.0895	69.11
Orbit "LONG-4-5"							
Earth	-	-	-	0.0000	-0.3727	0.3727	0.00
L_4	0.4929	0.8547	0.0133	-0.0099	0.0127	0.0161	29.46
L_3	-0.9652	-0.0004	0.0348	-0.0018	-0.0587	0.0588	87.74
L_5	0.4868	-0.8518	0.0191	0.0172	0.0226	0.0284	146.35
Earth	0.9999	-0.0000	0.0000	0.8086	-3.4852	3.5778	174.94

Conclusions

Trajectories in the planar restricted three-body problem with near-zero ΔV to move a spacecraft between any two points on the group formed by the Earth and the Lagrangian points L3, L4, L5 in the Earth-Sun system are found.

It is shown how to apply these results to build a cycler transportation system to link all the points in this group.

It is also shown how to use one or more "swing-by" with the Earth to build a cycler transportation system between the Earth and the Lagrangian points L4 and L5, with small ΔV required for maneuvers in nominal operation.

History of Swing-By (Comets)

- **Jean le Rond d'Alembert (1773): "On the Orbit of the Comets" and "On the Pertubations of the comets".**
- **Laplace (1795): "Mécanique Céleste".**
- **U. G. Leverrier (1847): "Comptes Rendu".**
- **H. A. Newton (1878): "On the Origin of Comets".**
- **F. Tisserand (1889): "Tisserand Criterion".**
- **M. O. Callandreau (1892): "Theory of Periodic Comets".**
- **E. Stromgren and collaborators (1914).**
- **G. V. Pirquet (1928): "Space Trajectories".**
- **E. Everhart, S. Yabushita, M. Valtonen (last 30 years).**

History of Swing-By (Astronautics)

- M. Minovitch (1961): “A Method for Determining Interplanetary Free-Fall Reconnaissance Trajectories”.
- G. Flandro (1966): “Fast Reconnaissance Missions to the Outer Solar System Utilizing Energy Derived From the Gravitational Field of Jupiter”.
- Farquhar, Muhonen, Church, Dunham, Davis, Efron, Yeomans and Schanzle (1985).
- E. A. Belbruno and J. K. Miller (1987 to present)

Aplications of Swing-By

- Inner Solar System: Use of Venus for trips to Mars.
- Tour to the Outer Solar System (Voyager).
- Multiple Swing-By (Earth, Venus, etc) to reach the Outer Solar System.
- Plane Change (Ulysses) to leave the ecliptic.
- Use of the Moon to escape from Earth.
- Use of the Moon to keep geometry.
- Tour to the Satellites of Jupiter or Saturn.

DEFINITION OF SWING-BY

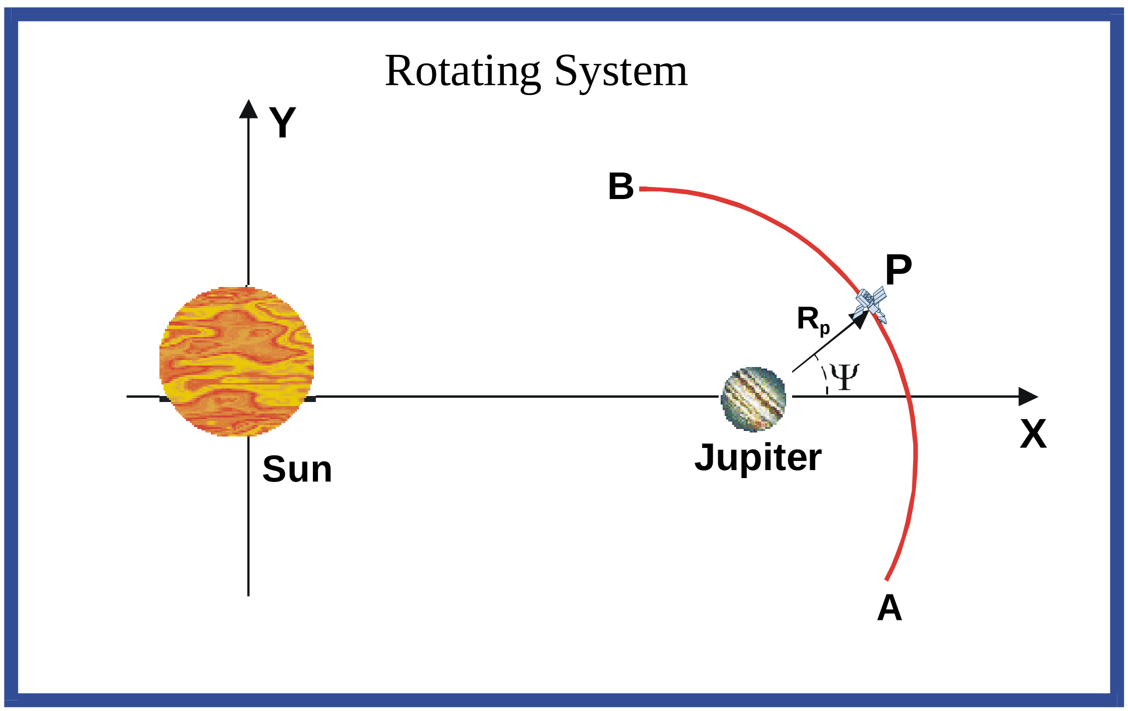
TWO BODY MODEL

- ⇨ We assume planar motion
- ⇨ Three parameters describe the Swing-by:

R_p = Periapse distance

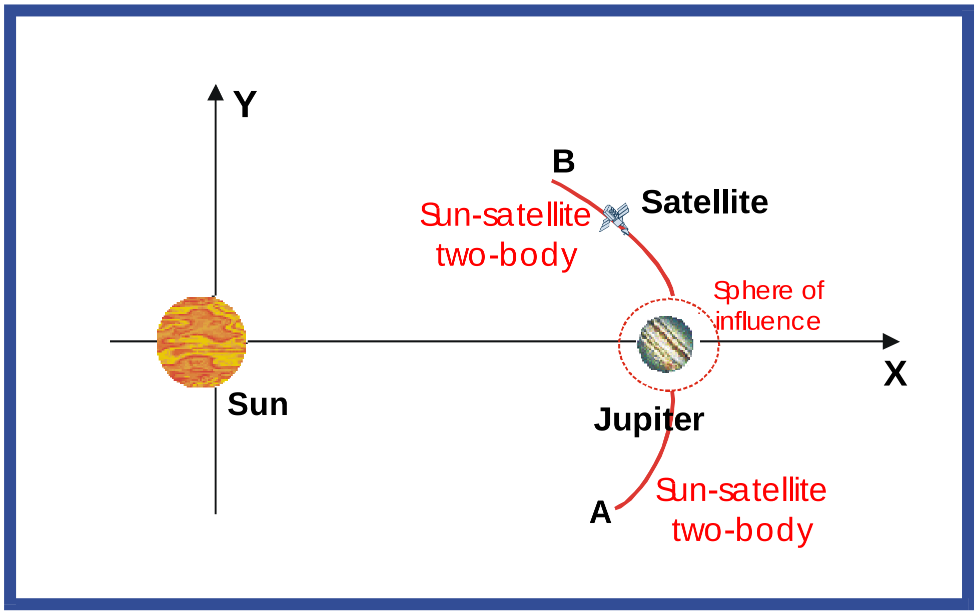
V_∞ = Hyperbolic Excess Velocity or J (Jacobian constant) or V_p (Periapsis velocity)

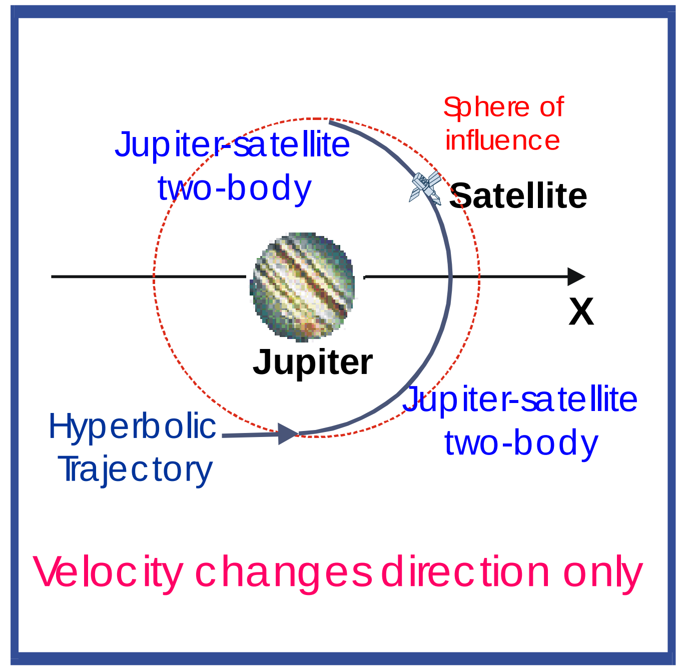
ψ = Angle of approach (ψ is also the angle between $\vec{V}_2$ and $\vec{V}_p$)



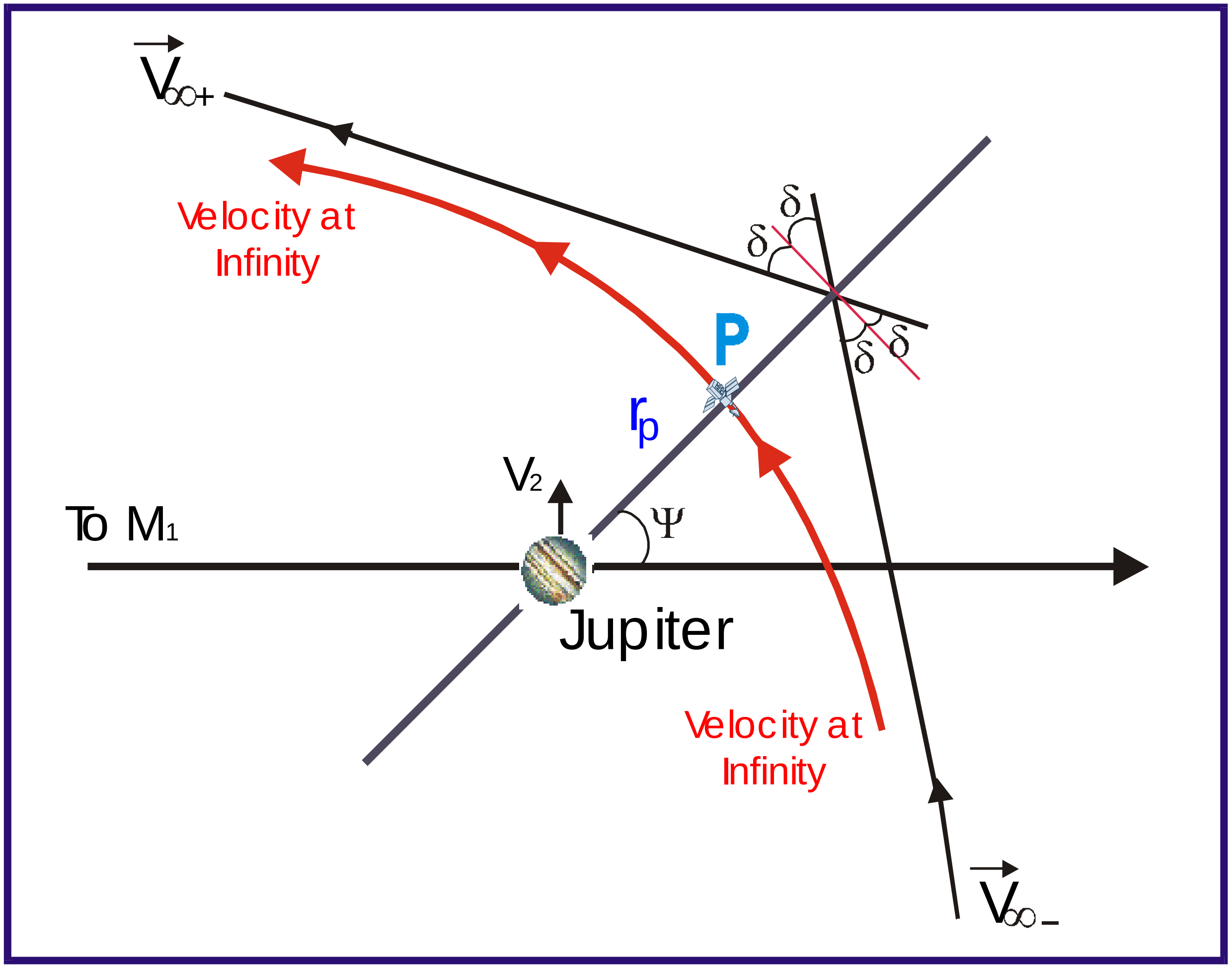
SWING-BY

⇨ Patched Conics for first approximation



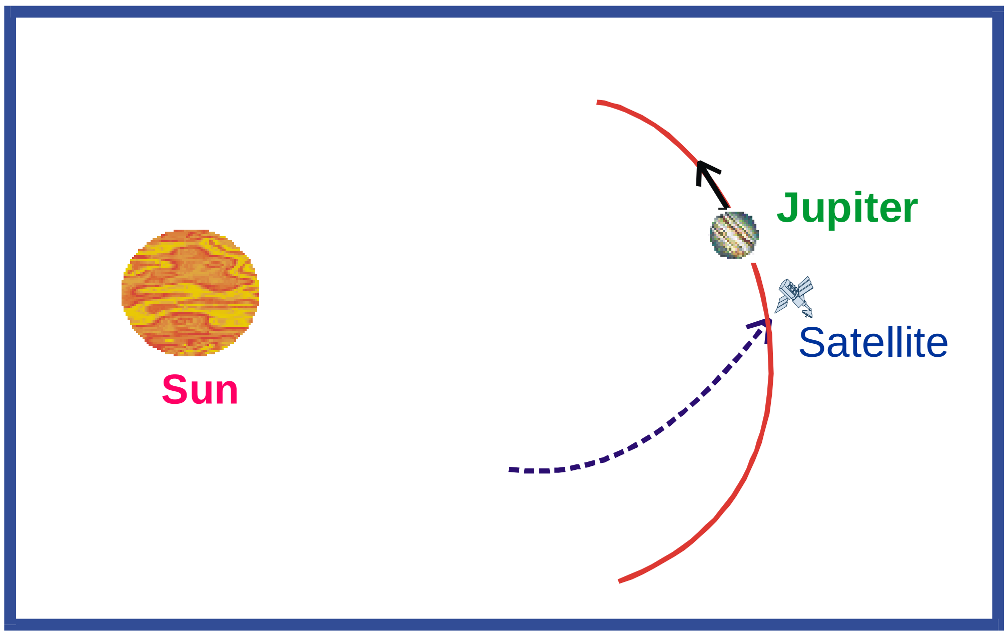


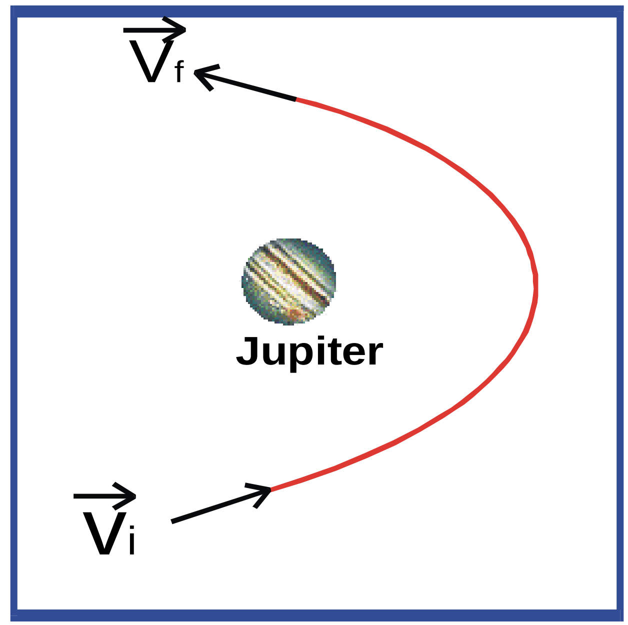
SWING-BY



THE SWING-BY MANEUVER AND SOME VARIABLES

SWING-BY





THE SWING-BY MANEUVER AND SOME VARIABLES

$\vec{V}_2$ = Inertial velocity of Jupiter

$\vec{V}_{\infty -}$ = Velocity with respect to Jupiter before Swing-by

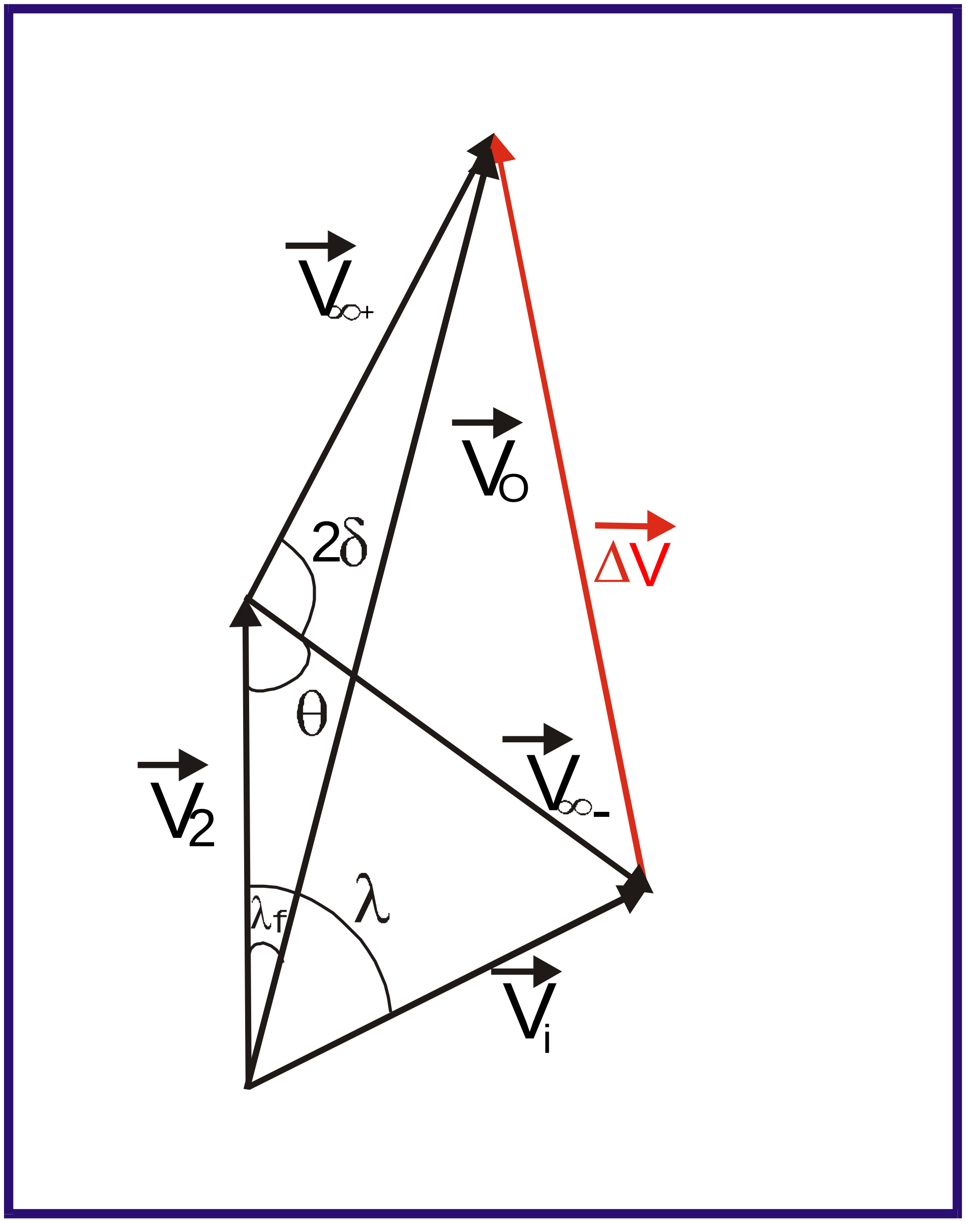
$\vec{V}_{\infty +}$ = Velocity with respect to Jupiter after Swing-by

r_p = periapse distance

Ψ = angle of approach

δ = half of the deflexion angle

VECTORIAL ADDITION



VECTORIAL ADDITION

V_i = Inertial velocity before Swing-By
V_0 = Inertial velocity after Swing-By
V_2 = Inertial velocity of Jupiter
$V_{\infty -}$ = Velocity with respect to Jupiter before Swing-By
$V_{\infty +}$ = Velocity with respect to Jupiter after Swing-By

$$\vec{V}_i = \vec{V}_{\infty -} + \vec{V}_2$$

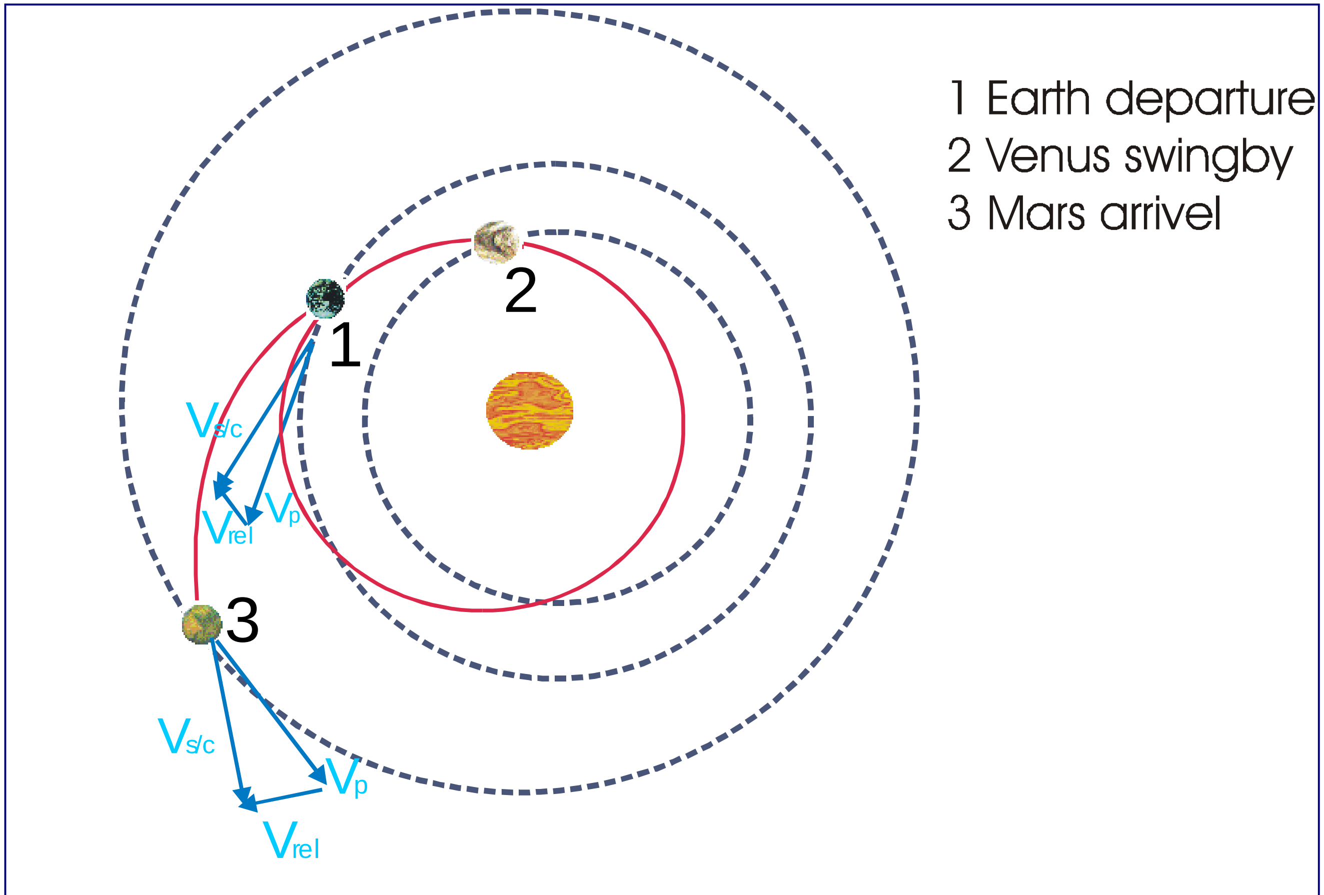
$$\vec{V}_0 = \vec{V}_{\infty +} + \vec{V}_2$$

$$\Delta\vec{V} = \vec{V}_0 - \vec{V}_i$$

So, $\left|\Delta\vec{V}\right| = 2\left|\vec{V}_{\infty -}\right| \mathrm{Sin}\,(\delta)$, where

$$\mathrm{Sin}(\delta) = \frac{1}{\left(1 + \frac{r_p V_\infty^2}{GM_2}\right)}$$

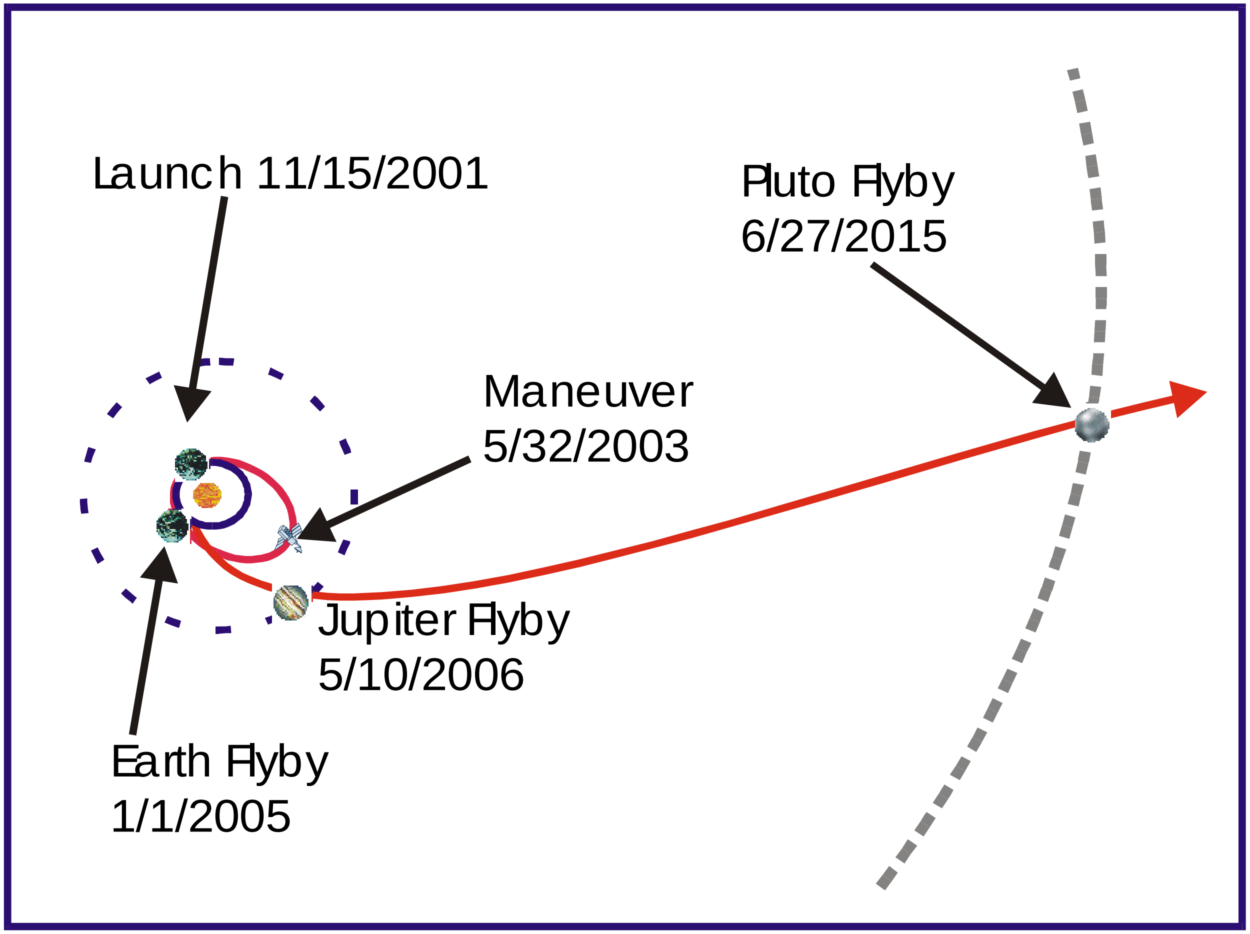
Use of Venus for Trips to Mars



3/1/2017 Earth Departure Unpowered Seingby Trajectory

From Scott, S. A. and Braun, R. D., 1991.

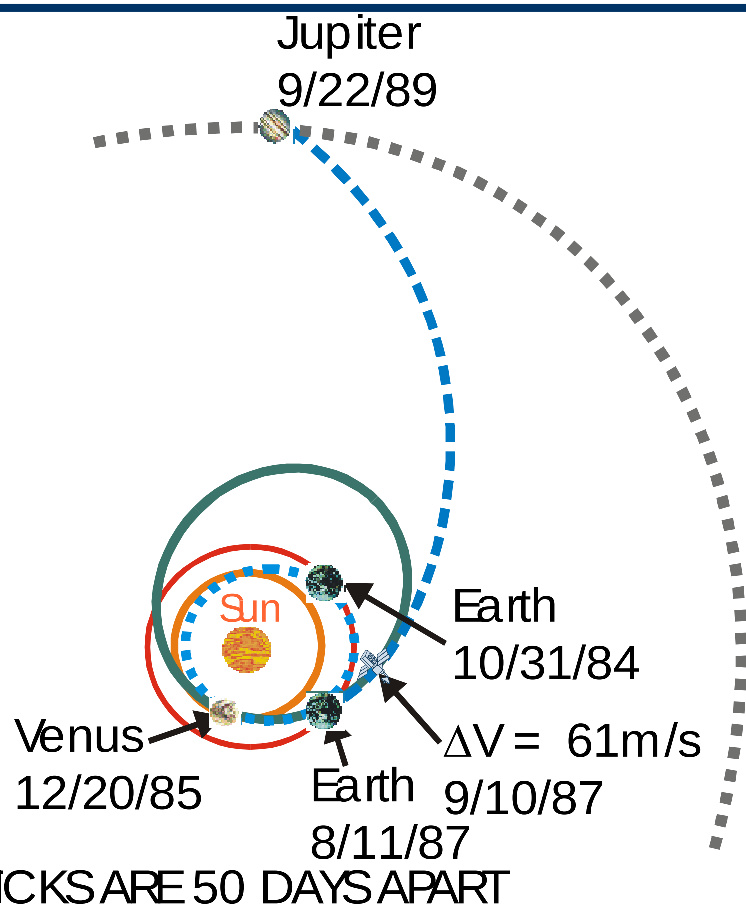
Multiple Swing-By



(Earth , Venus, etc) to reach the Outer Solar System

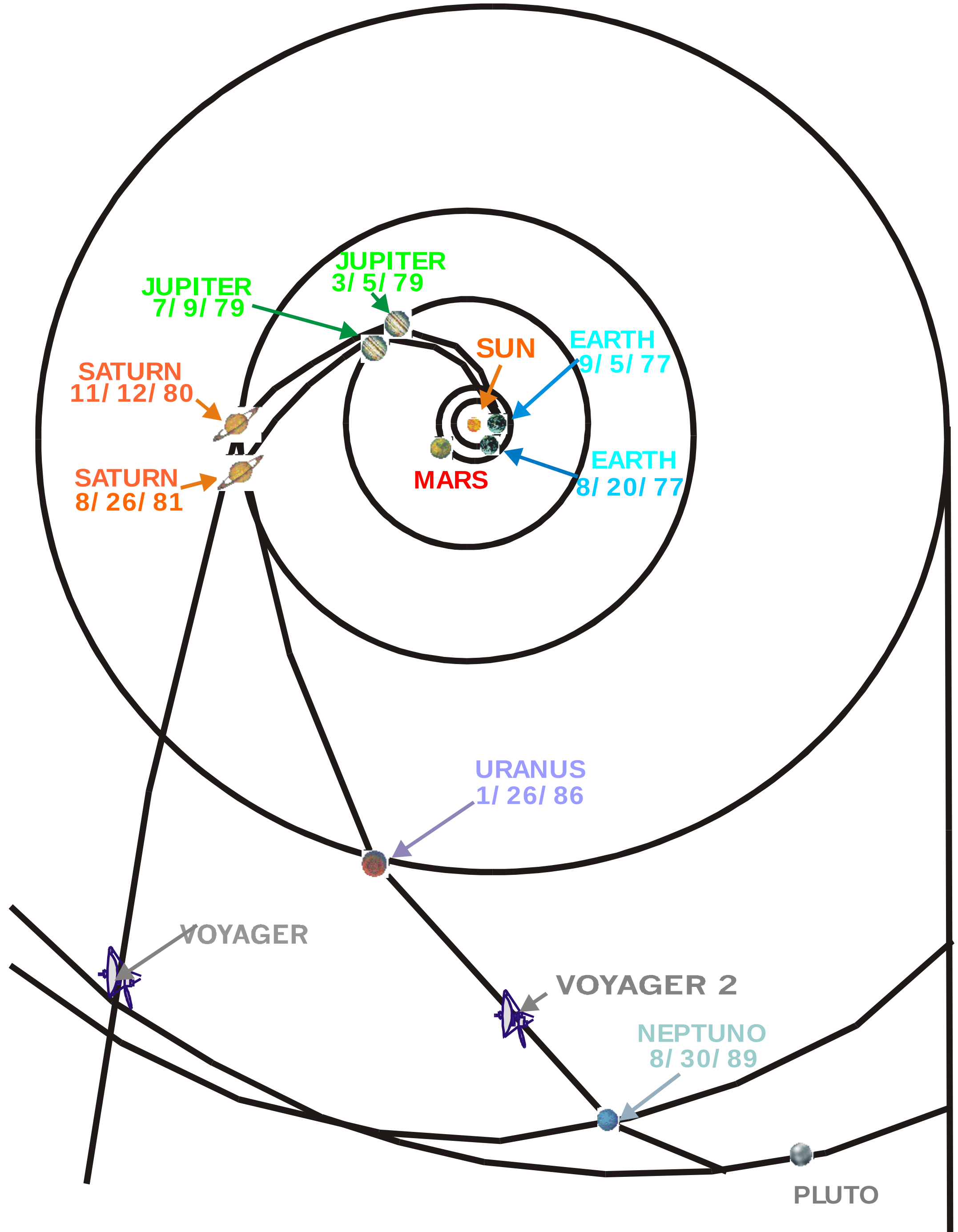
PF350: 3 + ΔVEJGA Trajectory (From Weinstein, 1992).

Tours to the Outer Solar System

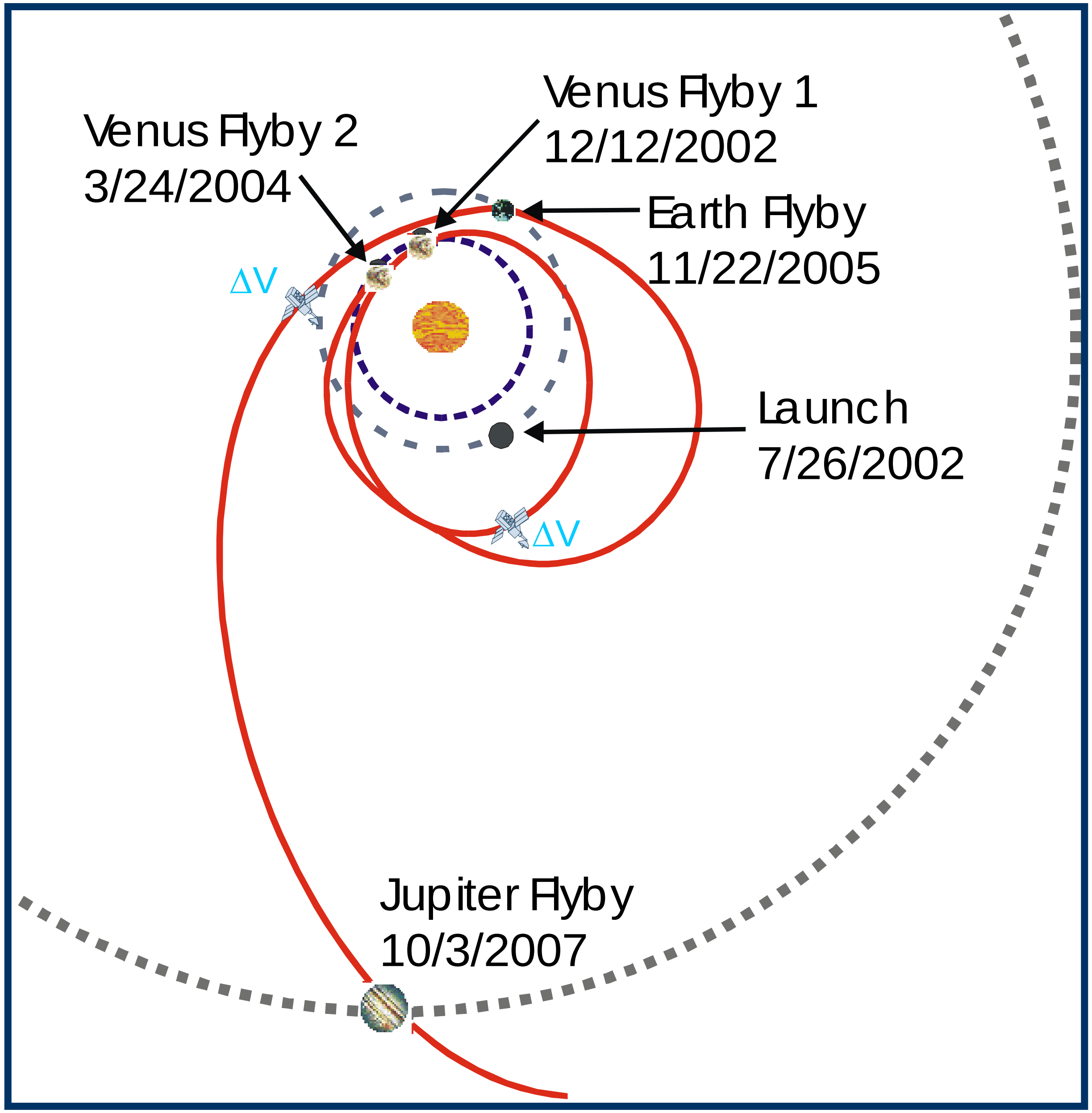


GALILEO 1984 Venus-Earth gravity-assist trajectory

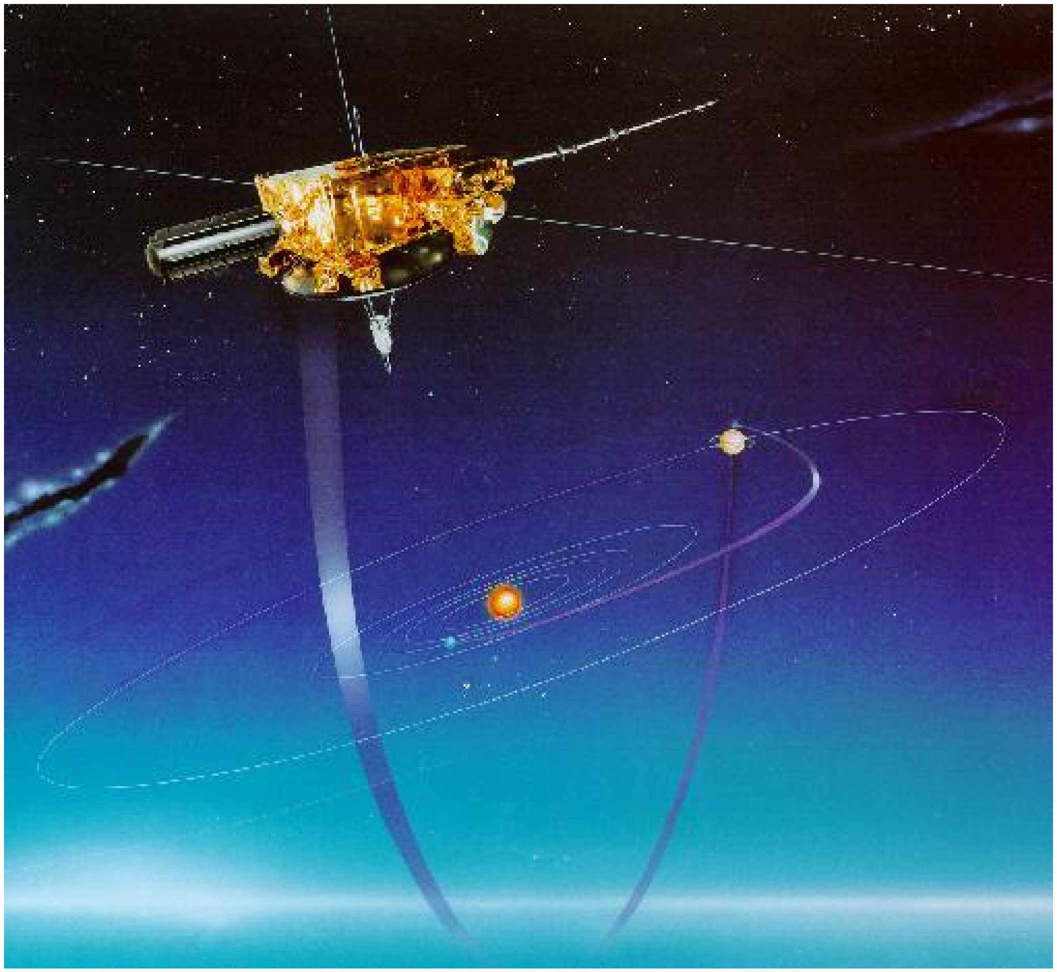
Voyager


JUPITER
3/ 5/ 79
JUPITER
7/ 9/ 79
SATURN
11/ 12/ 80
SATURN
8/ 26/ 81
SUN
EARTH
9/ 5/ 77
EARTH
8/ 20/ 77
MARS
URANUS
1/ 26/ 86
VOYAGER
VOYAGER 2
NEPTUNO
8/ 30/ 89
PLUTO

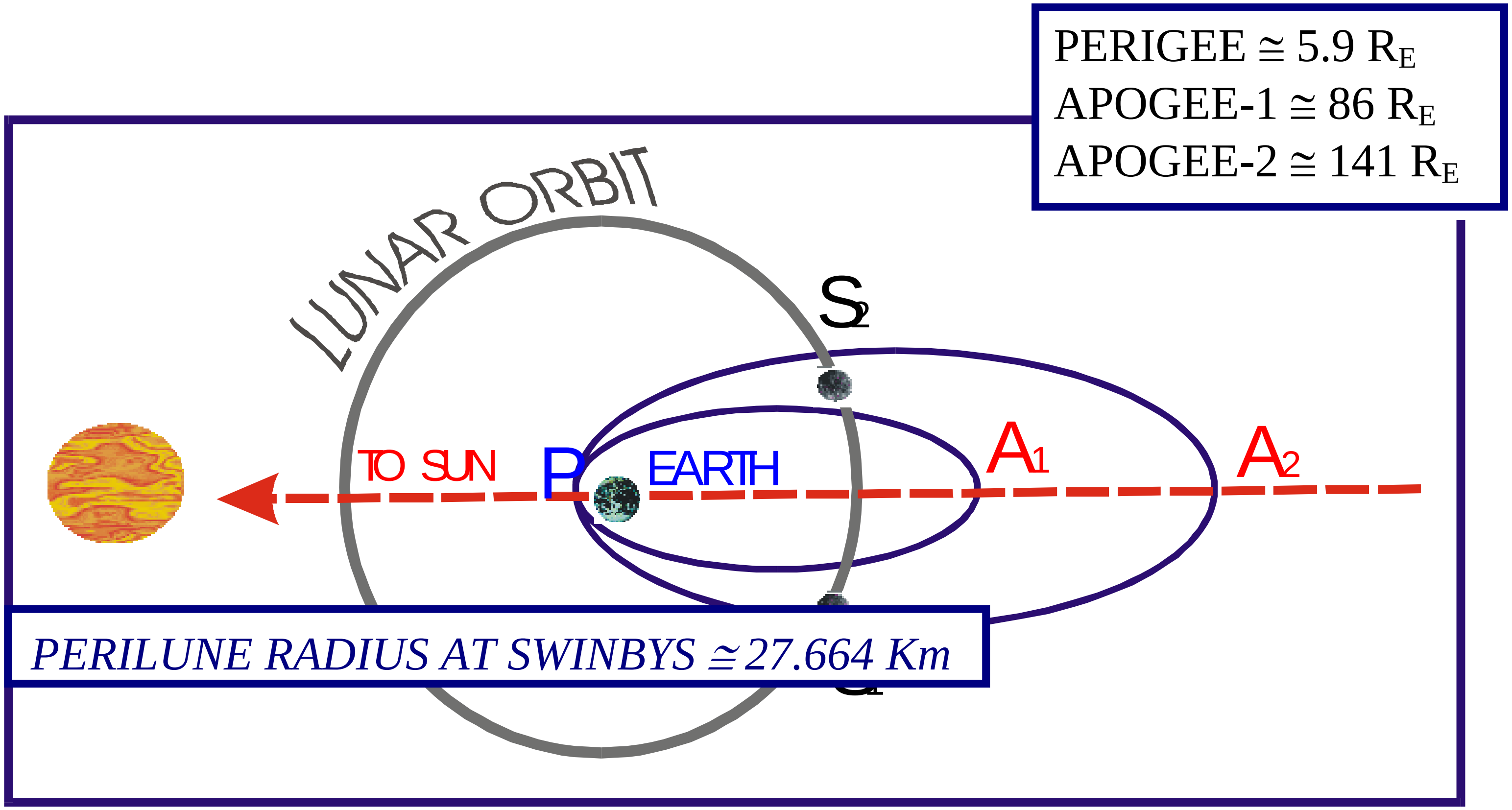
Mission to Jupiter



Plane change (ULYSSES) to leave the ecliptic



Use of the Moon to keep geometry



Sun-Synchronous periodic orbit using double lunar swing-by, ***[1,1,1] class****.*

From Farquhar and Dunham, 1980.

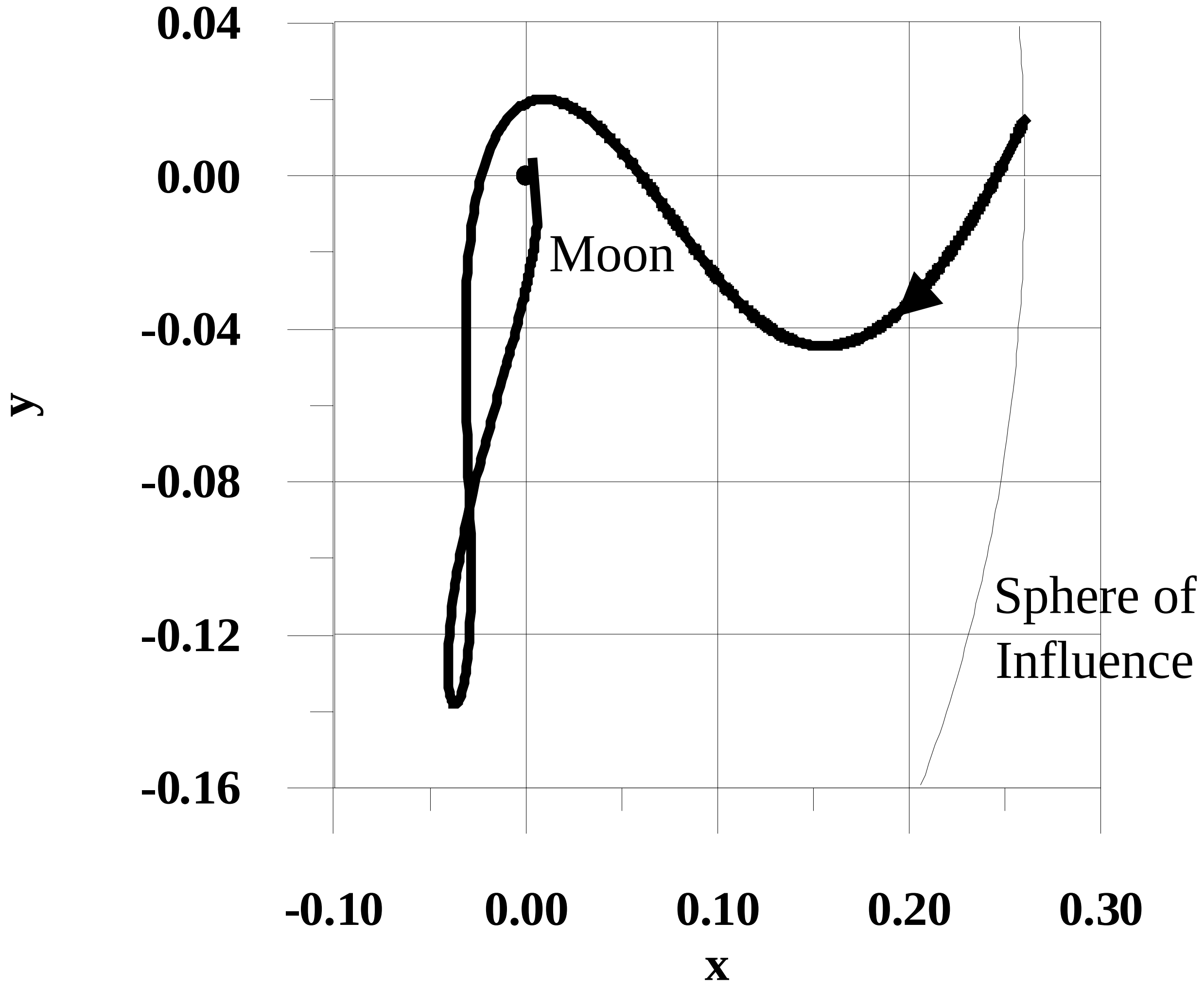
INTRODUCTION

The ballistic gravitational capture is a characteristic of some dynamical systems.

A spacecraft change from a hyperbolic orbit into an elliptic orbit with a small negative energy without the use of any propulsive system.

The force responsible is the gravitational force of the third body involved in the dynamics. So, this force is used as a zero cost control, equivalent to a continuous thrust applied in the spacecraft.

TRAJECTORIES TO THE MOON



MATHEMATICAL MODEL (RPTB)

The canonical system of units and the rotating frame are used.

Equations of motion are:

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y}$$

where Ω is the pseudo-potential given by

$$\Omega = \frac{1}{2}\left(x^2 + y^2\right) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2}$$

The Jacobian constant is:

$$J = \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} + (1-\mu)r_1^2 + \mu r_2^2 - V^2$$

APPROACH TO STUDY THIS PROBLEM

We study the two-body energy of the -Moon:

$$C_3 = V^2 - 2\mu/r$$

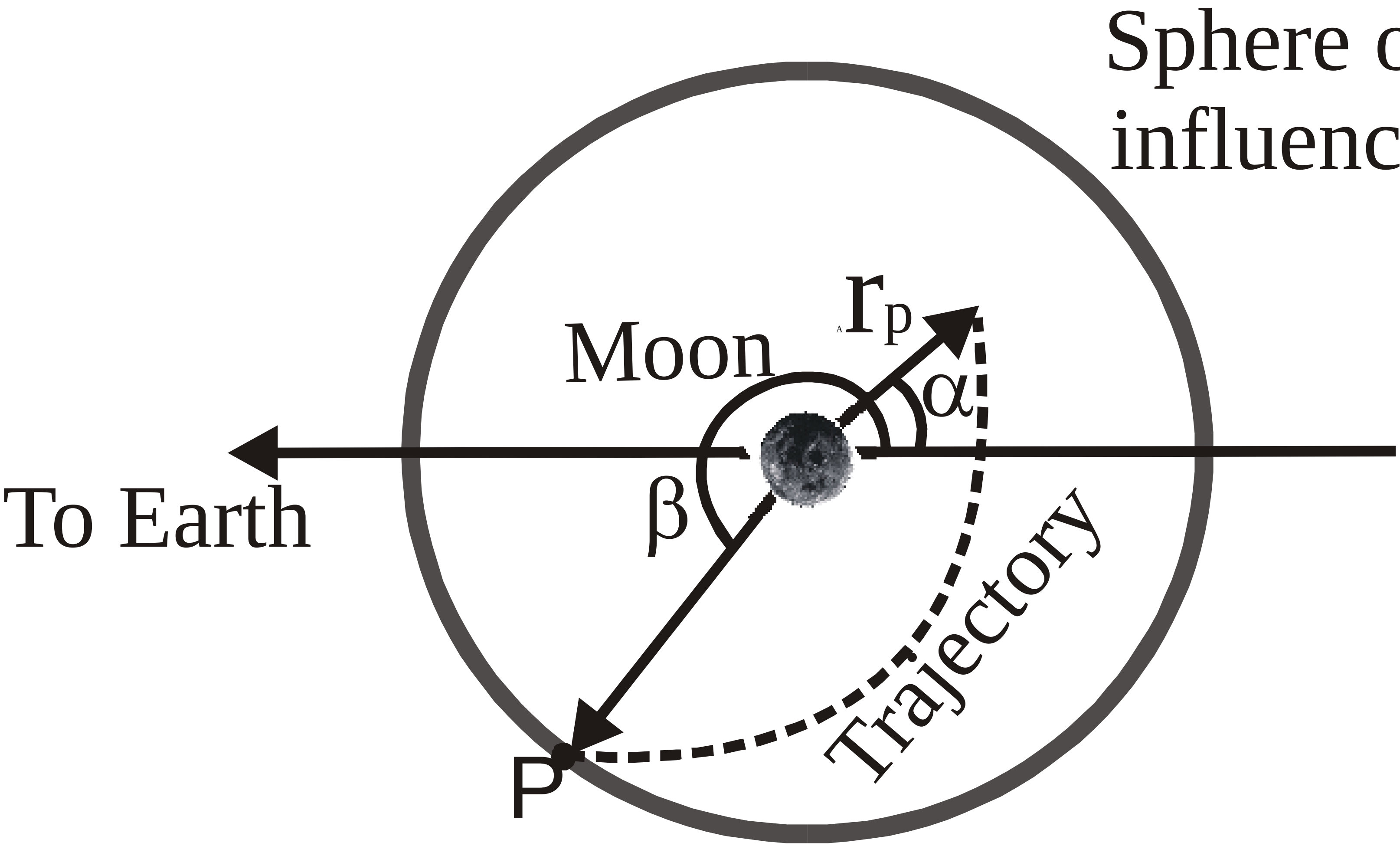
From C_3 we know if the orbit is elliptic ($C_3 < 0$), parabolic ($C_3 = 0$) or hyperbolic ($C_3 > 0$) with respect to the Moon.

For spacecrafts approaching the Moon, it is possible to use the gravitational force of the Earth to lower the value of C_3.

The search for trajectories arriving at the Moon with the maximum possible value for the reduction of C_3 is very important.

Usually, a numerical approach of verifying the values of C_3 is used to identify trajectories. If there is a change of sign in C_3 from negative to positive when leaving the Moon, it means that a ballistic gravitational capture occurs in the positive sense of time.

GRAVITATIONAL CAPTURE



STRATEGY TO FIND TRAJECTORIES

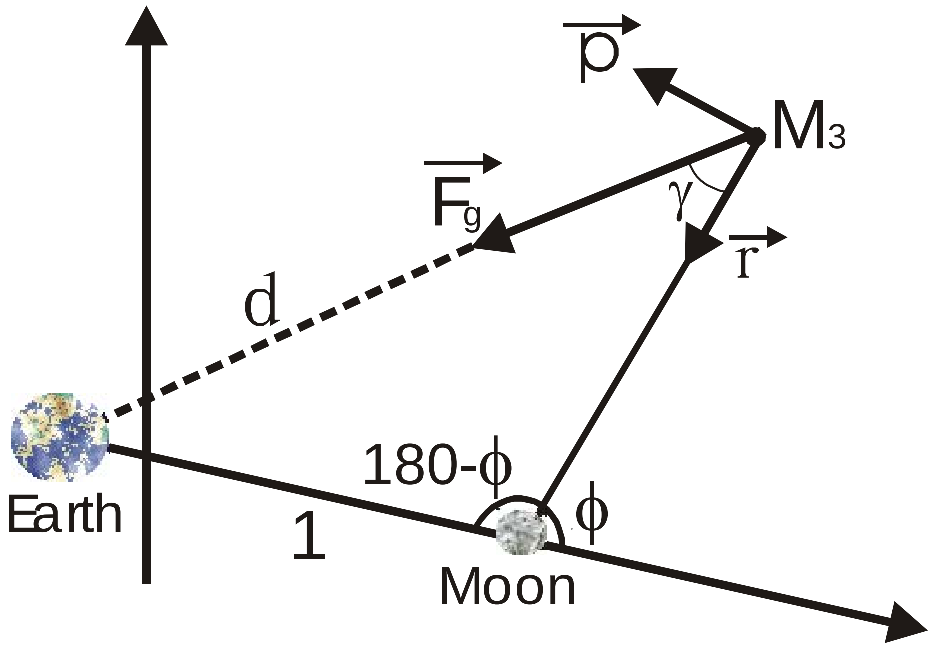
The spacecraft starts its motion close to the Moon and a negative time step is used to determine its motion before the closest approach.

The final conditions were converted into the initial conditions.

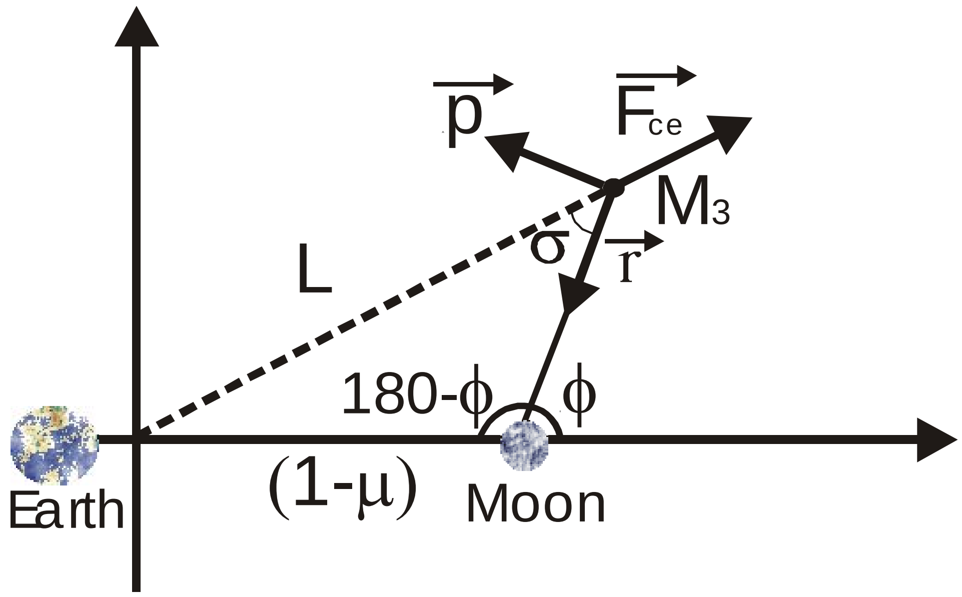
A trajectory is considered a ballistic gravitational capture when the distance from the Moon reaches 100,000 km in a time less than 50 days.

FORCES INVOLVED IN THE DYNAMICS

Gravitational force of the Earth



The centrifugal force

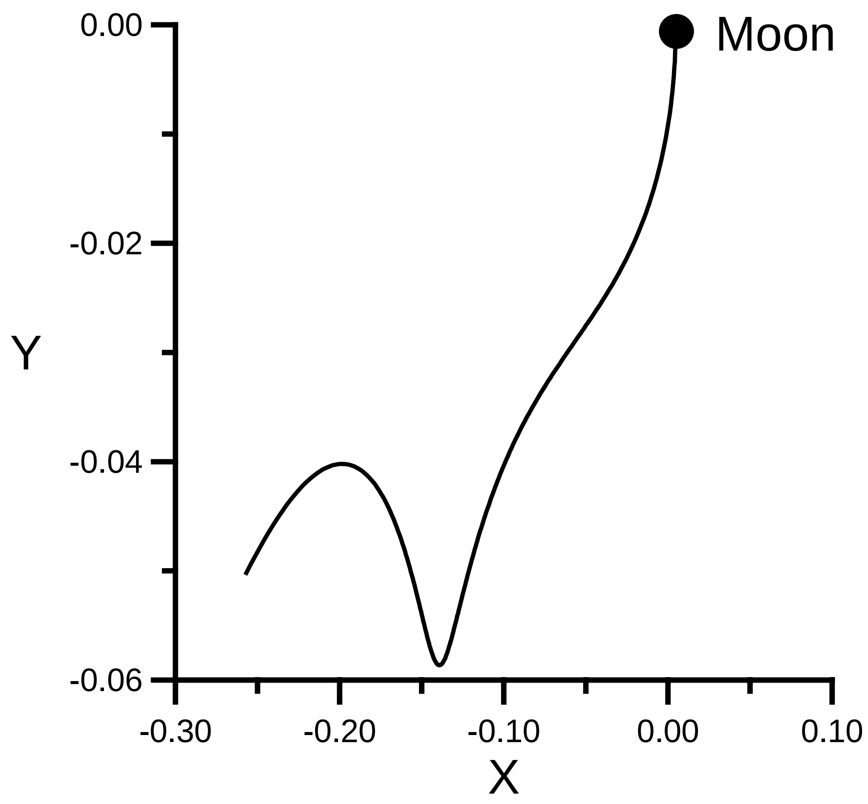


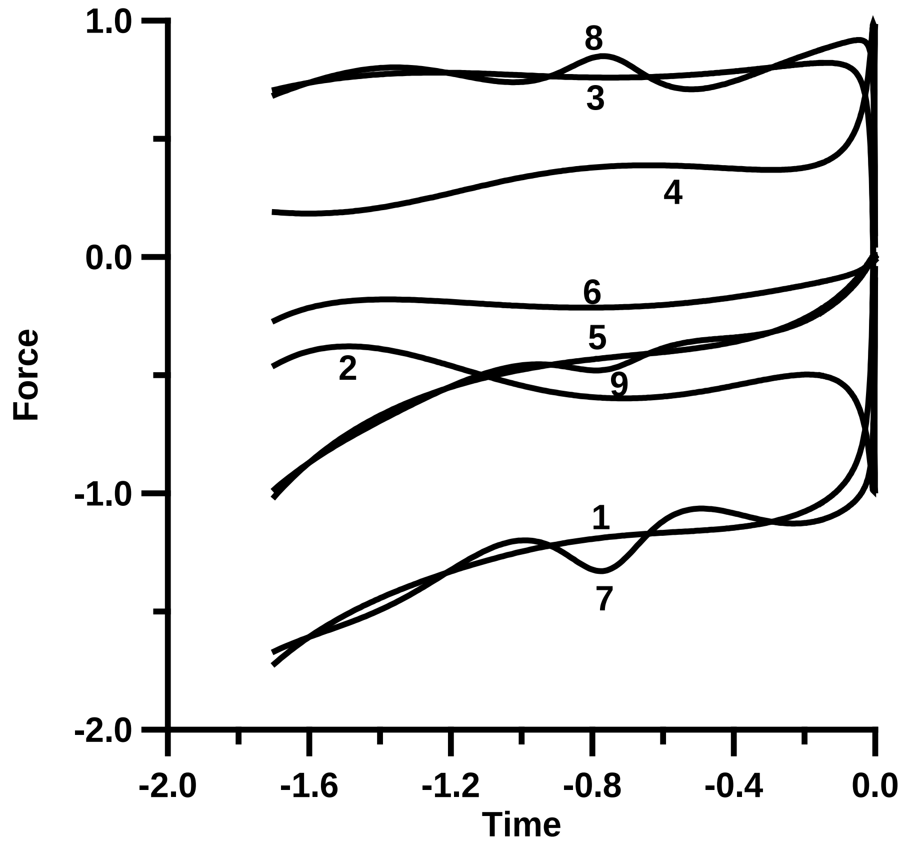
EXAMPLE OF TRAJECTORY

The curves are:

1: Gravitational radial force;
2: Gravitational transversal force;
3: Centripetal radial force;
4: Centripetal transversal force;
5: Resultant radial force;
6: Resultant transversal force;
7: Gravitational force in the direction of motion;
8: Centripetal force in the direction of motion;
9: Resultant force in the direction of motion.

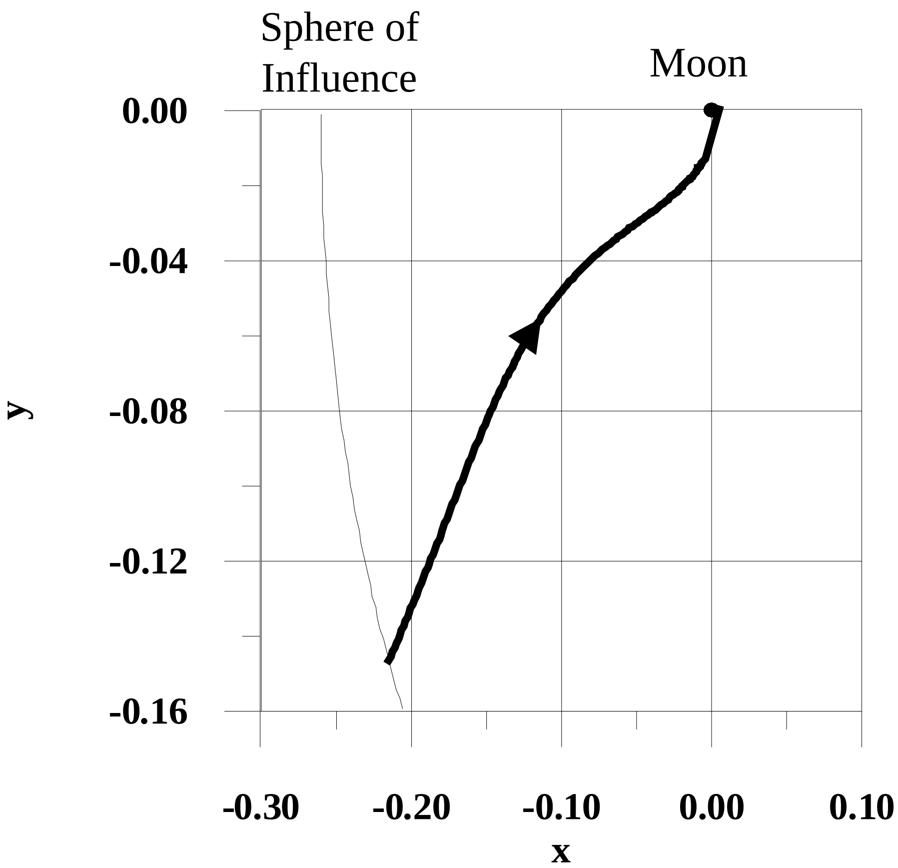
EXAMPLE OF TRAJECTORY

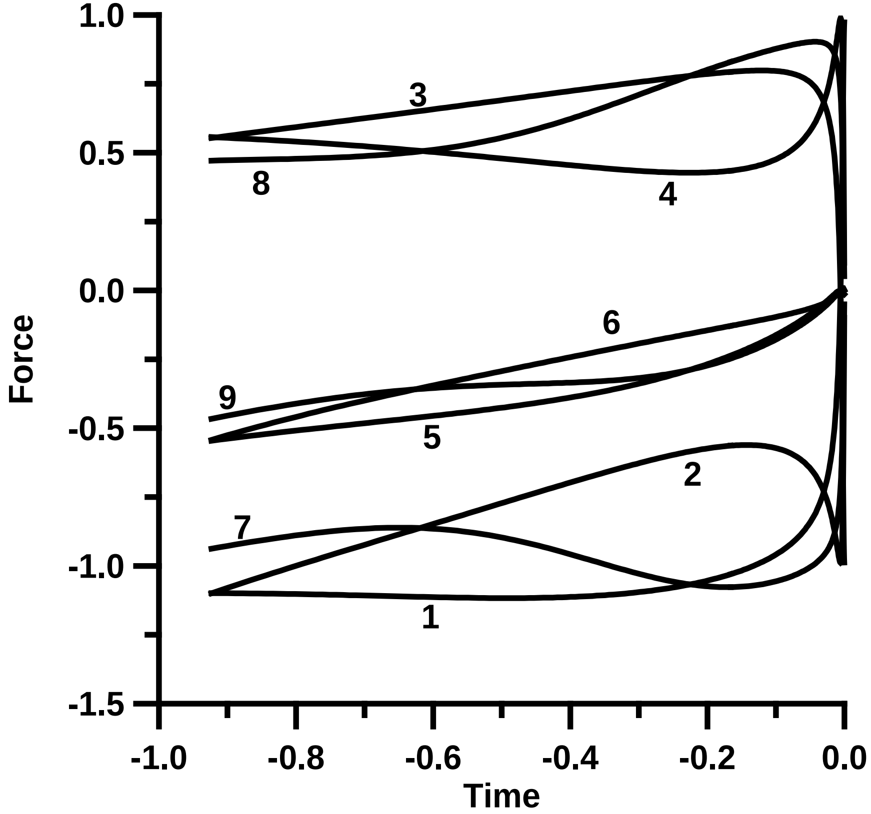




Trajectory with C_3 = -0.2 and $\alpha = 0^o$

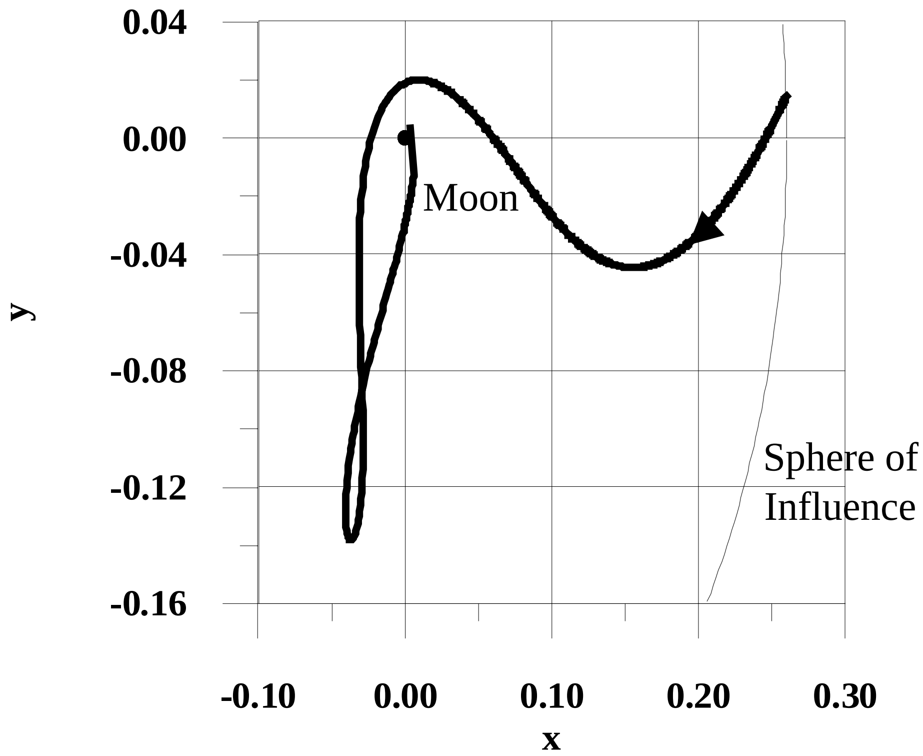
EXAMPLE OF TRAJECTORY

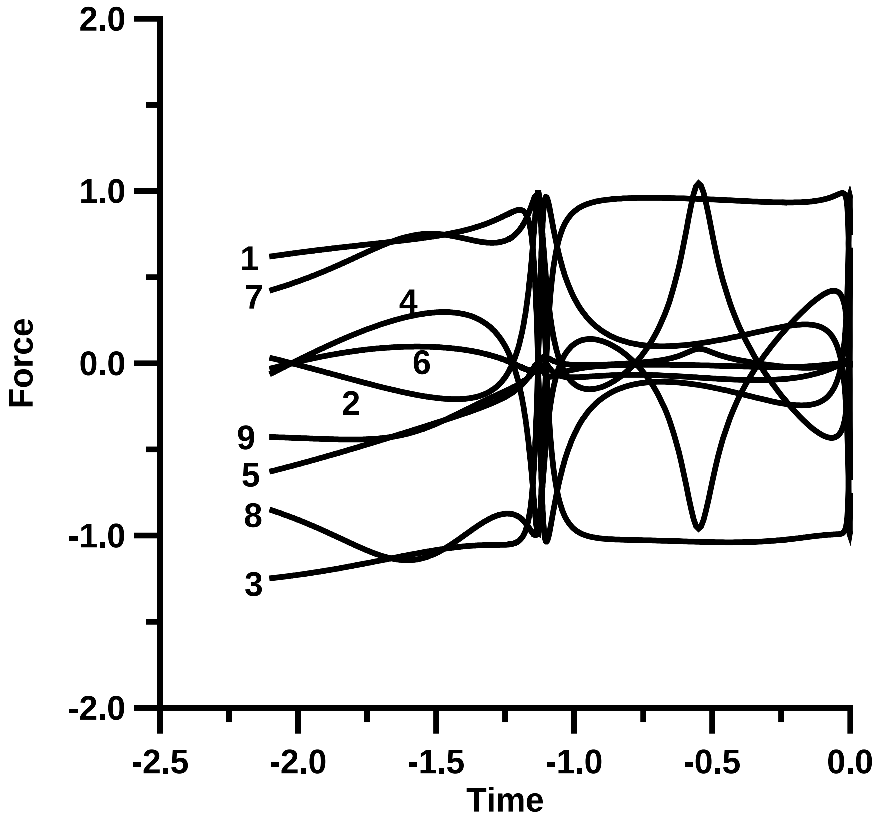




Trajectory with C_3 = -0.15 and $\alpha = 0^o$

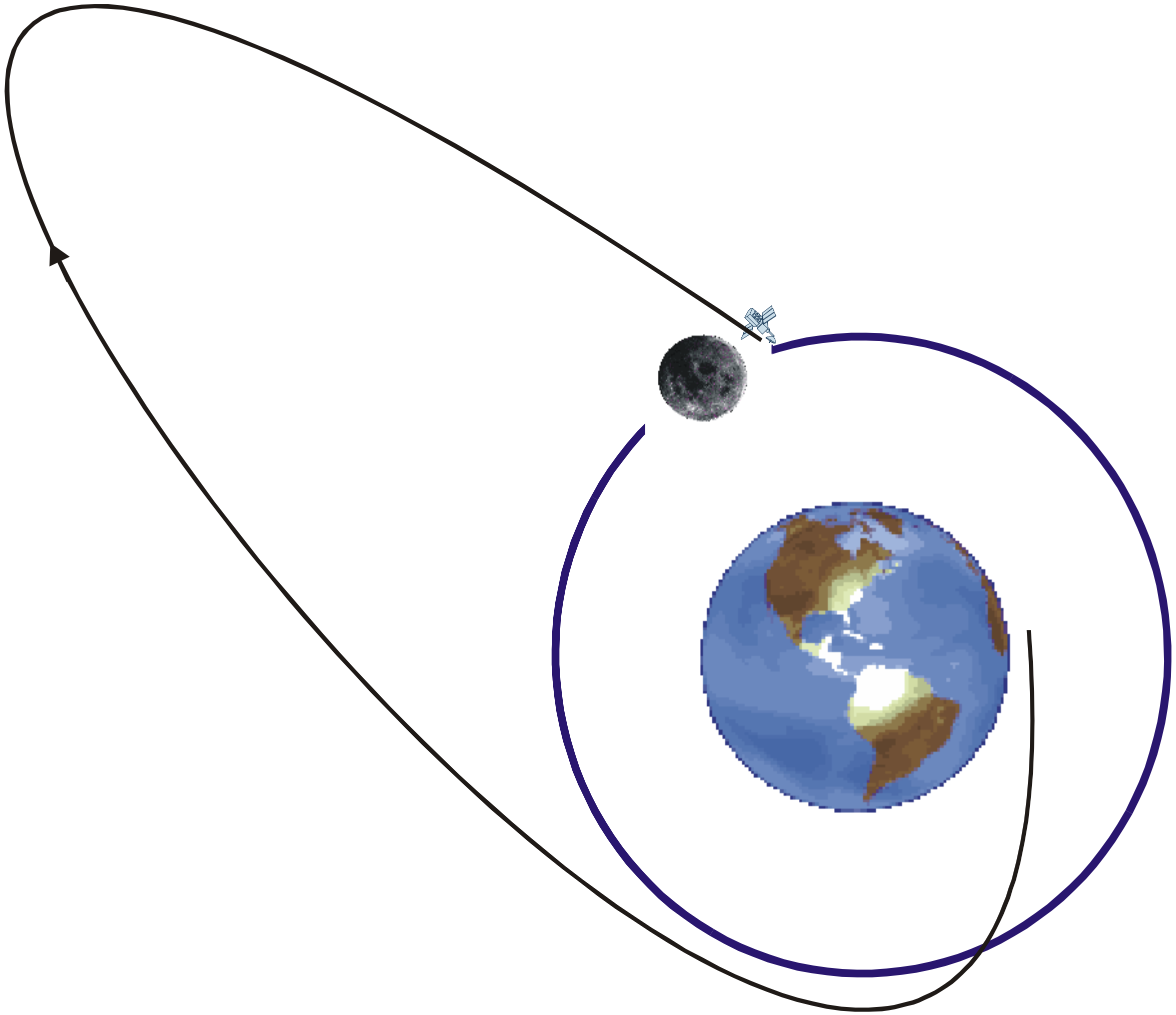
EXAMPLE OF TRAJECTORY



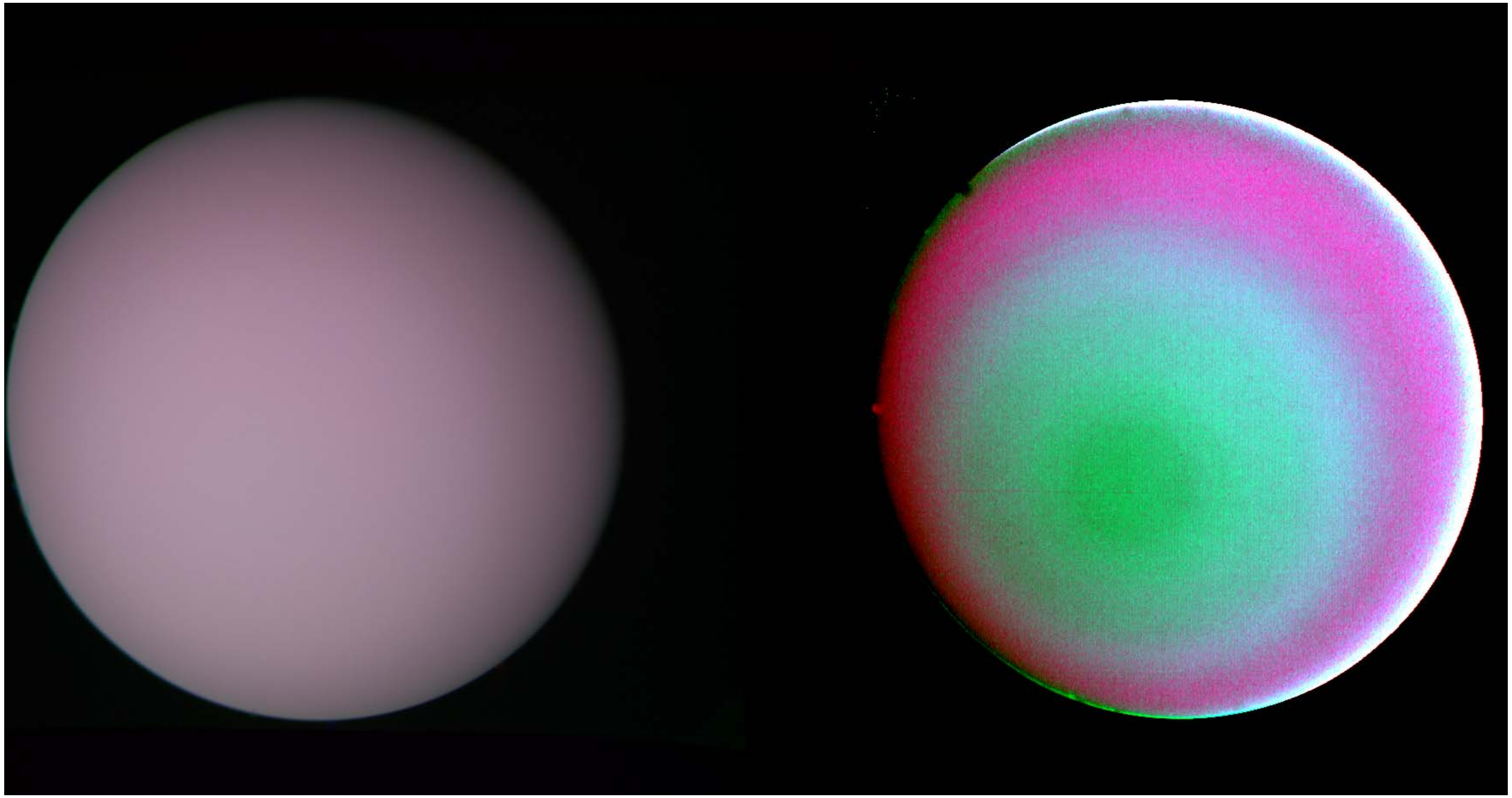


Trajectory with C_3 = -0.2 and $\alpha = 45^o$

EXAMPLE OF TRANSFER

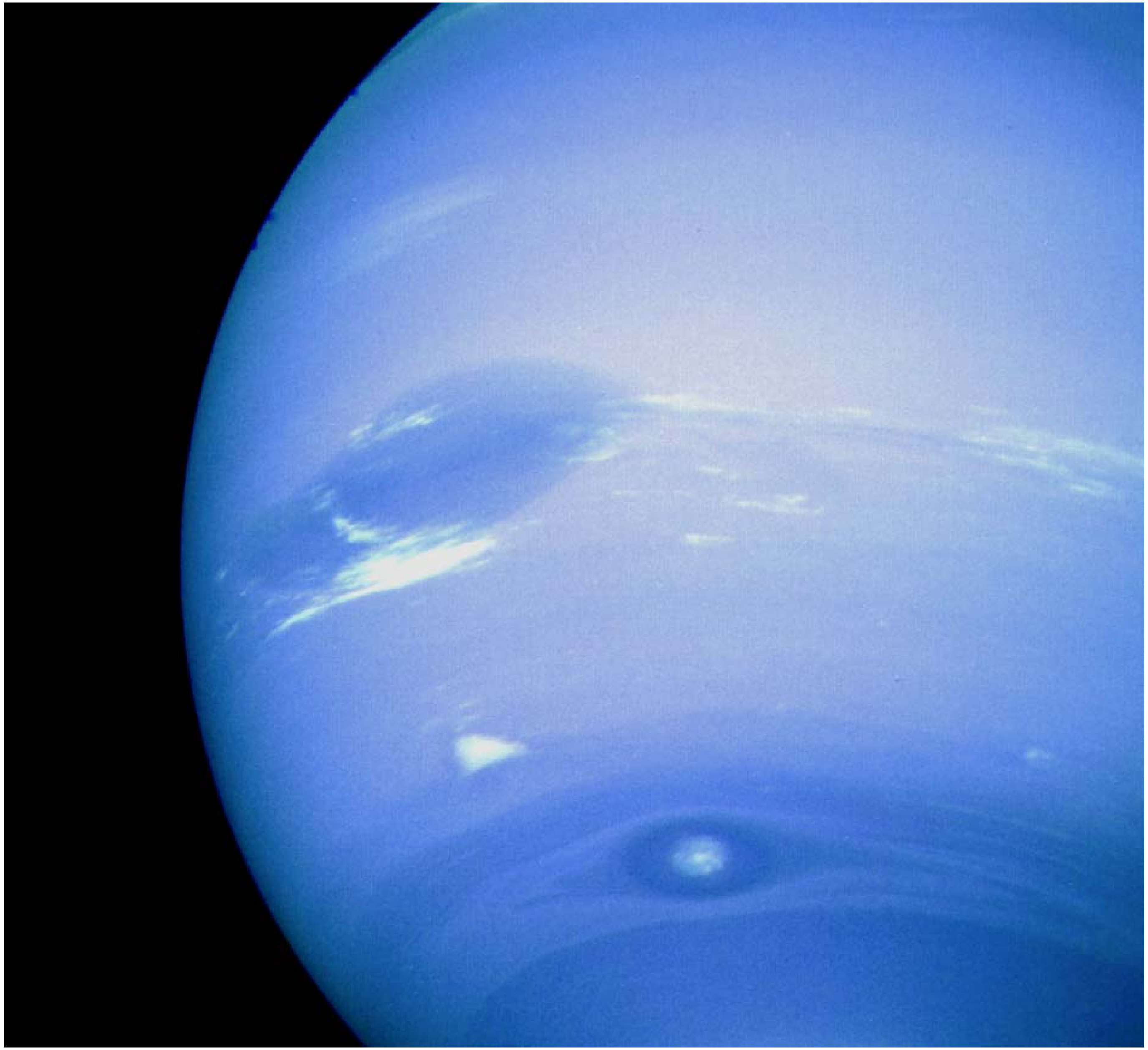


Uranus and Satellites from Voyager 2

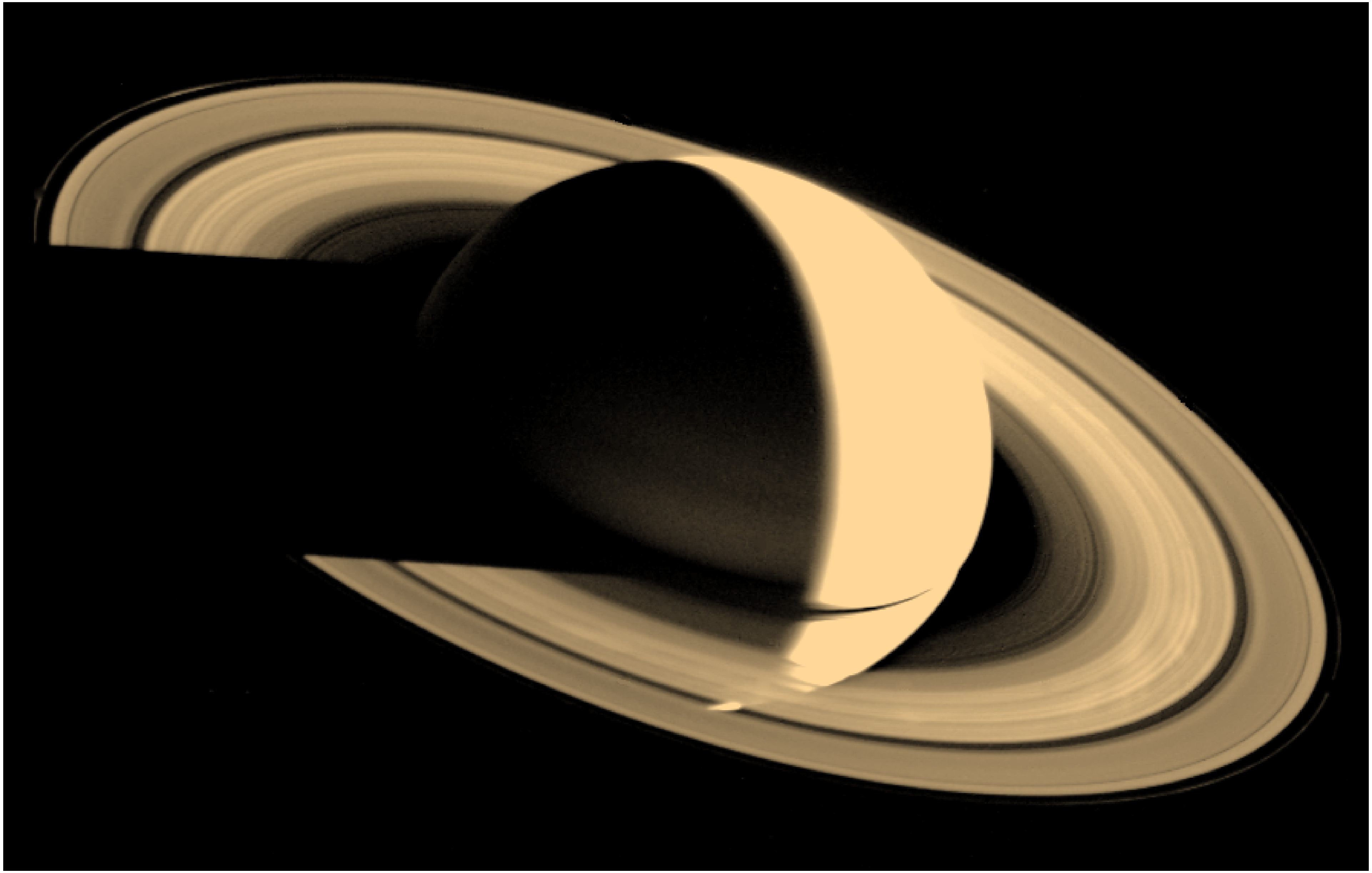


Titan from Voyager

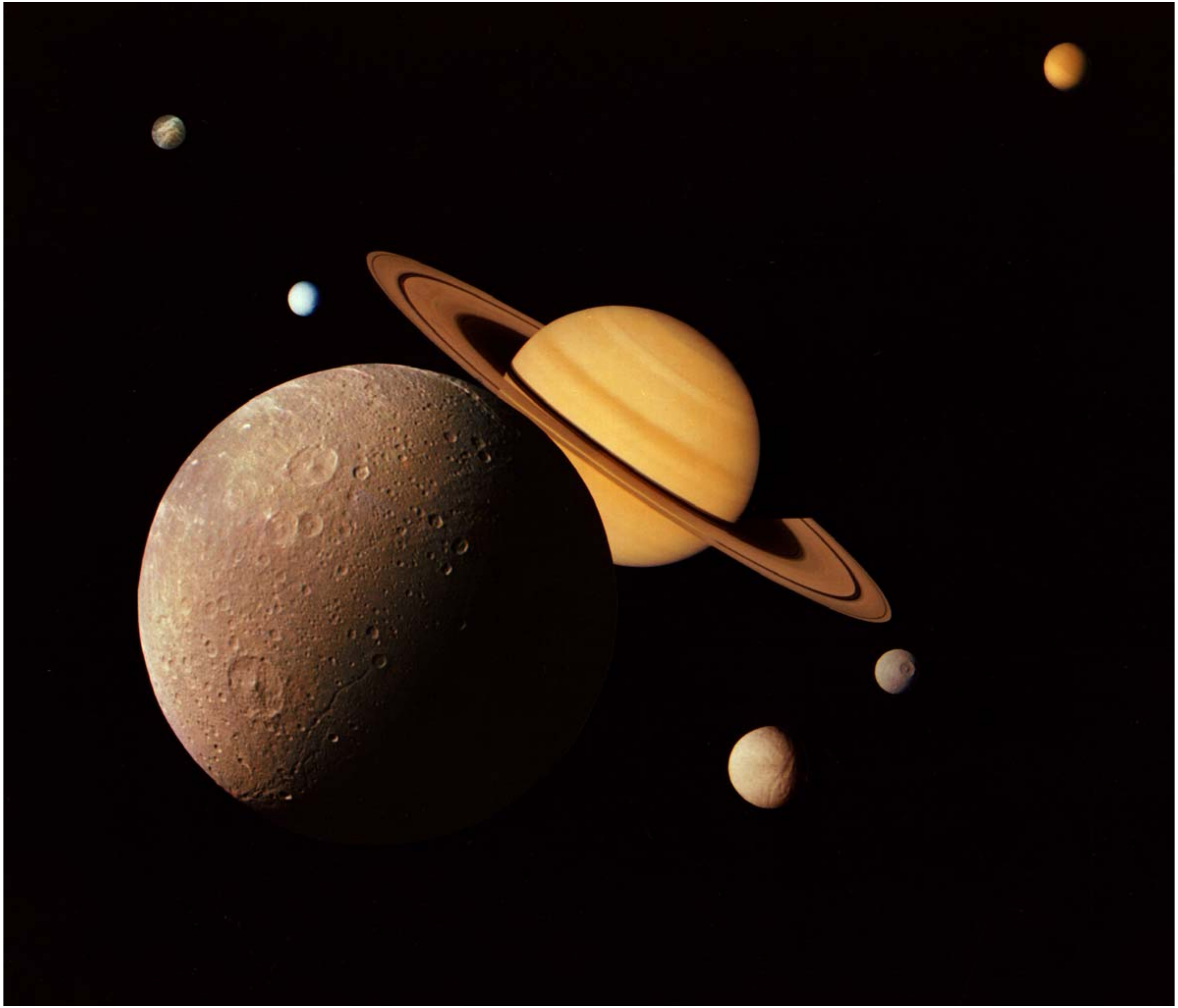
Neptune from Voyager 2

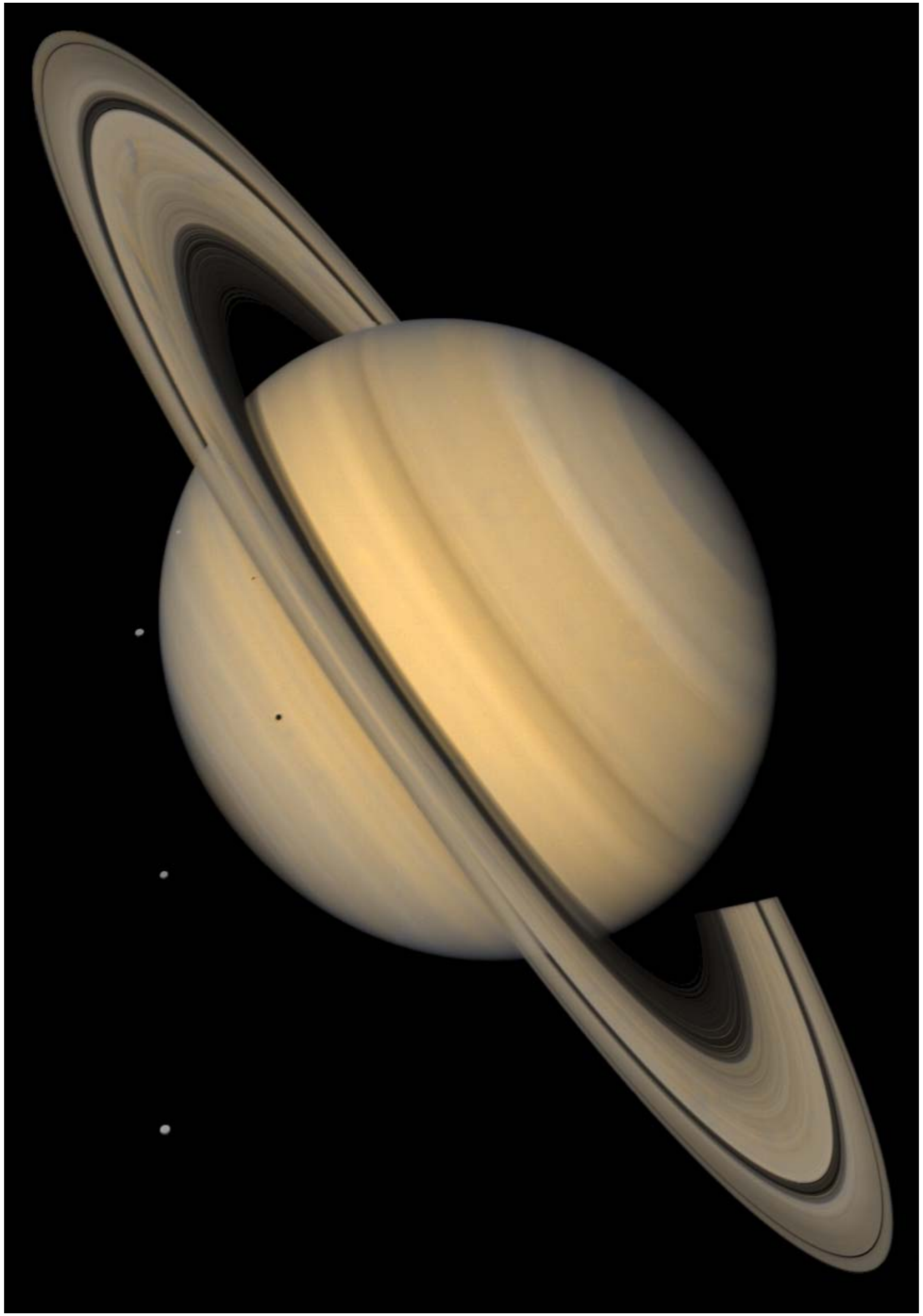


Saturn from Voyager 1



Saturn from Voyager 1

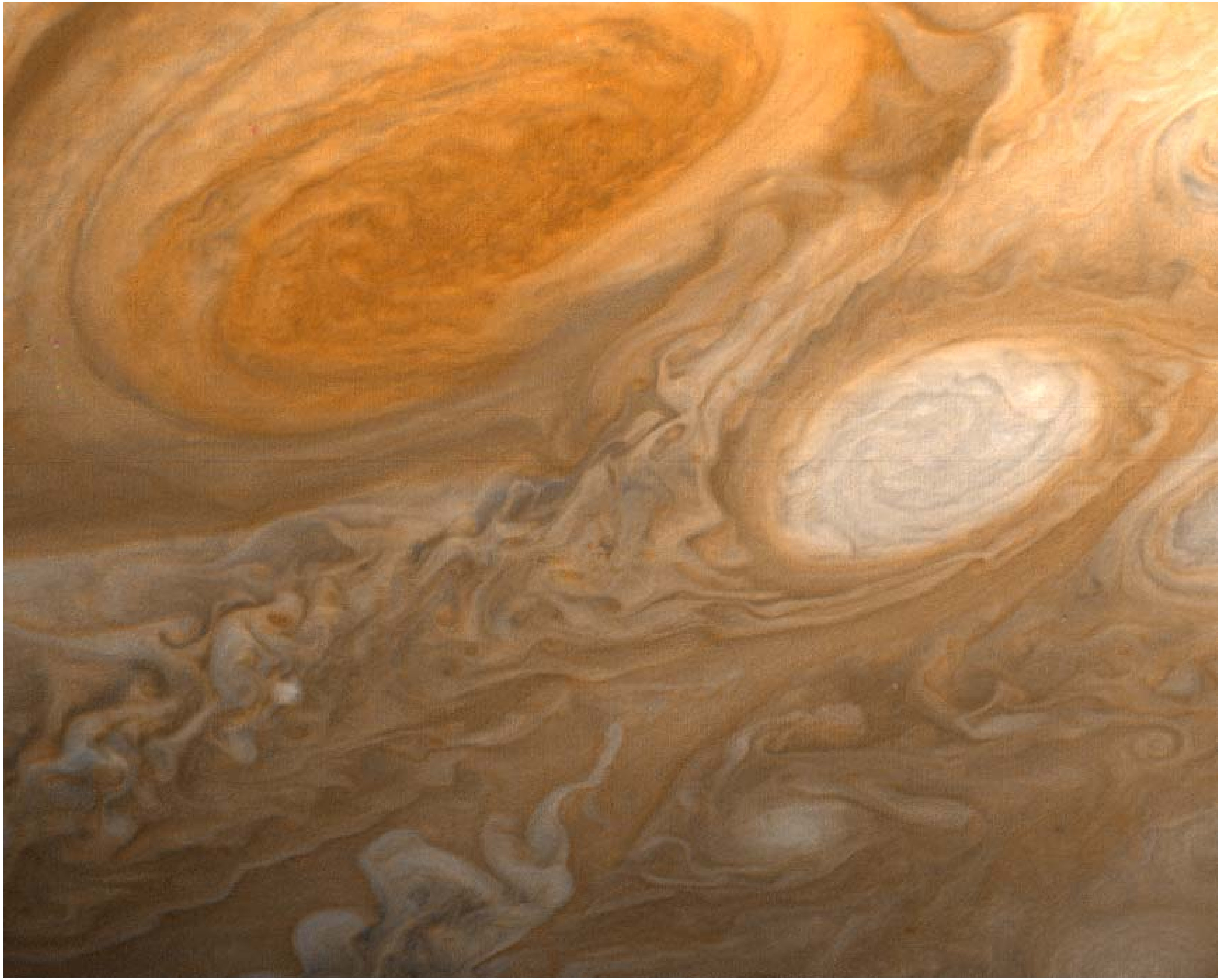




Saturn from Voyager 1

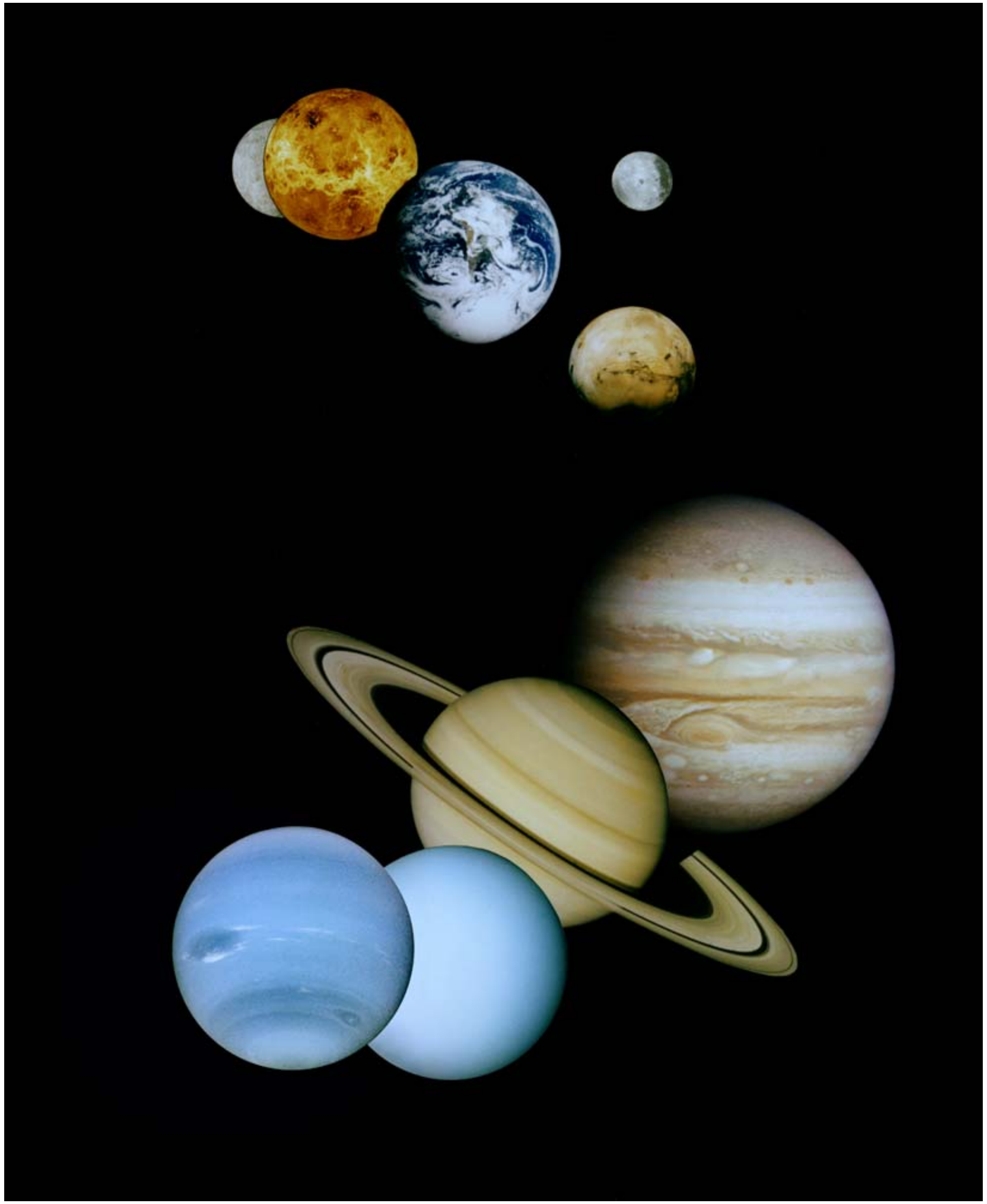
Jupiter from Voyager

Jupiter from Voyager

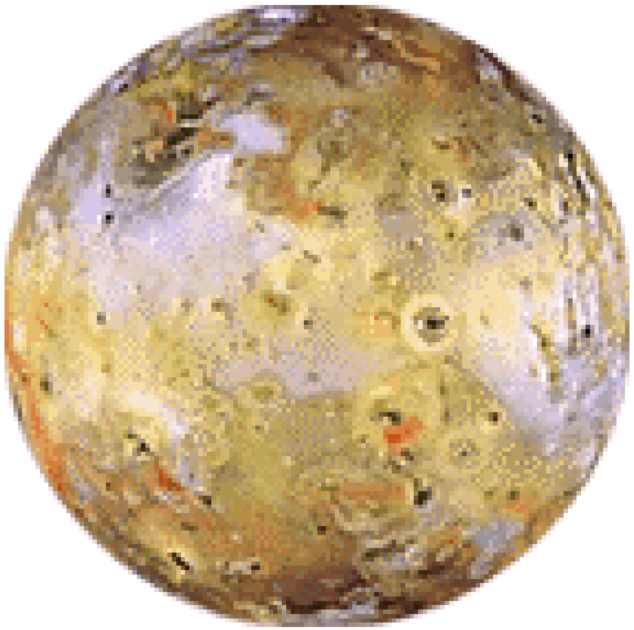


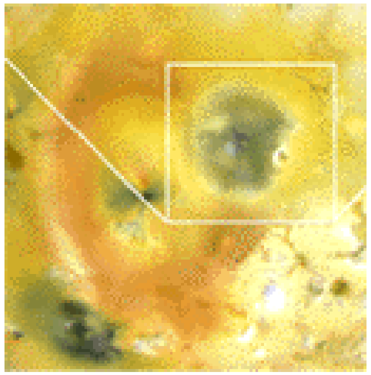
Jupiter and Io

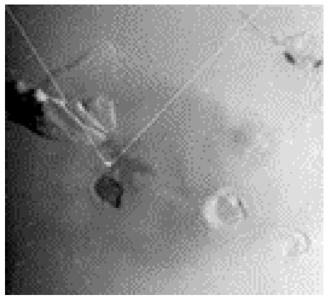
Solar System from Voyager

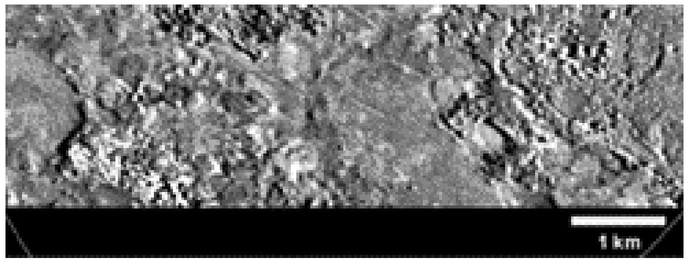


Io



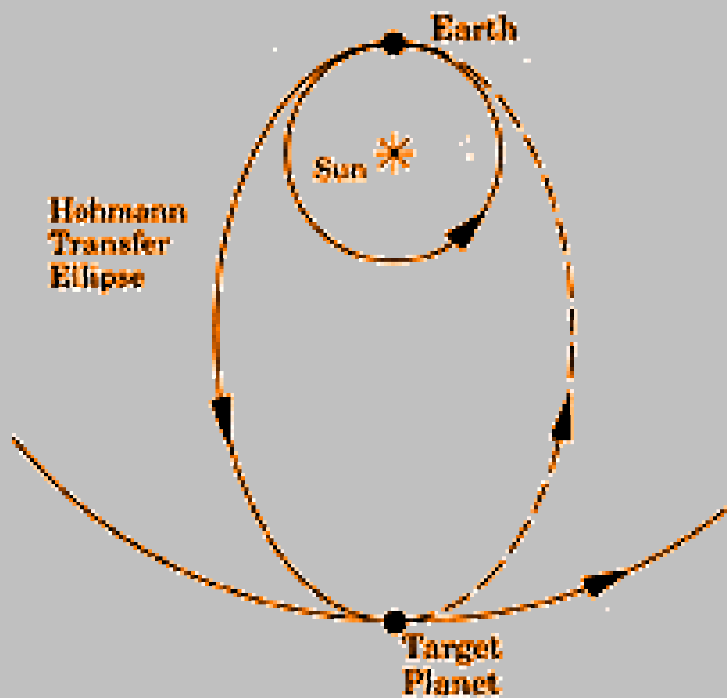


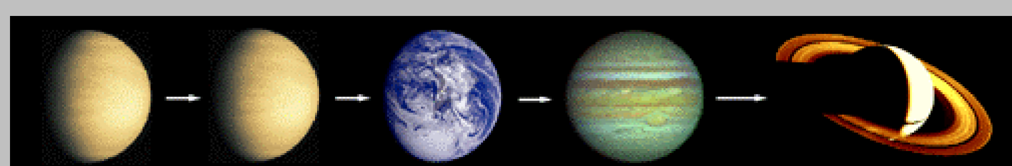


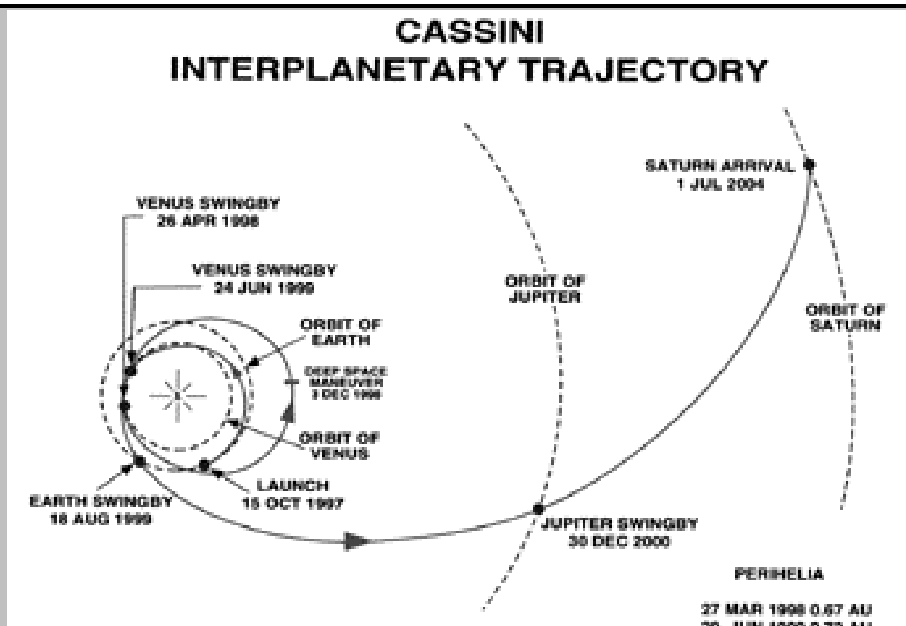


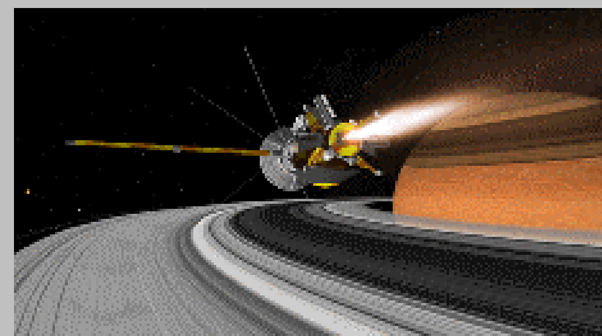
Jupiter

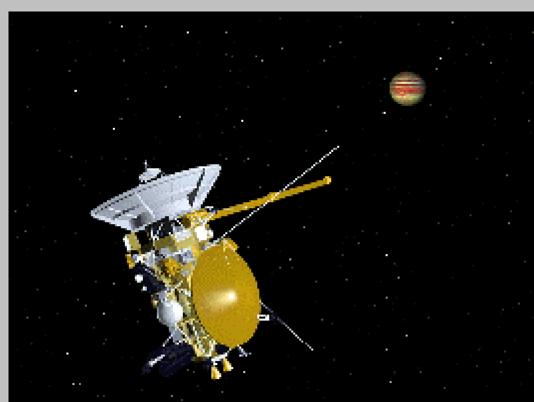
CASSINI MISSION

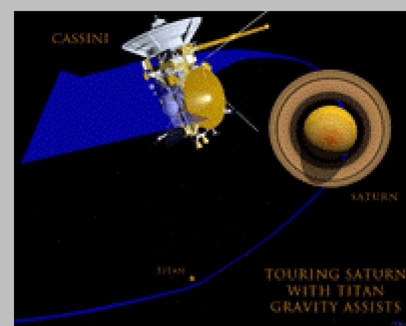





CASSINI
INTERPLANETARY TRAJECTORY
SATURN ARRIVAL
1 JUL 2004
VENUS SWINGBY
26 APR 1998
VENUS SWINGBY
24 JUN 1999
ORBIT OF
JUPITER
ORBIT OF
SATURN
ORBIT OF
EARTH
DEEP SPACE
MANEUVER
3 DEC 1998
ORBIT OF
VENUS
LAUNCH
15 OCT 1997
EARTH SWINGBY
18 AUG 1999
JUPITER SWINGBY
30 DEC 2000
PERIHELIA
27 MAR 1998 0.67 AU






CASSINI
SATURN
TITAN
TOURING SATURN
WITH TITAN
GRAVITY ASSISTS